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EIGENVECTORS AND EIGENVALUES ACHIEVEMENT OF THE DYNAMIC RESPONSE AT DAMPED VIBRATIONS

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ABSTRACT

This paper presents a general method to determine the eigenvectors and eigenvalues for the equation of any type of damped vibrations. By mean of the eigenvectors, one can write the general solution of the system describing the vibratory movement of a mechanic system, with n discrete masses, for different types of disturbing forces.

The method is simulated on the computer and the results achieved may be compared with those achieved by the approximate method.

1.DETERMINATION OF EIGENVECTORS AND EIGENVALUES FOR UNDAMPED STRUCTURE

The equation of motion for a structure with n degrees of freedom [1] are written in form of a matrix thus:

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{B} \cdot \dot{\mathbf{u}} + \mathbf{R} \cdot \mathbf{u} = \mathrm{F}(t) \qquad (1)$$

with the initial condition: $\mathbf{u}(0) = \mathbf{u}_0$ and $\dot{\mathbf{u}}(0) = \mathbf{v}_0$ (1')

The semnifications of the equation (1) are: **M** - masses matrix ($m_{ii} = m_i$, $m_{ij} = 0$ for $i \neq j$), **B** - damping coefficient matrix ($b_{ij} = b_{ji}$), **R** - rigidity coefficient matrix ($r_{ij} = r_{ji}$), F(t) - vector of disturbing forces, u(t) - vector of displacements.

The eigenvalues (eigenpulsation) of the undamped system ($\mathbf{B} = 0$) are given by characteristic equation:

$$\det(\mathbf{R} - p^2\mathbf{M}) = 0 \qquad (2)$$

which has real and positive roots: $p_1 < p_2 < \ldots < p_n$ (3)

The first eigenvalue p_1 is called fundamental pulsation. Eigenvectors (eigenforms) corresponding to each eigenvalues p_α are given by the system:

$$(\mathbf{R} - p_\alpha^2 \cdot \mathbf{M}) \cdot \mathbf{V}_\alpha = 0 \tag{4}$$

with

$$\mathbf{V}_\alpha = [A_{1\alpha}\ A_{2\alpha} \ldots\ldots A_{n\alpha}] \text{ for } \alpha \in \{1,2,\ldots,n\}. \tag{5}$$

Vectors $\mathbf{V}_\alpha$ with $\alpha \in \{1,2,\ldots,n\}$ form the modal matrix: $\mathbf{V} = [\mathbf{V}_1\ \mathbf{V}_2 \ldots\ldots \mathbf{V}_n]$ (6)

By mean of these eigenvectores and eigenvalues one may express:

a) Solution for the homogeneous equation ($\mathbf{B} = 0$, $\mathbf{F} = 0$) representing free oscillations:

$$u_h^l = \sum_{\alpha=1}^{n} A_{h\alpha} \cdot C_\alpha \cdot \cos(p_\alpha \cdot t - \theta_\alpha) \text{ cu } h \in \{1,2,\ldots,n\}. \tag{7}$$

where the constants C_α and θ_α $\alpha \in \{1,2,\ldots,n\}$ are determined from the initial (1').

b) Solution of nonhomogeneous equation ($\mathbf{B} = 0$ and $\mathbf{F} = 0$) representing the oscillations of the stabilized forces:

$$u_h^f = \sum_{\alpha=1}^{n} A_{h\alpha} \cdot q_\alpha \text{ with } h \in \{1,2,\ldots,n\}. \tag{8}$$

with q_α time function, depending on the disturbing forces $\mathbf{F}(t)$. In the case of the damped structures, it is supposed that the matrix $\mathbf{B}$ is a linear combination of matrices $\mathbf{M}$ and $\mathbf{R}$ of the form:

$$\mathbf{B} = \alpha \cdot \mathbf{M} + \beta \cdot \mathbf{R} \tag{9}$$

In this case solution of homogeneous equation (1) ($\mathbf{F}(t) = 0$) is called free vibration:

$$u_h^l = \sum_{\alpha=1}^{n} A_{h\alpha} \cdot C_\alpha \cdot e^{\nu_\alpha \cdot p_\alpha \cdot t} \cdot \cos(p_\alpha \cdot t - \theta_\alpha) \qquad \text{with } h \in \{1,2,\ldots,n\} \tag{10}$$

and the solution of the nonhomogeneous equation (1) is called the stabilized solution or forced solution:

$$u_h^f = \sum_{\alpha=1}^{n} A_{h\alpha} \cdot q_\alpha \text{ with } h \in \{1,2,\ldots,n\}.$$

