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TECHNIQUE OF THE FIXED POINT STRUCTURES

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The notion" fixed point structure"(see [4] and [5])is a generalization of some notions as "topological space with fixed point property"(Brouwer, Schauder, Tychonov,...),"ordered set with fixed point property"(Knaster, Tarski, Bourbaki-Birkhoff,...),"mapping with fixed point property on family of sets"(Jones, de Blasi,...),"object with fixed point property"(Lawvere, Lambek, Rus,...). In the paper [6]-[9] we use the technique of the fixed point structures to give some new fixed point theorems. Thus we obtain the following general results (we follow terminologies and notations in [5]):

THEOREM 1. Let (X,S,M) be a fixed point structures and (θ,η) ($\theta: Z \to R_+$, $\eta: P(X) \to P(X)$) a compatible pair with (X,S,M). We suppose that :

(i) $\theta|_{\eta(Z)}$ has the intersection property,

(ii) f is a (θ,φ)-contraction.

Then

(a) f has at least a fixed point,

(b) if $F_f \in Z$, then $\theta(F_f) = 0$.

THEOREM 2. Let (X,S,M) be a fixed point structures and (θ,η) a compatible pair with (X,S,M) . Let $Y \in \eta(Z)$ and $f \in M(Y)$. We suppose that:

(i) $A \in Z$, $x \in X$ imply $A \cup \{x\} \in Z$ and $\theta(A \cup \{x\}) = \theta(A)$,

(ii) f is θ-condensing.

Then

(a) f has at least a fixed point,

(b) if $F_f \in Z$, then $\theta(F_f) = 0$.

THEOREM 3. Let (X,S,M) be a fixed point structures and (θ,η) a compatible pair with (X,S,M). Let $Y \in \eta(Z)$, $f : Y \to X$ a mapping and $\rho : X \to Y$ a retraction. We suppose that :

(i) $\theta|_{\eta(Z)}$ is a mapping with the intersection property,

(ii) f is a strong (θ,φ)-contraction,

(iii) f is retractible onto Y by ρ and $\rho \circ f \in M(Y)$,

(iv) ρ is (θ,a)-Lipschitz ($a \in \mathbb{R}_+$),

(v) the function $a\varphi$ is a comparison function.

Then

(a) f has at least a fixed point,

(b) if $F_f \in Z$, then $\theta(F_f) = 0$.

THEOREM 4. Let (X,S,M) be a fixed point structures and (θ,η) a compatible pair with (X,S,M). Let $Y \in \eta(Z)$, $f : Y \to X$ a mapping and $\rho : X \to Y$ a retraction. We suppose that :

(i) $A \in Z$, $x \in X$ imply $A \cup \{x\} \in Z$ and $\theta(A \cup \{x\}) = \theta(A)$,

(ii) f is a strong θ-~~contraction~~ condensing mapping ,

(iii) f is retractible onto Y by ρ and $\rho \circ f \in M(Y)$,

(iv) ρ is a strong $(\theta,1)$-contraction.

Then

(a) f has at least a fixed point,

(b) if $F_f \in Z$, then $\theta(F_f) = 0$.

The object of the present paper is to give some applictions of these general results.

Example 1. Let (X,d) be a complete metric space, $S = P_{cp}(X)$, $M(Y) = \{ f: Y \to Y \mid d(f(x),f(y)) < d(x,y)$, for all $x,y \in X, x \neq y\}$, $\eta(A) = \overline{A}$ and $\theta = \alpha_{H}$. Then from the Theorem 2 we have

THEOREM 1.1. Let (X,d) be a bounded complete metric space and $f: X \to X$ a mapping. We suppose that

(i) f is α_{DP} -condensing,

(ii) $d(f(x),f(y)) < d(x,y)$, for all $x,y \in X, x \neq y$.

Then $F_f = \{x\}$.

Example 2. Let (X,d) be a complete metric space ,$S = P_{cp}(X)$, $M(Y) = \{f \in C(Y,Y) \mid$ f has at most a fixed point and f is asymptotically regular$\}$; $\eta(A) = \overline{A}$ and $\theta = \alpha_{DP}$. In this case ,from the Theorem 2 we have

THEOREM 2.2.Let (X,d) be a bounded complete metric space and $f : X \to X$ a continuous α_{DP} -condensing mapping.We suppose that

(i) f has at most a fixed point,

(ii) f is asymptotically regular.

Then $F_f = \{x\}$.

Example 3. Let X be Banach space $S = P_{cp,cv}(X)$, $M(Y) = C(Y,Y)$, $\eta(Y) = \overline{co}\, Y$,$\theta = \alpha_K$ and ρ , the radial retraction.In this case from the Theorem 3 we have

THEOREM 3.1.Let X be a Banach space and $f: \overline{B}(0;R) \to X$ a continuous mapping. We suppose that

(i) f is a strong (α_K, φ)-contraction,

(ii) f is retractible onto $\overline{B}(0;R)$ by radial retaction.

Then f has at least a fixed point.

Example 4.Let X be a Hilbert space , $S = P_{b,cl,cv}(X)$, $M(Y) = \{f : Y \to Y \mid f \text{ is nonexpansive}\}$, $\eta(A) = \overline{A}$, $\theta = \beta_{KL}$. In this case fromthe Theorem 1 we have

THEOREM 1.1.Let X be a Hilbert space , $Y \in P_{b,cl}(X)$ and $f : X \to X$. We suppose that

(i) f is a nonexpansiv mapping,

(ii) f is a (β_{KL}, φ)-cotraction.

Then $F_f = \{x\}$.

For other applictions of the theorems 1-4 see [5] - [9] .

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