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DYNAMICS OF IMMERSED TUBE TAKING INTO ACCOUNT THE TUBE BAFFLE IMPACT

Boris CIORNEIU and Marcel MIGDALOVICI

ABSTRACT This paper presents an analytical model for the immersed tube-baffle interaction. In this case both squeeze film and impact effect lead to the system equations coupling. Using the exponentials of operators the problem is solved in 2N spaces, where the lateral displacement and velocity of the tube are given. Finally the influence of some parameters related to the contact force is shown in Table II.

INTRODUCTION The tube vibrations against a baffle plate in a liquid is quite different from the unimmersed case. It is necessary now to consider the effects caused by the squeeze-film and the tube-to-fluid viscous damping. The former is the dominant mechanism among the others and its influence cannot be neglected. Further, we shall present, according to Fig.1, mathematical model of the immersed fixed-fixed taking into account its interaction with a baffle.

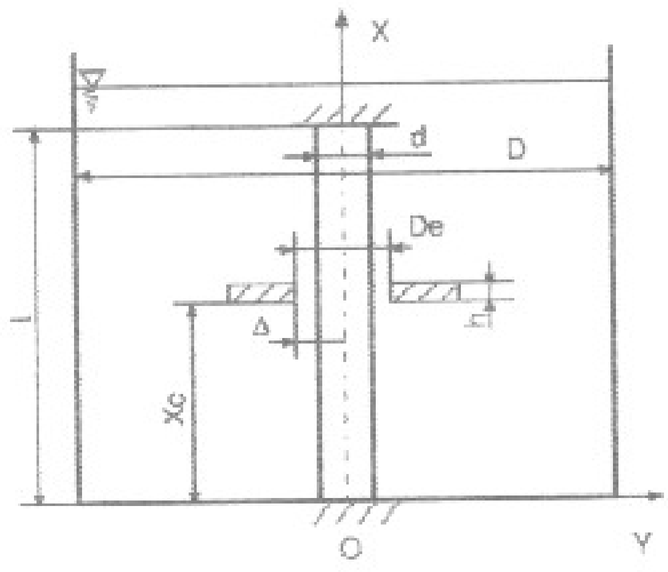

Fig.1 SYSTEM GEOMETRY

PROBLEM FORMULATION As it can be seen in Fig.1, the model is restricted to a fixed-fixed tube immersed in water. The physical characteristics of the system are:

l - length of the tube; EI - bending rigidity; m - mass density per unit length; ρ - mass density of the water; D - hydraulic diameter; Δ - radial clearance; h - baffle thickness.

The whole analysis is made under the following hypotheses:

- the tube lateral displacement is small, so that the small oscilations theory is applicable.
- the length of the tube is very large as compared to thickness h and we may assume that the contact force acts at $x=x_c$.
- the impact force is assumed to be an elastic one.

The partial differential equation of the motion for the transverse displacement is presented in eq. (1).

$$EI\frac{\partial^4 v}{\partial x^4}+\mu I\frac{\partial^5 v}{\partial x^4 \partial t}+[mg(l-x)-T(l)]\frac{\partial^2 v}{\partial x^2}-mg\frac{\partial v}{\partial x}+$$

$$+m\frac{\partial^2 v}{\partial t^2}+[C_v+\delta(x-x_c)HC_{af}]\frac{\partial v}{\partial t}+ \quad (1)$$

$$+\delta(x-x_c)[k_c(v(x,t)-\Delta)H(v(x_c,t)-\Delta)+$$

$$+k_0(-v(x,t)-\Delta)H(-v(x_c,t)-\Delta)] = f(x,t)$$

where:

$H = H(-v(x_c,t)-\Delta)-H(v(x,t)-\Delta)$; t are the space and time variable respectively; v(x,t) is the lateral displacement of the tube; T() is the axial tension; k_c - is the equivalent stiffness; C_v is the tube-to-fluid viscous damping coefficient; C_{af} - is the squeeze-film damping coefficient integrated over the h; m is the mass per unit length of the tube including the added mass; f(x,t) is the driving force per unit length; μ is the internal damping coefficient; g is the gravitational acceleration; $\delta()$ is Dirac's delta function and H() is Heaveside's unit step function.

