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A COMPUTATIONAL METHOD WITH FINITE ELEMENTS FOR A COUPLED SOLUTION BETWEEN MECHANIC AND THERMIC CONTACT PROBLEMS

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ABSTRACT. This paper proposes a computational method for analysing the behaviour of two elastic bodies coming into contact with friction, obeying volume forces of density, surface tractions of density, and thermal load. Thermal load results from the computation of the distribution of the thermal field in the two bodies, assuming the existence of a heat source. In this computation, attention is concentrated on contact problem in elasticity and on thermal transfer through the contact zone, using contact finite element and the thermal finite contact element.

Mechanical contact will be analysed in following incremental stages. Firstly, the displacement field will be determined from the forces and thermal loads. Here, too, attention is concentrated on the contact zone, where use will be made of finite contact element which will be a model of: the condition of contact, the law of friction and the geometry of the contact interfaces. Then there follows the alternative and incremental computation of the displacement field and thermal field, since the thermal contact resistence and geometry of the contact zone change. In the liniarization of the set of equations use is made of the Newton-Raphson method, and a consistent technique which implies computational efficiency.

1. CLASSICAL AND VARIATIONAL PROBLEM OF CONTACT PROBLEM IN ELASTICITY AND OF CONTACT PROBLEM IN HEAT TRANSFER PROBLEM

We shall now formulate a class of initial-value problems in elastostatic which include sliding friction effects. Let $\Omega^{\alpha}\subset R^{N}$, $\alpha=1,2$, $N=2,3$, the domains occupied by two elastic bodies that come into contact with friction.

Let us denote by Γ^{α} the boundary of Ω^{α} and let $\Gamma^{\alpha}_{o}, \Gamma^{\alpha}_{1}, \Gamma^{\alpha}_{2}$ be open and disjoint parts of Γ^{α} so that $\Gamma^{\alpha}=\Gamma^{\alpha}_{o}\cup\Gamma^{\alpha}_{1}\cup\Gamma^{\alpha}_{2}$ with $\alpha=1,2$.

Assume that the bodies Ω^α are subjected to volume forces of density $f^\alpha = (f_1^\alpha,...,f_N^\alpha)$ on Ω^α, to surface tractions of density $t^\alpha = (t_1^\alpha,...,t_N^\alpha)$ on Γ_1^α, the heat flux q on Γ_1^α and are hold fixed on Γ_o^α. We shall usse the following notations for the normal and tangential components of the displacements and of the stress vector:

$$u_n^\alpha = u_i^\alpha n_i^1, u_{t_i}^\alpha = u_i^\alpha - u_n^\alpha n_i^1, \sigma_n^\alpha = \sigma_{ij}^\alpha n_i^\alpha n_j^1, \sigma_{t_i}^\alpha = \sigma_{ij}^\alpha n_j - \sigma_n^\alpha n_i^1,$$

where i,j = 1,...,N, $n^\alpha = (n_1^\alpha,...,n_N^\alpha)$ is the outward normal unit vector on Γ^α and the summation convention is used for i and j.

Find the field of displacements $u^\alpha = (u_1^\alpha,...,u_N^\alpha)$, defined on Ω^α which satisfy the following equations and conditions:

- the equilibrium equation

$$\sigma^\alpha_{ij/j}(u^\alpha) + f_i^\alpha = 0 \text{ in } \Omega^\alpha \tag{1}$$

- the constitutive equation

$$\sigma_{ij}^\alpha = a^\alpha_{ijkh}\varepsilon_{kh}(u^\alpha) \quad in\ \Omega^\alpha \tag{2}$$

where $a^\alpha_{ijkh} = a^\alpha_{jikh} = a^\alpha_{khij}$ and $a^\alpha_{ijkh}\xi_{ij}\xi_{kh} \geq c|\xi|$, $\xi = (\xi_{ij})$, and $\varepsilon_{kh}(u^\alpha) = \frac{1}{2}(\frac{\partial u_k}{\partial x_h} + \frac{\partial u_h}{\partial x_k})$, f_i = the components of body force per unit volume, assumed to be sufficiently smooth functions of $x = (x_1,...,x_N)$;