(11)

where

$$\begin{cases} q_\alpha = \dfrac{1}{p_\alpha^*} \int_0^t S_\alpha(\tau) \cdot e^{\nu_\alpha \cdot p_\alpha \cdot (t-\tau)} \sin p_\alpha^* (t-\tau) d\tau \\ S_\alpha(\tau) = \sum_{h=1}^{n} F_h(\tau) \cdot A_{h\alpha} \end{cases} \tag{12}$$

In the formulas (10) and (12) the following values have appeared:

ν_α - damping factor for eigenform p_α , p_α- unmapped eigenvalue (eigenpulsation) , $\overset{*}{p}_\alpha$ - damped eigenpulsation. Among all these values the following relation exists:

$$\overset{*}{p}_\alpha = p_\alpha \cdot \sqrt{1-\nu_\alpha^2} \qquad (13).$$

This method so called modal method is the bases of the almost all the programmes of dynamic calculus of the structures. This method has the great disadvantage that the damping factors ν_α are approximated.

2.DETERMINATION OF THE EIGENVECTORS AND EIGENVALUES FOR DAMPED STRUCTURES

In the case of the damped structures ($\mathbf{B} \neq 0$) the eigenvalues and eingenvectors are determined beginning with the equation:

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{B} \cdot \dot{\mathbf{u}} + \mathbf{R} \cdot \mathbf{u} = 0 \qquad (14)$$

which describes the free oscillations. Searching solutions in the form of $\mathbf{u}^l = \mathbf{A} \cdot \mathbf{e}^{\lambda t}$ for (14) it may be obtained:

$$(\lambda^2 \cdot \mathbf{M} + \lambda \cdot \mathbf{M} + R) \cdot \mathbf{A} = 0 \qquad (15)$$

representing a linear and homogeneous system. To obtain solutions different from the common ones, for the system (15), it must be accomplished the conditions:

$$\det(\lambda^2 \cdot \mathbf{M} + \lambda \cdot \mathbf{B} + \mathbf{R}) = 0 \qquad (16)$$

representing the characteristic equation of damped structures. Equation (16) has complex conjugate roots with real part negative, if the conditions of stability [3] are accomplished and represents the eigenvalues of the damped structures:

$$\lambda_k = a_k + i \cdot b_k \text{ and } \lambda_{n+k} = a_k - i \cdot b_k \qquad \text{with } a_k < 0 \text{ for } \mathbf{k} \in \{1,2,\ldots,n\}. \qquad (17)$$

The imaginary parts of the eigenvalues represent damped eigenpulsations:

$$\overset{*}{p}_\alpha = b_k \text{ with } \mathbf{k} \in \{1,2,\ldots,n\} \qquad (18)$$

The modules of the eigenvalues of the equation (16) are the eigenvalues of the equation (2) [2]

$$p_k = |\lambda_k| = \sqrt{a_k^2 + b_k^2} \;;\; \mathbf{k} \in \{1,2,\ldots,n\} \qquad (19)$$

For the eigenvalues λ_k and λ_{n+k} the eingenvectors are determined, noted correspondingly with $\mathbf{U}_k$ and $\mathbf{U}_{n+k}$ of the form:

$$\mathbf{U}_k = \mathbf{Y}_k + i \cdot \overline{\mathbf{Y}}_k ; \quad \mathbf{U}_{n+k} = \mathbf{Z}_k + i \cdot \overline{\mathbf{Z}}_k \quad \text{with } \mathbf{k} \in \{1,2,\ldots,n\}. \qquad (20)$$