The boundary conditions for a fixed-fixed tube are:

$$v(x,t)|_{x=0} = v(x,t)_{x=l} = 0 \tag{2}$$

$$\frac{\partial v(x,t)}{\partial x}\|_{x=0} = \frac{\partial v(x,t)}{\partial x}\|_{x=l} = 0$$

and the initial conditions:

$$v(x,t)\|_{t=0} = f(x)\,,\quad \frac{\partial v(x,t)}{\partial t}\|_{t=0} = g(x) \tag{3}$$

For purposes of the analysis it is more convenient to use dimensionless parameters:

$$\xi = \frac{x}{l};\ \tau = \left(\frac{EI}{m}\right)^{1/2}\frac{t}{l^2};\ w = \frac{v}{l}$$

$$\alpha = \frac{\mu}{l}\left(\frac{I}{mE}\right)^{1/2};\ \beta = \frac{l^3 mg}{EI};\ \Gamma = \frac{T(l)\,l^2}{EI} \tag{4}$$

$$\gamma = \frac{C_v l}{(EIm)^{1/2}};\ \varepsilon = \frac{C_{sf} l}{(EIm)^{1/2}};\ \theta = \frac{\Delta}{l}$$

$$F = \frac{f l^3}{EI};\ \Omega = \left(\frac{m}{EI}\right)^{1/2}\omega l^2;\ \eta = \frac{k_c l^3}{EI}$$

Taking into account the first interaction and substituting eqs (4) into eqs.(1), (2), (3) we obtain:

$$\frac{\partial^4 w}{\partial \xi^4} + \alpha\frac{\partial^5 w}{\partial \xi^4 \partial \tau} + [\beta(1-\xi)-\Gamma]\frac{\partial^2 w}{\partial \xi^2} - \beta\frac{\partial w}{\partial \xi} +$$

$$+ \frac{\partial^2 w}{\partial \tau^2} + [\gamma + \delta(\xi-\xi_c)\varepsilon]\frac{\partial w}{\partial \tau} + \tag{5}$$

$$+ \eta\delta(\xi-\xi_c)(w-\theta)H(w-\theta) = F(\xi,\tau)\ .$$

The new boundary conditions:

$$w(\xi,\tau)|_{\xi=0} = w(\xi,\tau)|_{\xi=1} = 0 \tag{6}$$

$$\frac{\partial w(\xi,\tau)}{\partial \xi}\|_{\xi=0} = \frac{\partial w(\xi,\tau)}{\partial \xi}\|_{\xi=1} = 0$$

and the new initial conditions:

$$w(\xi,\tau)\big|_{\tau=0} = l^{-1}f(x); \tag{7}$$

$$\frac{\partial w}{\partial \tau}\Big|_{\tau=0} = l\left(\frac{EI}{m}\right)^{\frac{1}{2}} g(x).$$

We search the solution in the form

$$w(\xi,\tau) = \sum_{n=1}^{\infty} \psi_n(\xi)\, q_n(\tau), \tag{8}$$

where $\psi_n(\xi)$ is the n^{th} natural function wich satisfies the following system:

$$\frac{d^4\psi}{d\xi^4} + U\frac{d^2\psi}{d\xi^2} - \lambda\psi = 0 \tag{9}$$

$$\psi\big|_{\xi=0} = \psi\big|_{\xi=1}; \quad \frac{d\psi}{d\xi}\Big|_{\xi=0} = \frac{d\psi}{d\xi}\Big|_{\xi=1} = 0,$$

and the orthogonality conditions:

$$\frac{1}{M_n}\int_0^1 \psi_m \cdot \psi_n d\xi = \delta_{mn} \tag{10}$$

These functions have the form:

$$\psi_n(\xi) = c_{1n}\sin s\xi + c_{2n}\cos s\xi + c_{3n}\, sh\, r\xi + c_{4n}\, ch\, r\xi \tag{11}$$

where:

