

THE STARLIKENESS OF A PARTICULAR CLASS OF INTEGRAL OPERATORS

Valeria SELINGER

Let A be the class of all analytic functions f in the unit disc $U = \{z \in \mathbb{C} / |z| < 1\}$, with $f(0) = 0$ and $f'(0) = 1$.

A function $f \in A$ is said to be starlike if $\operatorname{Re} [zf'(z)/f(z)] > 0$ in U . We denote the class of starlike functions by S^* .

Let $g \in A$ with $g(z)/z \neq 0$ in U and let c be complex number with $\operatorname{Re} c > 0$. Consider the integral operators J_g^c on A defined by $F = J_g^c(f)$, where

$$(1) \quad F(z) = \frac{c+1}{[g(z)]^c} \int_0^z f(t) \frac{[g(t)]^c}{t} dt, \quad z \in U, f \in A.$$

Making the substitution $w=uz$, (1) can be rewritten:

$$F(z) = (c+1) \left[\frac{g}{g(z)} \right]^c \int_0^1 f(uz) \left[\frac{g(uz)}{uz} \right]^c u^{c-1} du,$$

where all powers are the principale ones. This shows that the integral operators J_g^c is well defined.

In this article we employ the method of admissible functions due to P.T.Moceanu and S.S.Miller [1], [2], [4], to obtain sufficient conditions on g , so that $J_g^c(S^*) \subset S^*$.