Substituting λ_k and $\mathbf{U}_k$ in (15) and separating the real part from the imaginary part, we obtain:

$$\begin{aligned} &\left\{\left[\left[a_k^2 - b_k^2\right] \cdot \mathbf{M} + a_k \cdot \mathbf{B} + \mathbf{R}\right] \cdot \mathbf{Y}_k - \left[2a_k \cdot b_k \cdot \mathbf{M} + b_k \cdot \mathbf{B}\right] \cdot \overline{\mathbf{Y}}_k\right\} + \\ &+ i \cdot \left\{\left[2a_k \cdot b_k \cdot \mathbf{M} + b_k \cdot \mathbf{B}\right] \cdot \mathbf{Y}_k + \left[\left(a_k^2 - b_k^2\right) \cdot \mathbf{M} + a_k \cdot \mathbf{B} + \mathbf{R}\right] \cdot \overline{\mathbf{Y}}_k\right\} = 0 \end{aligned} \qquad (21)$$

If we note with: $\mathbf{D}=\left(a_k^2-b_k^2\right)\cdot\mathbf{M}+a_k\cdot\mathbf{B}+\mathbf{R}$ and $\mathbf{E}=2\cdot a_k\cdot b_k\cdot\mathbf{M}+b_k\cdot\mathbf{B}$ (22)

then the equation (21) becomes: $\mathbf{D}\cdot\mathbf{Y}_k-\mathbf{E}\cdot\overline{\mathbf{Y}}_k+i\cdot\left(\mathbf{E}\cdot\mathbf{Y}_k+\mathbf{D}\cdot\overline{\mathbf{Y}}_k\right)=0+0\cdot i$ (21')

By identification we may obtain the *linear* and homogeneous system:

$$\begin{cases}\mathbf{D}\cdot\mathbf{Y}_k-\mathbf{E}\cdot\overline{\mathbf{Y}}_k=0\\ \mathbf{E}\cdot\mathbf{Y}_k+\mathbf{D}\cdot\overline{\mathbf{Y}}_k=0\end{cases} \tag{23}$$

Solving the system (23) it results the vector $\mathbf{U}_k$. If we do the same thing for λ_{n+k} and $\mathbf{U}_{n+k}$ we obtain the algebraic homogeneous system (23')

$$\begin{cases}\mathbf{D}\cdot\mathbf{Z}_k-\mathbf{E}\cdot\overline{\mathbf{Z}}_k=0\\ \mathbf{E}\cdot\mathbf{Z}_k+\mathbf{D}\cdot\overline{\mathbf{Z}}_k=0\end{cases}$$

(23')

which has the solution: $\mathbf{Z}_k=\mathbf{Y}_k;\ \overline{\mathbf{Z}}_k=-\overline{\mathbf{Y}}_k$ (24)

Hence, the eigenvectors $\mathbf{U}_k$ and $\mathbf{U}_{n+k}$, corresponding to eigenvalues λ_k and λ_{n+k}, which are complex conjugate, are complex conjugate, too.

$\mathbf{U}_k=\mathbf{Y}_k+i\cdot\overline{\mathbf{Y}}_k;\ \mathbf{U}_{n+k}=\mathbf{Y}_k-i\cdot\overline{\mathbf{Y}}_k$ with $\mathbf{k}\in\{1,2,\ldots,n\}$. (25)

If we note: $\mathbf{V}_k=\mathbf{Y}_k;\ \mathbf{V}_{n+k}=\overline{\mathbf{Y}}_k$ with $\mathbf{k}\in\{1,2,\ldots,n\}$ (26)

then the complex eigenvectors become: $\mathbf{U}_k=\mathbf{V}_k+i\cdot\mathbf{V}_{n+k};\ \mathbf{U}_{n+k}=\mathbf{V}_k-i\cdot\mathbf{V}_{n+k}$. By mean of eigenvalues λ_k, λ_{n+k} and eigenvectors $\mathbf{U}_k$ and $\mathbf{U}_{n+k}$ the solution of homogeneous equation (14) may be written, representing the free oscillation of the system:

$$\mathbf{u}^l=\sum_{k=1}^{n}e^{a_k\cdot t}\left[C_k\cdot\mathbf{V}_k\cdot\cos(b_k\cdot t)+C_{n+k}\cdot\mathbf{V}_{n+k}\cdot\sin(b_k\cdot t)\right] \tag{26'}$$

The constants C_k, C_{n+k}, $k\in\{1,2,\ldots,n\}$ are determined from the initial conditions (1').