$$c_{1n} = (r/s\ sh\ r + \sin s)/(ch\ r - \cos s);\ c_{2n} = -1$$

$$c_{3n} = -(sh\ r + r/s \sin s)/(ch\ r - \cos s);\ c_{4n} = 1$$

and

$$r = (-U/2 + ((U/2)^2 + \lambda)^{1/2})^{1/2}$$

$$s = (U/2 + ((U/2)^2 + \lambda)^{1/2})^{1/2},\ U = \beta - \Gamma$$

The harmonic eigenvalue problem is a self-adjoint one and λ_n eigenvalue must satisfy the characteristic equation

$$1-1/2(\cos r+\frac{U}{2\sqrt{\lambda}}\sin r)\exp s-1/2(\cos s-\frac{U}{2\sqrt{\lambda}}\sin r)\exp(-s) = 0 \qquad (12)$$

Substituting eq.(8) into eq.(5), multiplying the resulting equation by ψ_m and then integrating with respect to ξ from 1 we obtain:

a) If $-\theta<w(\xi_c,\tau)<\theta$

$$\ddot{q}_n+\sum_{m=1}^{\infty}(\alpha_n a_{mn}+y_n\delta_{mn}+s_{mn})\dot{q}_m + \qquad (13)$$

$$+\sum_{n=1}^{\infty}(\lambda_m\delta_{mn}-\beta b_{mn})q_m = Q_n \qquad n=1,2,\ldots,N$$

where:

$$a_{mn} = \frac{1}{M_n}\int_0^1\frac{d^4\psi_m}{d\xi^4}\psi_n d\xi;\; b_{mn} = \frac{1}{M_n}\int_0^1\frac{d\psi_m}{d\xi}\psi_n d\xi;$$

$$Q_n = \frac{1}{M_n}\int_0^1 F\psi_n d\xi;\quad \Omega_n^2 = \lambda_n-\beta b_{nn}$$

$$y_n = \pi\sqrt{2}\rho d l\frac{\nu^{\frac{1}{2}}\Omega_n^{\frac{1}{2}}}{(EI)^{\frac{1}{4}}m^{\frac{3}{4}}}\frac{1+(\frac{d}{D})^3}{[1-(\frac{d}{D})^2]^2}$$

$$e_{nm} = \frac{12\pi}{(EIm)^{\frac{1}{2}}}\frac{\rho g h k\nu}{M_n}(\frac{d}{De-d})^3\psi_n(\xi_c)\psi_m(\xi_c)$$

$$k = 1-\frac{d}{h}\tanh(\frac{h}{d})$$

where:

ν - kinematic viscosity, D - hydraulic diameter, D_e - hole diameter of the baffle.

As the internal damping is small α_n coefficient was chosen so thet the damping ratio be 0,1% over all the modes.

Under these circumstances eq.(13) can be rewritten under matrix form:

$$\{\ddot{q}\}+[D]\{\dot{q}\}+[E]\{q\} = \{Q\} , \qquad (14)$$

where:

$$d_{ij} = \alpha_i a_{ij}+\gamma_i \delta_{ij}+e_{ij}$$

$$e_{ij} = \lambda_i \delta_{ij}-\beta b_{ij} \qquad i,j = 1,2,\ldots,N$$

we put eq.(14) under the following form:

$$\{\dot{Z}\} = [A]\{Z\}+[B(\tau)] , \qquad (15)$$

where:

$$\{Z\} = \begin{matrix}\dot{q}\\ q\end{matrix} \; ; \; \{B(\tau)\} = \begin{matrix}Q\\ O\end{matrix} ,$$

$$[A] = \begin{bmatrix} -[D] & -[E] \\ [I] & [O] \end{bmatrix} \qquad (16)$$

Making the transformation

$$\{Z\} = [U]\{\varphi\} , \qquad (17)$$

we obtain:

$$\{\dot{\varphi}\}+[J]\{\varphi\} = \{G(\tau)\} , \qquad (18)$$

where [J] matrix is a quasidiagonal one:

$$[J] = [U]^{-1}[A][U] \; ; \; J_{ij} = [D_j]\delta_{ij} ;$$

$$[D_j] = \begin{bmatrix} Re\beta_j & -Im\beta_j \\ Im\beta_j & Re\beta_j \end{bmatrix} \qquad (19)$$

Here [U] matrix is defined as:

$$[U] = [\{Im\Phi^1\}\{Re\Phi^1\}\ldots\{Im\Phi^J\}\{Re\Phi^J\}] , \qquad (20)$$

where:

$\{Re\Phi^j\}$; $Re\beta_j$ and $\{Im\Phi^j\}$; $Im\beta_j$ are real imaginary values of $\{\Phi^j\}$ eigen vector, eigenvalue respectively wich belong to [A] matrix.

