Bul.St.Univ.Baia Mare
Seria B, Matematică-Informatică, Vol.VII, Nr.1-2, 115-118

Q - OPTIMAL FORMULAS OF QUADRATURE WITH WEIGHT FUNCTION OF THE LAGUERRE TYPE

Dumitru ACU

In this paper the Q-optimal formulas of quadrature (see [5],[1]) are constructed, reffering to the weight function $w:(0,+\infty)\to(0,+\infty)$, $w(x)=x^{\alpha}e^{-x}$, $\alpha>-1$, named the weight function of the Laguerre type.

We consider the quadrature formulas of the type

$$(1)\quad \int_0^{\infty} x^{\alpha}e^{-x}f(x)dx=\sum_{i=1}^{n} A_i f(x_i)+R_{n-1}(f),\ \alpha>-1,$$

so that the algebrical degree of exactity to be n-1.

We have

$$A_i=\int_0^{\infty} x^{\alpha}e^{-x}\,l_i(x)dx,\quad i=\overline{1,n},$$

$$(2)\quad R_{n-1}^{(f)}=\int_0^{\infty} x^{\alpha}e^{-x}u(x)[x_1,x_2,\dots,x_n,x;f]\,dx,$$

where

$$l_i(x)=\frac{u(x)}{u'(x_i)(x-x_i)},\quad i=\overline{1,n},$$

$$u(x)=\prod_{i=1}^{n}(x-x_i)$$

and $[x_1,\dots,x_n,x;f]$ is the divided difference of f(x) on the nodes $x_1,x_2,\dots,x_n,x$ (see [4]).

Consider the function Q(x) defined by the relation

$$(3)\quad Q(x)=\int_0^{x} x^{\alpha}e^{-x}u(x)dx$$

or, equivalentr,

$$(4)\quad Q'(x)=x^{\alpha}e^{-x}u(x)$$