3. PARTICULAR SOLUTION OF THE NONHOMOGENEOUS EQUATION

It should be determined the particular solution of equation (1) in the case of certain particular disturbing forces, with a large practicability.

3.1 CASE OF DISTURBING FORCES $\mathbf{F}(t) = \mathbf{F}(\alpha, \beta, m, t)$

It is considered the vector of the disturbing forces of the form:

$$\mathbf{F}(t)=\mathbf{F}(\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{m},t)=e^{\alpha t}[\mathbf{P}_m(t)\cdot\cos(\beta\cdot t)+\mathbf{Q}_m(t)\cdot\sin(\beta\cdot t)] \tag{27}$$

where:

$$\mathbf{P}_m(t)=\sum_{k=0}^{m}t^k\cdot\mathbf{G}_k;\quad \mathbf{Q}_m(t)=\sum_{k=0}^{m}t^k\cdot\mathbf{H}_k \tag{28}$$

are polynomial vectors of the maximum **m** degree and the vectors $\mathbf{G}_k$ and $\mathbf{H}_k$, $k\in\{1,2,\ldots,n\}$are given. The expression of the vector **F** given by (27) contains 8 distinct cases depending on the disturbances $\alpha,\beta\in\mathbf{R}$ and $\mathbf{m}\in\mathbf{N}$. Thus. For $\alpha = 0$, $\beta\neq0$, and $\mathbf{m} = 0$, then vector **F** is written as follows:

$$\mathbf{F}(t) = \mathbf{G}_0\cdot\cos(\beta t) + \mathbf{H}_0\sin(\beta t) \qquad (29)$$

a case very often meet in practice. The particular solution of equation (1) would have the following form according to the complex number $\lambda_0 = \alpha + i\cdot\beta$.

a) If $\lambda_0 = \alpha + i\cdot\beta$ is not the real root of the characteristic equation (16), then the particular solution of the equation (1) will have the same form as the vector of the disturbing force:

$$\mathbf{u}^f = e^{\alpha t}\left[\mathbf{P}_m^*(t)\cdot\cos(\beta t) + \mathbf{Q}_m^*(t)\cdot\sin(\beta t)\right] \qquad (30)$$

where
$$\mathbf{P}_m^*(t) = \sum_{k=0}^{m} t^k\cdot\mathbf{G}_k^*;\quad \mathbf{Q}_m^*(t) = \sum_{k=0}^{m} t^k\cdot\mathbf{H}_m^* \qquad (31)$$

are the undetermined polynomial vectors. They may be determined by substituting (30) to (1) and by identification.

b) If $\lambda_0 = \alpha + i\cdot\beta$ is a root of the characteristic equation (16) of the degree **h**, then:

$$\mathbf{u}^f = t^h e^{\alpha t}\left[\mathbf{P}_m^*(t)\cdot\cos(\beta t) + \mathbf{Q}_m^*(t)\cdot\sin(\beta t)\right] \qquad (32)$$

where polynomial vectors $\mathbf{P}_m^*(t)$ and $\mathbf{Q}_m^*(t)$ are determined analogously with the previous case.

In this case the general solution of the equation (1) so called complete dynamic response is:

$$\mathbf{u} = \mathbf{u}^l + \mathbf{u}^f \qquad (33)$$

where $\mathbf{u}^l$ is given by (26) and $\mathbf{u}^f$ is given by (30) or (32).