We assume here that for j=2k-1, k=1,2,...,N $Im\beta_j>0$.

The solution of eq.(18) is:

$$\{\varphi\} = \exp([J]\tau)\,[\int_0^\tau \exp(-[J]s)\{G(s)\}ds + \{K\}] \tag{21}$$

where:

$$\{G(s)\} = [U]^{-1}\{B(s)\}$$

The solution of eq.(18) can be rewritten as:

$$\{\varphi\} = \{\varphi^0\} + \{\varphi^p\} \tag{22}$$

where:

$\{\varphi^0\}$ - is the solution of the homogeneous eq. related to eq.(18).

$\{\varphi^p\}$ - is the particular solution of eq.(18).

$$\varphi_j^0(\tau) = \exp(\tau Re\beta_j)\,[k_j \cos(\tau Im\beta_j) - k_{j+1}\sin(\tau Im\beta_j)] \qquad j = 2k-1,\ k=1,2,\dots,N \tag{23}$$

$$\varphi_j^0(\tau) = \exp(\tau Re\beta_{j-1})\,[k_{j-1}\sin(\tau Im\beta_{j-1}) + k_j \cos(\tau Im\beta_{j-1})] \qquad j = 2k,\ k = 1,2,\dots,N$$

Here kj constants are evaluated taking into account the initial conditions (7);

$$\varphi_j^p = \sum_{l=1}^{N}\int_0^\tau \exp[(\tau-s)Re\beta_j]\,[\cos(\tau-s)Im\beta_j] * u_{j,l}Q_l - \sin[(\tau-s)Im\beta_j]\,u_{j+1,l}Q_l]\,ds \quad j=2k-1,\ k=1,2,\dots,N$$

$$\varphi_j^p = \sum_{l=1}^{N}\int_0^\tau \exp[(\tau-s)Re\beta_{j-1}]\,[\sin[(\tau-s)Im\beta_{j-1}] * u_{j-1,l}Q_l + \cos[(\tau-s)Im\beta_{j-1}]\,u_{j,l}Q_l]\,ds \quad j=2k,\ k=1,2,\dots,N \tag{24}$$

Finally, the dimensionless lateral displacement and velocity are:

$$w(\xi,\tau) = \sum_{i=1}^{N}\sum_{j=1}^{N}\psi_i(Im\Phi_i^{2j-1}+N\varphi 2j-1+Re\Phi_i^{2j-1}+N\varphi 2j)$$
$$w(\xi,\tau) = \sum_{i=1}^{N}\sum_{j=1}^{N}\psi_i(Im\Phi_i^{2j-1}\varphi 2j-1+Re\Phi_i^{2j-1}\varphi 2j) \quad (25)$$

b) If

$$w(\xi_c,\tau) \geq \theta \ or \ w(\xi_c,\tau), \leq -\theta$$
$$\ddot{q}_n+\sum_{m=1}^{N}(\alpha_n a_{nm}+y_n\delta_{nm})\dot{q}_m + \sum_{m=1}^{N}(\lambda_n\delta_{nm}-\beta b_{nm})q_m = Q_n \quad (26)$$

where:

$$Q_n = \int_0^1 F\psi_n(\xi)\,d\xi + \Theta\eta\theta\psi_n(\xi_c)$$

Here Θ function is defined as;

$$\Theta = \begin{cases} 1 \ if \ w(\xi_c,\tau)\geq\theta \\ -1 \ if \ w(\xi_c,\tau)\leq -\theta \end{cases} \quad (27)$$

The solution of eq.(26) has the same form as eq.(22). Now the integration into φ_j^p will be done from τ_c to τ and the new k_j constants of φ_j^c will be evaluated from the continuity conditions of displacement and velocity at $\tau=\tau_c$. Here τ_c is the dimensionless contact time wich can be evaluated from eq.:

$$w(\xi_c,\tau)=\theta \quad (28)$$

NUMERICAL RESULTS AND DISCUSSIONS

The numerical results of this study are based on the above model. To keep the analysis sufficiently simple we set $f(x)=g(x)=0$ and the driving force as:

$$f(x,t) = \delta(x-x_e) P_0 \sin\omega t \tag{29}$$

Moreover, the evolution of the ξ_c contact point is considered till the first moment when a trend of getting out of the contact appears.

