Bul.St.Univ.Baia Mare
Seria B, Matematică-Informatică, Vol.VII, Nr.1-2, 119-120

ON AN INEQUALITY

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In [2] I have proved:

<u>Proposition</u>. Be $f,g:[a,b]\to R$, $a<b$, two functions which satisfy the conditions:

i) f and g continue on $[a,b]$;

ii) f - increasing on $[a,b]$;

iii) g second derivative on $[a,b]$ and

$$g''(x)\geq 0 \quad \text{on } [a,b];$$

iv) $g(a)=g'(a)=0$.

If

$$F(x)=\int_a^x f(t)g''(t)dt, \quad x\in[a,b],$$

then the inequality,

$$(1) \quad \int_a^b F(t)dt \geq g(b)f(a)$$

is verified.

<u>Remark 1</u>. For $g(x)=(x-a)^2/2$, from (1) results the inequality proved by Gh.Miculescu in [1].

1. The inequality (1) can be thus generalized:

Be $f,g:[a,b]\to R$, $a<b$, two functions which satisfy the conditions:

i) f and g continue on $[a,b]$;

ii) g derivative of $(k+1)$ on $[a,b]$, k natural number;

iii) $g^{(k+1)}(x)\geq 0$, $(\forall)$ $x\in[a,b]$;

then the inequality

$$(2) \quad \int_a^b (b-t)^k f(t)g^{(k+1)}(t)dt \geq k!m\left[g(b)-g(a)-\frac{b-a}{1!}g'(a)-\dots-\frac{(b-a)^k}{k!}g^{(k)}(a)\right],$$

where $m=\min\limits_{x\in[a,b]} f(x)$, is true.

For the proof the function

$h: [a,b] \to R$

$$h(y)=\int_a^y (y-t)^k f(t) g^{(k+1)}(t)dt - km\left[g(y)-\sum_{j=0}^{k}\frac{(y-a)^j}{j!}g^{(j)}(a)\right]$$

is considered.

2. Inequality (1) can be extended to the function of two variables.

Be $f,g:[a,b]\times[c,d]\to R$ two functions of two variables which satisfy the conditions

i) f and g continue on the rectangular $[a,b] \times [c,d]$;

ii) there is g''_{xy} on $[a,b] \times [c,d]$ and $g''_{xy}(x,y) \geqslant 0$ for any $(x,y) \in [a,b]\times[c,d]$;

then the inequality

$$(3)\quad \int_a^b \int_c^d f(x,y)g''_{xy}(x,y)dxdy \geqslant m\left[g(b,d)+ g(a,c)- g(b,c)- g(a,d)\right],$$

where $m = \min\limits_{(x,y) \in [a,b] \times [c,d]} f(x,y)$, is

For the proof the helping function

$$h(s,t)=\int_a^s \int_c^t f(x,y)g''_{xy}(x,y)dxdy -m\left[g(s,t)+g(a,c)-g(s,c)-g(a,t)\right]$$

$(s,t) \in [a,b] \times [c,d]$, is considered.

1. Gh. Miculescu, On the problem 8319, G.M.5, 1982, p.166.

2. D. Acu, On an inequality, G.M.3, 1988, p.97-98.

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