3,2 CASE OF DISTURBING FORCES UNDER THE FORM OF SOME LINEAR COMBINATIONS

It is considered the vector of disturbing forces of the form:

$$\mathbf{F}(t) = \sum_{j=1}^{p}\mathbf{F}(\alpha_j,\beta_j,m_j,t) \qquad (34)$$

where
$$\mathbf{F}(\alpha_j,\beta_j,m_j,t) = e^{\alpha_j t}\left[\mathbf{P}_{m_j}(t)\cdot\cos(\beta_j t) + \mathbf{Q}_{m_j}(t)\cdot\sin(\beta_j t)\right] \qquad (35)$$

In this case the particular solution is:
$$\mathbf{u}^f = \sum_{j=1}^{p}\mathbf{u}^{f,j} \qquad (36)$$

where: $\mathbf{u}^{f,j}$ is the particular solution for the problem:
$$\mathbf{M}\cdot\ddot{\mathbf{u}} + \mathbf{B}\cdot\dot{\mathbf{u}} + \mathbf{R}\cdot\mathbf{u} = \mathbf{F}(\alpha_j,\beta_j,m_j,t) \qquad (37)$$

which may be obtained with (30) or (32).

4.ALGORITHM FOR DETERMINATION OF THE GENERAL SOLUTION

To obtain the general solution of the equation (1) with the initial conditions (1') by the method of eigenvectors and eigenvalues complexly, we must perform 3 steps:

- determination of the eigenvalues of the characteristic equation (16), then determination of the eigenvectors corresponding to each eigenvalues (25). By mean of them the solution of the homogeneous equation (14) is written, given by (26) where the constants $\mathbf{C}_1$, $\mathbf{C}_2$,...., $\mathbf{C}_{2n}$ are undetermined.
- determination of the particular solution of the equation (1), when the disturbing forces of the form (27) or (34) exist, by mean of the identification method, under the form of (30) or (32).
- determination of general solution $\mathbf{u(t)} = \mathbf{u}^l(t) + \mathbf{u}^f(t)$ where the constants $\mathbf{C}_1$, $\mathbf{C}_2$,...., $\mathbf{C}_{2n}$ are determined from the initial conditions (1').

5. CONCLUSIONS

A great disadvantage of this method lies in that the equation (1) has coefficients of order 10^{10} or greater. In this case the determination of eigenvalues of the equation (16) could not be obtained with an accuracy of order 10^{-10},.....10^{-15}, and consequently the corresponding eigenvectors could not be obtained very exactly.

The eigenvalues of the equation (16) are obtained with an accuracy of 10^{-3},...,10^{-5} and consequently the great errors are introduced. It means that this method is more efficiently then the approximate method. It is simulated on the computer together the approximate method to compare the results. The dynamic response obtained by the method of eigenvectors and eigenvalues is called the real dynamic response and that obtained by the approximate method is called the approximate response.

6.EXAMPLES FOR COMPUTATION

Let be a structure characterized by matrices:

$$\mathbf{M} = \begin{pmatrix} 2000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1500 \end{pmatrix};\ \mathbf{B} = \begin{pmatrix} 50 & -50 & 0 \\ -50 & 150 & -100 \\ 0 & -100 & 150 \end{pmatrix};\ \mathbf{R} = \begin{pmatrix} 6000 & -2000 & 0 \\ -2000 & 3000 & -1000 \\ 0 & -1000 & 4000 \end{pmatrix}$$

$$\mathbf{u}_0 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};\ \mathbf{v}_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix};\ \mathbf{F}(t) = \cos(t)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \sin(t)\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Determine the dynamic response within the range (0;10s) both by the approximate method and by the eigenvalues and eigenvectores method, and call it the real dynamic response. The characteristic equation of the undamped eigenpulsations (2) is: $\lambda^6 - 8,(6)\ \lambda^4 + 22,(3)\ \lambda^2 - 16,(16) = 0$ which has roots:

$p_1 = 1,1477539$;

p_2= 1,658984; p_3= 2,1441406. The eigenvaectors are damped each other:

$$\mathbf{V}_1' = \begin{pmatrix} 1,2020 \\ 2,0239 \\ 1 \end{pmatrix};\ \mathbf{V}_2' = \begin{pmatrix} -0,5150 \\ -0,1279 \\ 1 \end{pmatrix};\ \mathbf{V}_3' = \begin{pmatrix} 1,813 \\ -2,896 \\ 1 \end{pmatrix}$$

The characteristic equation, giving the eigenpulsations of damped structure (16) is:

$$\lambda^6 + 0,275\lambda^5 + 8,68\lambda^4 + 1,358416\lambda^3 + 22,35916\lambda^2 + 1,4166\lambda + 16,(6) = 0$$