The parameters used in this computation are given in Table I, also some numerical results as $F_c=\max\{\eta[w(\xi_c,\tau)-\Theta]\}$ the maximum of the contact force and $\Delta\tau$ the duration of the contact are presented in Table II. We mention here that all the computation is made in considering the first five modes.

The evolution of $w(\xi_c,\tau)$ the dimensionless lateral displacement and of $w(\xi_c,\tau)$ the dimensionless velocity of ξ_c the contact point are displayed in Figs. 2,3 and 4 taking into account dofferent stiffnesses.

The contributions $w^0(\xi_c,\tau)$ coming from solution (23) and $w^{p\Delta}(\xi_c,\tau)$, $w^{pF}(\xi_c,\tau)$ coming from solution (24) to the $w(\xi_c,\tau)$ are presented in Figs. 5,6 and 7. Here $w^{p\Delta}(\xi_c,\tau)$ take into account the baffle stiffness and the driving force effects respectively, while $w^0(\xi_c,\tau)$ considers the initial conditions.

Theses figures reveal that the lateral displacement of ξ_c during the contact is virtually controlled by $w^{p\Delta}(\xi_c,\tau)$ and $w^0(\xi_c,\tau)$ for small values of η.

As shown in Table II the influence of h parameter to F_c the contact force is weak, but in order to diminish the contact pressure the contact surface should be large.

Other effect of the h parameter is that τ_c the contact time increases due to the squeeze-film so that τ_c=0,7005E-01 when h=0,01m and τ_c=0,7211E-01 for h=0,03m (ω=94,25 rad/s).

The influence of ω, the forcing frequency, as it can be seen from Table II, is weak when this was increased from ω=94,25 rad/s to 157 rad/s.

The most important parameter for this model is η the dimensionless stiffness wich controls the values of F_c. In this respect for small values of η it is necessary to consider more than five modes for an accurate calculation.

Finally, as a particular case of this topic we present in Figs. 8 and 9 the evolution of ξ_c when ρ=0, h=0,01m, ω=157 rad/s and η=196900 where it can be seen that the velocity decreases sharply (see Fig.8) as compared with Fig. 2.

TABLE I. PARAMETERS USED IN COMPUTATION

PARAMETER	SYMBOL	DIMENSION	VALUE
Length of the tube	l	[m]	2.14
Tube diameter	d	[m]	19.1×10^{-3}
Hydraulic diameter	D	[m]	0.5
Mass density of water	ρ	[Kgm^{-3}]	10^3
Mass / unit length	m	[Kgm^{-1}]	0.504
Kinematic viscosity	ν	[m^2s^{-1}]	10^{-6}
Bending rigidity	EI	[Nm^2]	497.8
Baffle thickness	h	[m]	10^{-2}; 3×10^{-2}
Equivalent stiffness	k_c	[Nm^{-1}]	10^5
Tube baffle clearance	Δ	[m]	0.25×10^{-3}
Baffle location	x_c	[m]	1.29
Location of the excitation	x_e	[m]	0.5
Driving force intensity	P_o	[N]	10
Forcing frequency	ω	[$rad.s^{-1}$]	94.25; 157

TABLE II. NUMERICAL RESULTS

Ω_n	a_{mn}	b_{mn}	g_{mn}
.2234E+02	.6037E+03 .2927E-01	-.3714E+01 .3952E+01	.3837E+00 -.7603E-01
.6164E+02	.3134E-01 .4286E+04	-.3356E+01 -.6747E+01	-.8141E-01 .1613E-01

