Buletinul Ştiinţific al Universităţii din Baia Mare
Seria B, Matematică-Informatică, vol.IX(1993), 45-52

GENERALIZED CONTRACTIONS IN UNIFORM SPACES

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1. INTRODUCTION

Banach's fixed point principle in metric spaces was generalized in [9], [10] to uniform spaces as follow (see also [12]).

THEOREM 1. (N. Gheorghiu's theorem).

Let X be a Haussdorf sequentially complete uniform space with uniformity defined by a saturated family of pseudometrics $(d_i)_{i\in I}$, I being an index set. Let $f: X \to X$ a map with the properties

1) There exists $a: I \to I$, $q: I \to \mathbb{R}_+$ such that

$$d_i(f(x), f(y)) \le q_i d_{a(i)}(x,y) \ , \ \forall\, i\in I \ , \ \forall\, x,y\in X;$$

2) The series

$$\sum_{n=1}^{\infty} q_i q_{a(i)} \cdots q_{a^n(i)}(x,y)$$

is convergent for each $i\in I$ and each $x,y\in X$.

Then f has a unique fixed point.

Also, the same fixed point principle was extended in [3] to generalized contraction principle.

THEOREM 2.

Let (X,d) be a complete metric space and $f: X \to X$ a φ-contraction with φ a (c)-comparison function, i.e. a map which satisfies the following condition

$$d(f(x), f(y)) \le \varphi(d(x,y)) \ , \ \forall x,y\in X. \qquad (1)$$