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GENERALISED T-BIRECURRENT AFFINE CONNECTIONS

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Let A_n be a space with affine connection γ. In a local coordinate system, we denote by γ^i_{jk}, the components of the affine connection, by T^i_{jk} the components of the torsion tensor of the connection γ and by $T_k = T^i_{ik}$ the components of the torsion vector.

The space A_n is called space with birecurrent torsion or T-birecurrent space, [5] if there exists a covariant tensor of second order ω_{rs} so that:

$$T^i_{jk,rs} = \omega_{rs} T^i_{jk} \qquad (1)$$

where comma denotes the covariant derivation with respect to γ.

Definition 1. The spaces A_n are called generalized T-birecurrent, if we have:

$$T^i_{jk,rs} = \varphi_{rs} T^i_{jk} + a_{rs} Q^i_{jk} \qquad (2)$$

where φ_{rs} and a_{rs} are covariants tensor of second order and Q^i_{jk} a skew-symmetric tensor in j and k.

Remark 1. The relation (2) is a natural generalisation of the relation (1), because relation (1) can be written

$$T^i_{jk,rs} = \varphi_{rs} T^i_{jk} + (\omega_{rs} - \varphi_{rs}) T^i_{jk} \qquad (3)$$

and the space is generalized T-birecurrent, with an arbitrary φ_{rs} and

$$a_{rs} = \omega_{rs} - \varphi_{rs} \qquad (4)$$
$$Q^i_{jk} = T^i_{jk}$$

we have therefore

Proposition 1. The A_n T-birecurrent spaces are also generalized T-birecurrent space, with an arbitrary φ_{rs} and a_{rs} and Q^i_{jk} given by (4).

Derivating covariantly (10) twice and taking (2) and (5) into account, we have:

$$Q^i_{jk} = \frac{1}{n-1}(\delta^i_j Q_k - \delta^i_k Q_j) \qquad (11)$$

relation of the same kind as (10), therefore:

Proposition 4. In a generalized T-birecurrent A_n space, n>1, with semi-symmetric connection, the tensor $Q^i{}_{jk}$ and his contracted $Q_k = Q^i{}_{ik}$ satisfy the relation (11).

Transvecting (11) by $_i$ we have

$Q^i{}_{jk\ i} = 0$ (13)

and therefore:

Proposition 5. In the generalized T-birecurrent A_n spaces with semi-symmetric connection (13) takes place.

The relation (10) and (11) give for these spaces, the answer to the remark 2. Indeed from (10), derivating covariantly twice and taking (5) and (10) into account, it follows (2) with $Q^i{}_{jk}$ given by (11).

We have therefore the converse assertion:

Proposition 6. The A_n spaces with semi-symmetric connection and with generalized T-birecurrent torsion vector, are also generalized T-birecurrent spaces with the same φ_{rs} and with $Q^i{}_{jk}$ given by (11).

From the relation of S.Golab [6] for semi-symmetric connections:

$$T^i_{sj}T^s_{kh} + T^i_{sk}T^s_{hj} + T^i_{sh}T^s_{jk} = 0 \qquad (14)$$

$$T_s T^s_{kh} = 0 \qquad (15)$$

derivating covariantly twice and taking count of (2), (5) and 14), (15) we have:

$$a_{rp}(Q^s_{kh}T^i_{sj} + Q^s_{hj}T^i_{sk} + Q^s_{jk}T^i_{sh} +$$
$$+ T^s_{kh}Q^i_{sj} + T^s_{hj}Q^i_{sk} + T^s_{jk}Q^i_{sh} +$$
$$+ T^s_{kh,r}T^i_{sj,p} + T^s_{hj,r}T^i_{sk,p} + T^s_{jk,r}T^i_{sh,p} +$$
$$+ T^s_{kh,p}T^i_{sj,r} + T^s_{hj,p}T^i_{sk,r} + T^s_{jk,p}T^i_{sh,r} = 0 \qquad (16)$$

and

$$a_{rs}(Q^i_{jk}T_i + T^i_{jk}Q_i) + T^i_{jk,r}T^i_{i,s} + T^i_{jk,s}T_{i,r} = 0 \qquad (17)$$

Therefore:

Proposition 7. In a generalized T-birecurrent semi-symmetric A_n spaces, (16) and (17) take place.

If the semi-symmetric connection of the A_n space is an E-connection, therefore [2]

$$T_{i,j} - T_{j,i} = 0 \qquad (18)$$

from

$$T^i_{jk,r} + T^i_{kr,j} + T^i_{rj,k} = 0 \qquad (19)$$

by covariant derivation and taking count of (2) we have:

$$\varphi_{rs}T^i_{jk} + \varphi_{js}T^i_{kr} + \varphi_{ks}T^i_{rj} + a_{rs}Q^i_{jk} + a_{js}Q^i_{kr} + a_{ks}Q^i_{rj} = 0 \qquad (20)$$

and in this applying a contraction in i and r and taking count of (5) and (18) it follows:

$$\varphi_{is}T^i_{jk} + a_{is}Q^i_{jk} = 0 \qquad (21)$$

we have therefore:

Proposition 8. In the generalized T-birecurrent semi-symmetric E-connection, the tensors φ_{rs} and a_{rs} are solutions of the n linear sistems (21) and verifies (20).

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