α_n	γ_n	η	ω	Δz	h	F_c
.1063E-14	.2169E+00	.1969E+04	.9425E+02	.6465E-01	.1E-01	.173E+00
.3636E-14	.3603E+00	.1969E+04	.9425E+02	.6479E-01	.3E-01	.170E+00
—	—	.1969E+04	.157E+03	.5815E-01	.1E-01	.183E+00

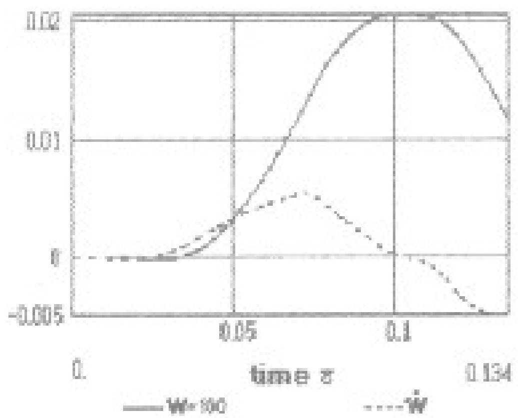

Fig.2 Evolution of contact point
$\eta=1969$

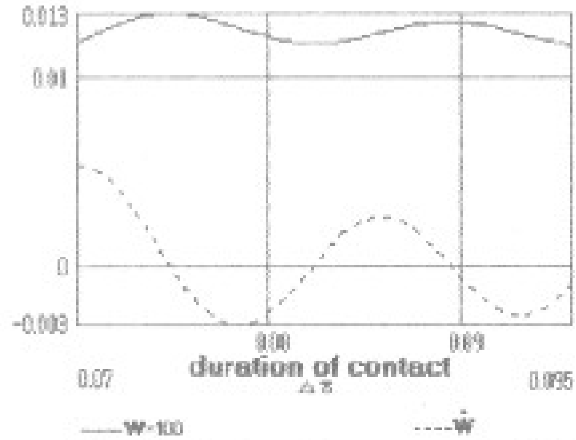

Fig.3 Evolution of contact point during the impact , $\eta=19690$

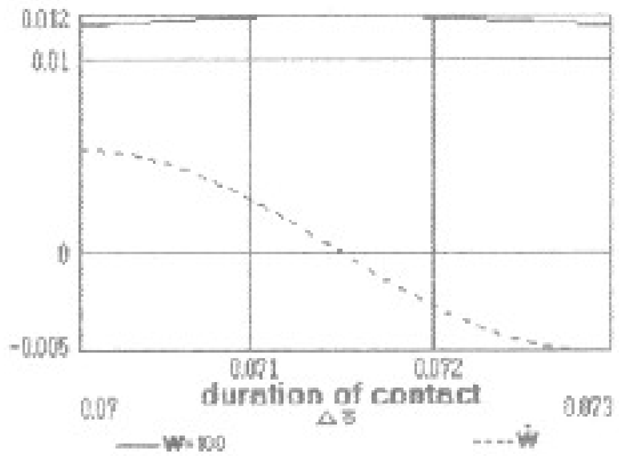

Fig.4 Evolution of contact point during the impact , $\eta=196900$

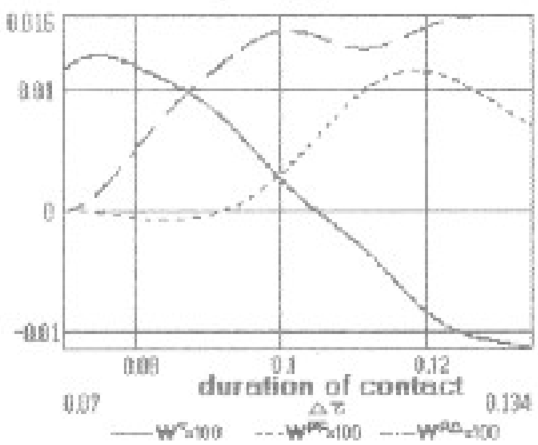