- the boundary conditions

$$u_i^\alpha = 0 \quad \text{on} \quad \Gamma_o^\alpha$$

$$\sigma_{ij}^\alpha(u^\alpha)n_j = t_i^\alpha \quad \text{on } \Gamma_1^\alpha \tag{3}$$

- the normal normal interface response

$$\sigma_n(u^\alpha) = -c_n(u_n^1 - u_n^2 - g)_+^{m_n} \text{ on } \Gamma_2^\alpha \tag{4}$$

with c_n and m_n material parameters (see [2]),

$$\text{or } \sigma_n = \frac{c_1(1617646.152\,\sigma/m)^{c_2}}{5.589^{1+0.0711c_2}}\exp[-\frac{1+0.0711c_2}{(1.363\sigma)^2}d^2]$$

where ξ is the initial mean plan distance $d = \xi g$, g is the initial gap between Γ_2^1 and Γ_2^2, c_1 and c_2 are mechanical constants expressing the nonlinear distribution of the surface hardness, σ and m are statistical parameters of the surface profile, representing respectively the RMS surface roughness and the mean asperity slope.

- the friction and contact conditions :

$$u_n^1 - u_n^2 \le g \Rightarrow \sigma_T(u^\alpha) = 0$$

$$|\sigma_T(u^\alpha)| \le c_T(u_n^1 - u_n^2 - g)_+^{m_T}$$

$$u_n^1 - u_n^2 > g \Rightarrow |\sigma_T(u^\alpha)| < c_T(u_n^1 - u_n^2 - g)_+^{m_T} \Rightarrow \dot{u}_T^1 - \dot{u}_T^2 = \dot{U}_T^C \qquad (5)$$

$$|\sigma_T(u^\alpha)| = c_T(u_n^1 - u_n^2 - g)_+^{m_T} \Rightarrow \exists \lambda \ge 0, \dot{u}_T^1 - \dot{u}_T^2 - \dot{U}_T^C = -\lambda \sigma_T \text{ on } \Gamma_2^\alpha$$

Where c_n, m_n, c_T, m_T are material constants depending on interface proprerties, $b_+ = \max(0, b)$, $\dot{u}_T^\alpha$ is the tangential velocity of material particles on Γ_2^α ,$\dot{U}_T^C$ is the prescribed tangential velocity of the Γ_2^1 with which Γ_2^2 comes in contact and g is the initial gap between Γ_2^1 and Γ_2^2 measured along the outward normal direction to Γ_2^1.

The friction law (5) is a generalization of the Coulomg's friction law, which is recovered if $m_n = m_T$. In such a case, $\mu = c_T/c_n$ is the usual coefficient of friction. The law (5) allows for a dependence of the friction coefficient on normal contact pressure.

The thermal equation in classical form will be:

$$\text{div}(K(T) \text{ grad } T) + Q = 0 \quad \text{in } \Omega_1 \cup \Omega_2 \qquad (6)$$

where K(T) is the thermal conductivity, T is the thermical field, Q is the thermal source and the boundary condition will be:

- $$K(T)\frac{\partial T}{\partial n} + c_t(T - \bar{T}) = 0 \quad \text{on} \quad \Gamma_0^\alpha \qquad (7)$$

where C_T is the coefficient of transmission between the, inside and outside of the bodies,and $\bar{T}$ is the outside temperatura

- $$K(T)\frac{\partial T}{\partial n} = q \quad \text{on} \quad \Gamma_1^\alpha \qquad (8)$$

where q is the heat flux through the Γ_1^α part of the boundary

- $$K(T)\frac{\partial T}{\partial n} + c_h(u, T)(T^2 - T^1) = 0 \quad \text{pe} \quad \Gamma_2^\alpha \qquad (9)$$

where $c_h(u, T)$ is the thermal conductivity of contact, and T^1 and T^2 are the temperatures of the contact boundary of the two bodies.

Thermal contact conductivity is presumed to be $c_h(u,T) = c'_h(T)(1+u)$ where u is the field of displacements.

