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THE COMPLETION OF A GAUSS TYPE VALUATION FIELD

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1.Let (K, v) be a *local field*, K being a commutative field and v a discrete and rank one valuation on K, such that K is complete with respect to v. For a $\delta \in \mathbb{Q}$, one denotes by $P_\delta(K,v)$ the set of all Laurent series $\alpha = \sum_{n\in\mathbb{Z}} a_n X^n$, $a_n \in K$ for every $n \in \mathbb{Z}$, and

a) there exist $M \in \mathbb{R}$ with $v(a_n) + n\delta \geq M$, for every $n \in \mathbb{Z}$.

b) $\lim_{n\to-\infty}(v(a_n) + n\delta) = \infty$.

If we define $u_\delta(\alpha) = \inf_{n\in\mathbb{Z}}(v(a_n) + n\delta)$, u_δ is a rank one and discrete valuation on $P_\delta(K,v)$ and this one is a local field with respect to u_δ. We call $P_\delta(K,v)$ a *Parshin field* ([4]).

In [1] we find a description of the completion of the rational function field $K(X)$ with respect to the so called Gauss valuation u_0, where $u_0(a_0 + a_1X + ... + a_nX^n) = \inf_n(v(a_n))$,as a subfield in the Parshin field $P_0(K,v)$.Now, if we introduce on $K(X)$ a Gauss valuation u_δ,where $u_\delta(a_0 + a_1X + ... + a_nX^n) = \inf_n(v(a_n) + n\delta)$, it is not difficult to do the same type of reasoning as in [1] in order to describe the completion of $K(X)$ with respect to u_δ in the Parshin field $P_\delta(K,v)$.

2.Let a be an algebraic element over K and $K' = K(a)$ the corresponding algebraic extension of the field K.For a $\delta \in \mathbb{Q}$,we denote by $P_\delta(K,v,a)$ the set of all Laurent series $\alpha = \sum_{n\in\mathbb{Z}} a_n(X-a)^n$, $a_n \in K'$ with the following properties:

c) there exist $M \in \mathbb{R}$ with $v'(a_n) + n\delta \geq M$, for every $n \in \mathbb{Z}$

d) $\lim_{n\to-\infty}(v'(a_n) + n\delta) = \infty$,where $v'$is the unique extension of the valuation v to K'.

If we define $u_\delta = \inf_{n\in\mathbb{Z}}(v'(a_n) + n\delta)$, $P_\delta(K,v,a)$ becomes a local field with respect to u_δ.

3.If $P(X) = a_0 + a_1X + ... + a_nX^n$ is a polynomial in $K[X]$,we consider $P(X) = a'_0 + a'_1(X-a) + ... + a'_n(X-a)^n$, the Taylor's expansion of $P(X)$ in the element $a \in K'$.

Let us define $u_\delta(P(X)) = \inf_n(v'(a'_n) + n\delta)$. It is not difficult to prove that u_δ is a valuation on $K(X)$. The problem is how to describe the completion of $K(X)$ with respect to u_δ. It is clear enough that u_δ is a Gauss type valuation on $K'(X)$, and the completion of $K'(X)$ with respect to this last valuation is possible to describe as a subfield in $P_\delta(K,v,a)$([1]). Using Taylor's expansions in a, it is possible to construct an embedding of $K(X)$ in $K'(X)$ and then in $P_\delta(K,v,a)$.So the completion of $K(X)$