Fig.5 Displacement contribution
$\eta=1969$

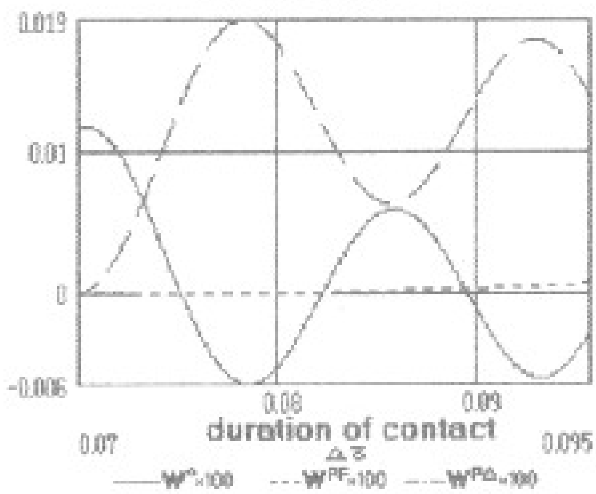

Fig.6 Displacement contribution
$\eta=19690$

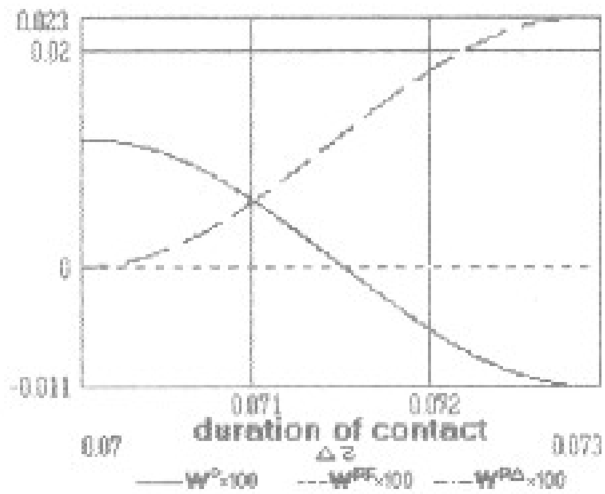

Fig.7 Displacement contribution
$\eta=196900$

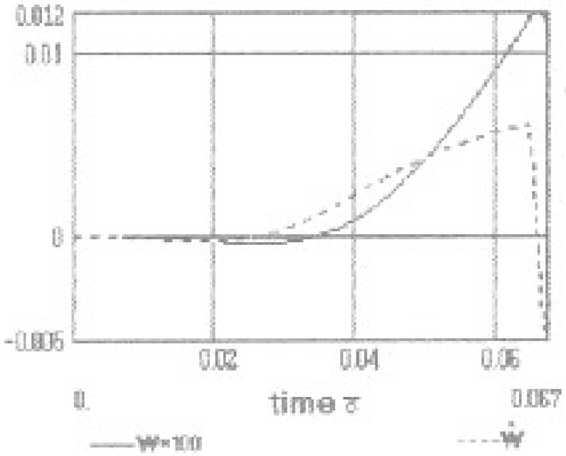

Fig.8 Evolution of contact point
$\rho=0$, $\eta=196900$

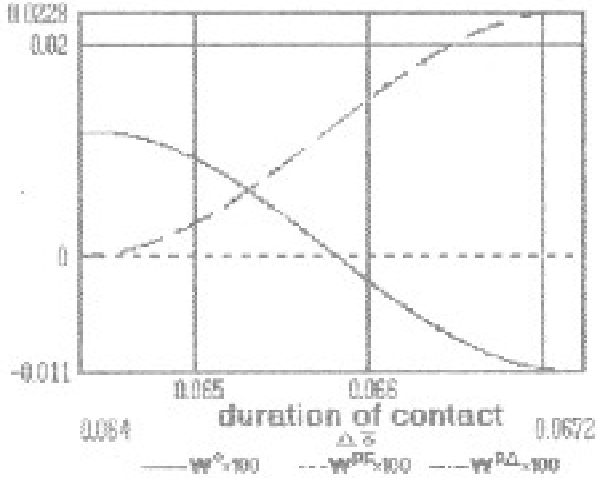

Fig.9 Displacement contribution
$\rho=0$, $\eta=196900$

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INSTITUTE OF SOLID MECHANICS
BUCHAREST
ROMÂNIA