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APPROXIMATION OF FUNCTIONS BY TYPICAL MEANS OF THEIR FOURIER SERIES

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Summary. The representation of a deviation for integrable functions and typical means for Fourier series of these functions is obtained. An appropriate remainder is estimated from both above and below by modulus of smoothness of even orders for given functions.

Let a function $f \in L_p$, $1 \le p \le \infty$, 2π -periodic and

$$f(x) \sim \sum_{k=-\infty}^{\infty} C_k e^{ikx} = \sum_{k=-\infty}^{\infty} A_k(x) \sim \tag{1}$$

Fourier expansion for f(x). Let us consider typical means for (1) i.e.

$$\sigma_n(f;x)=\sum_{|k|\le n}\left(1-\frac{|k|}{n+1}\right)A_k(x).$$

Let Δ and ω are symmetric difference and modulus of smoothness (of appropriate orders and steps) respectively.

The deviation $f(x)-\sigma_n(f;x)$ was investigated by some authors in different directions.Here is one of these results due to Lebed'H.K. and Avdienko A.A.[1]:

$$f(x)-\sigma_n(f;x)=-\frac{1}{2\pi}\int_1^\infty\Delta^2_{t/(n+1)}f(x)\,t^{-2}dt+\tau_n(f;x),$$

$$\|\tau_n(f;x)\|_p\le C\omega_2(f;1/(n+1))_p,\quad C>0.$$

In [2] we have represented $f(x)-\sigma_n(f;x)$ as a sum of appropriate improper integrals of differences for f and conjugated function of second-fourth orders and with a remainder estimated from above and below by a fourth order modulus of smoothness for f $\exists\, C_j>0$,

$$f(x)-\sigma_n(f;x)=C_1^*\int_1^\infty\Delta^2_{t/(n+1)}f(x)\,t^{-3}dt+C_2^*\int_1^\infty\Delta_{t/(n+1)}\tilde f(x)\,t^{-3}dt+$$

$$+C_3^*\int_1^\infty\Delta^4_{t/(n+1)}f(x)\,t^{-4}dt+\tau_n(f;x),$$

$$C_1\omega_4(f;1/(n+1))_p\le\|\tau_n(f;x)\|_p\le C_3\omega_4(f;1/(n+1))_p$$

Now we give a new result in this direction.

$$\exists\ C_1>0,\ C_2>0:\left(\sum_1^0=0\right): f(x)-\sigma_n(f;x)=$$

$$=\sum_{j=1}^{m} C_j^{*}\int_1^{\infty}\Delta^{2j}_{t/(n+1)} f(x)\, t^{-2j}dt+\sum_{j=1}^{m-1} C_j^{*}\int_1^{\infty}\Delta^{2j+1}_{t/(n+1)} \tilde{f}(x)\, t^{-(2j+1)}dt+\tau_n(f;x),$$

$$C_1\omega_{2m}(f;1/(n+1))_p \le \|\tau_n(f;1/(n+1))_p \le \|\tau_n(f;x)\|_p \le C_2\omega_{2m}(\ldots),\ m\in\mathbb{N}$$

The idea of the proof is the same as in [2]: the principle of comparison due to Trigub R.M. [3] and the theorems on multiplicators.

REFERENCES

1. **LEBED'H.K., AVDIENKO A.A.**, Concerning the approximation of periodical functions by Feier sums.- *Izv. AN SSSR*, ser. mat 1971 v.35, pp 83 - 92 (in Russian)

2. **NOSENKO Yu.L.**, Aproximation of functions by Riesz, (C,α) and typical means for Fourier series of these functions, *Bul.Şt.al Univ. Baia Mare*, ser.B, Vol.X,1994, pp. 89 - 92

3. **TRIGUB,R.M.**, Absolute convergence of Fourier integrals, *Izv.AN SSSR*, ser.mat.-1980 - 44 no, pp 1379 - 1409 (in Russian)

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