The roots of this equation are:

$$\lambda_{1,4} = -0,0108173 \pm 1,1477217\, i = a_1 \pm i\cdot b_1$$

$$\lambda_{2,5} = -0,044294 \pm 1,658334 \quad i = a_2 \pm i\cdot b_2$$

$$\lambda_{3,6} = -0,080776 \pm 2,141623 \quad i = a_3 \pm i\cdot b_3$$

The corresponding eigenvectors are:

$$\mathbf{V}_1 = \begin{pmatrix} 1,221 \\ 2,105 \\ 1 \end{pmatrix};\ \mathbf{V}_2 = \begin{pmatrix} -0,457 \\ -0,180 \\ 1 \end{pmatrix};\ \mathbf{V}_3 = \begin{pmatrix} 2,041 \\ -3,001 \\ 1 \end{pmatrix};\ \mathbf{V}_4 = \begin{pmatrix} 1,165 \\ 1,905 \\ 1 \end{pmatrix};\ \mathbf{V}_5 = \begin{pmatrix} -0,582 \\ -0,091 \\ 1 \end{pmatrix};\ \mathbf{V}_6 = \begin{pmatrix} 1,239 \\ -2,402 \\ 1 \end{pmatrix}$$

The solution of the homogeneous equation (14) is:

$$\mathbf{u}^1(t) = \sum_{k=1}^{3} e^{a_k t}\left[C_k \cdot \mathbf{V}_k \cdot \cos(b_k \cdot t) + C_{3+k} \cdot \mathbf{V}_{3+k} \cdot \sin(b_k \cdot t)\right]$$

The disturbing factor **F**(t) may be written under the form (27) where $\lambda_o = \alpha + i\beta = 0 + i$ which is not solution for the damped characteristic equations and consequently, the particular solution has the same form:

$\mathbf{u}^f(t) = \cos(\mathbf{t})\, \mathbf{G}_0 + \sin(\mathbf{t})\, \mathbf{H}_0$ where

$$\mathbf{G}_0 = \begin{pmatrix} 6,6346 \\ 8,2604 \\ 3,3228 \end{pmatrix} \cdot 10^{-4};\ \mathbf{H}_0 = \begin{pmatrix} 3,1581 \\ 6,7226 \\ 1,3785 \end{pmatrix} \cdot 10^{-5}$$

The general solution of the equation (1) with (1') with the given matrices is:

$$\mathbf{u}(t) = \mathbf{u}^1(t) = \mathbf{u}^f(t) = \sum_{k=1}^{3} e^{a_k t}\left[C_k \cdot \mathbf{V}_k \cdot \cos(b_k \cdot t) + C_{3+k} \cdot \mathbf{V}_{3+k} \cdot \sin(b_k \cdot t)\right] +$$

$$+ \cos(\mathbf{t})\, \mathbf{G}_0 + \sin(\mathbf{t})\, \mathbf{H}_0.$$

where the constants $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_6$ have the following values: $\mathbf{C}_1$ =0,436; $\mathbf{C}_2$=0,273; $\mathbf{C}_3$ =0,289; $\mathbf{C}_4$ =0,475; $\mathbf{C}_5$ = -0,343; $\mathbf{C}_6$ =0,3.

These values have been obtained after the initial conditions (1') had been stated for the general solution. Because we are interested in a comparison between the two methods, we choose an equidistant division from time range (0;10s) of the value $\Delta(t)$ = 0,2s and problem (1) solution is presented with the conditions (1') for various moments t = 0, t = 3s, t = 5,8s, t = 8,8s in the table below:

