OH THE SOLVING OF OPERATIONAL EQUATIONS DEPENDING ON A PARAMETER

Abstract

Our purpose in this paper is to solve the equation

$$P(x, \mu) = 0 \tag{1}$$

by interative methods

$$x_{n+1} = x_n - \Gamma_\mu^n P(x_n, \mu)$$

$$x_{n+1} = x_n - \Gamma_\mu^0 P(x_n, \mu)$$

where

$$\Gamma_\mu^n = [P'(x_n, \mu)]^{-1}$$

without using the majorant principle.

The operator $P(x, \mu)$ is defined in the product space $X \times M$ and has its values in X, where X is a Banach space and M is normed linear space. We suppose that $P(x, \mu)$ is continuous in x, admits Fréchet derivatives until the second order, $P'(x, \mu)$, $P''(x, \mu)$ in x and also partial Fréchet derivatives $\frac{\partial}{\partial\mu} P(x, \mu)$, $\frac{\partial}{\partial\mu} P'(x, \mu)$, $\frac{\partial}{\partial\mu} P''(x, \mu)$.

The paper contains several theorems concerning the existence of the solution $x^*(\mu) = \lim x_n(\mu)$ of the equation (1), where the sequence $x_n(\mu)$ is obtained by the above- mentioned methods. We give also theorems on unicity of the solution and some remarks.