Following steps similar to those of Duvaut and Lions [1], the nonlinear quasistatic elasticity problem and the distribution of the thermal field problem can be shown to be formally equivalent to the following variational problem :

The variational forms of mecanic and thermic equations for two bodies in contact are:

$$\sum_{\beta=1}^{2} \left\{ \int_{V^\beta} \sigma \delta\varepsilon \, dV - \int_{V^\beta} b \, \delta u \, dV - \int_{\Gamma_1^\beta} \bar{t} \delta u \, dA \right\} + \int_{\Gamma_2^\beta} c_n^*(u) \, g \delta g \, d\Gamma = 0 \qquad (10)$$

$$\sum_{\beta=1}^{2} \left\{ \int_{V^\beta} K \nabla T . \nabla \delta T \, dV - \int_{V^\beta} Q \delta T \, dV - \int_{\Gamma_1^\beta} q \delta T \, dA \right\} + \int_{\Gamma_2^\beta} c_h^*(u,T) \gamma \delta\gamma \, d\Gamma = 0 \qquad (11)$$

where we note with c_n^* the contact rigidity, g the gaps, c_h^* is the thermical contact conductivity and γ is the difference of temperature of the surfaces in contact.

2. THE FINITE ELEMENT OF MECHANIC AND THERMIC CONTACT

We will define the next geometrical parameters of the contact element (see Fig. 1).

$$l = ||x_2 - x_1||, \; t = \frac{1}{l}(x_2 - x_1), \; n = e_3 x t, \; g_n = (x_s - x_1) n \qquad (12)$$

l - the master segment length 1-2, t and n unit tangent and normal vector to the master segment and g_n normal gaps between the slave nod S and the master segment.

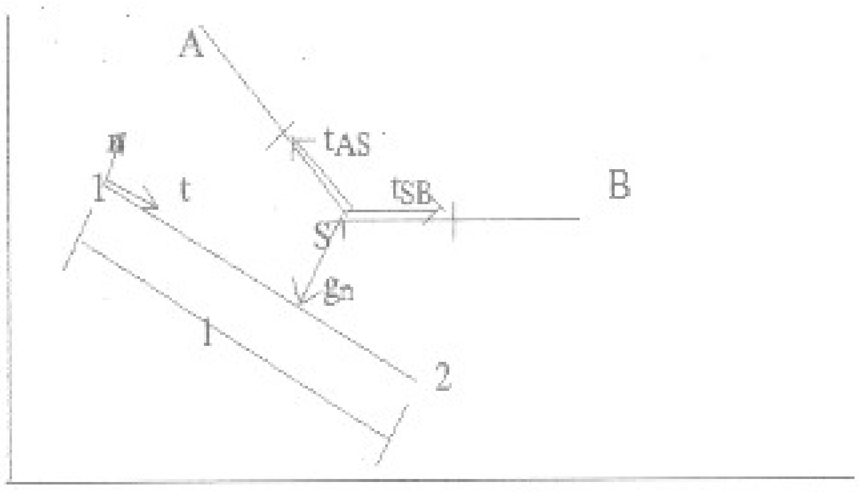

Fig. 1 Contact element geometry

The ortogonal projection of the slave nod S on the master segment and area of the contact element are given by:

$$\xi = \frac{1}{l}(x_S - x_1)t,\ A = \frac{1}{2}(\,||x_S - x_A|| + ||x_S - x_B||\,) \tag{13}$$

With this geometrical elements we can approximate the desplacements and thermical fields in the contact area following MEF technique.

Finaly, after the discrete formulation within the framework FEM, the following will be obtained:

$K_M U = R_M$

$K_T T = R_T$

where $K_M = K_M' + \sum_{s=1}^{S} K_{MC}^s$, $R_M = R_M' + \sum_{s=1}^{S} R_{MC}^s$, K_M', R_M' are mechanical global stiffness matrix and residual matrix, K_{MC}^s, R_{MC}^s are mechanical contact contributions of contact nod s, and $K_T = K_T' + \sum_{s=1}^{S} K_{TC}^s$, $R_T = R_T' + \sum_{s=1}^{S} R_{TC}^s$ with K_T', R_T' are thermal global stiffness matrix and residual matrix, K_{TC}^s, R_{TC}^s are thermal contact contributions of contact nod s, U and T are displacements and thermal fields vectors S is the total number of the slave nodes.

Through the alternative and increments solution of the two equations, we can approximate better the distribution of the thermical field and of the displacement field attention is concentrated on the contact zone, where use will be made of finite contact element which will be a model of the condition of contact, the law of friction and the geometry of the contact interfaces (see [4]).

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