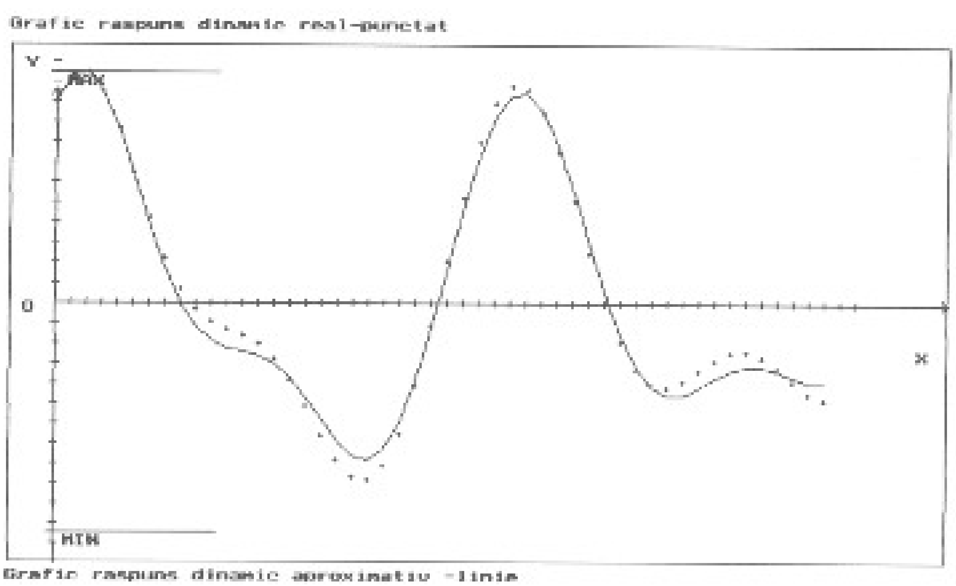
Table

The dynamic response is:

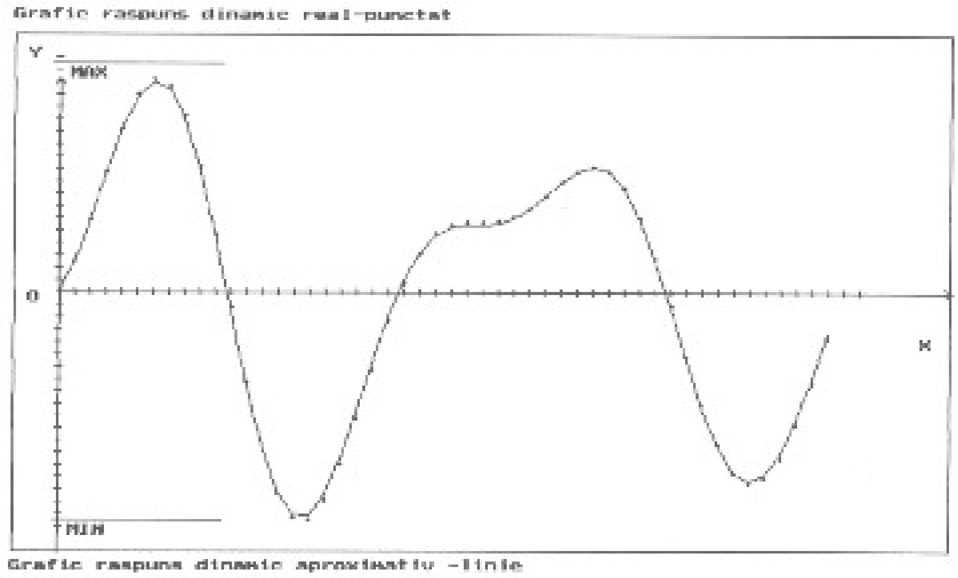
Time (s)	Approximate method (displacement mm)	Eigenvectors method (displacement mm)
t = 0	u(1) = 1	u(1) = 1
	u(2) = 0	u(2) = 0
	u(3) = 1	u(3) =1
t = 3	u(1) = -0,3744	u(1) = -0,3865
	u(2) = -1,8112	u(2) = -1,8348
	u(3) = -0,0776	u(3) = -0,0350
t = 5,8	u(1) = 1,0364	u(1) = 1.0872
	u(2) = 0,6012	u(2) = 0,6171
	u(3) = 0,6288	u(3) = 0,5580
t = 8,8	u(1) = -0,3173	u(1) = -0,2433
	u(2) = -1,5491	u(2) = -1,5647
	u(3) = -0,6323	u(3) = -0,7362

A better intuitive image is given by the plottings of the solutions for the equation (1) with conditions (1') by the both methods.

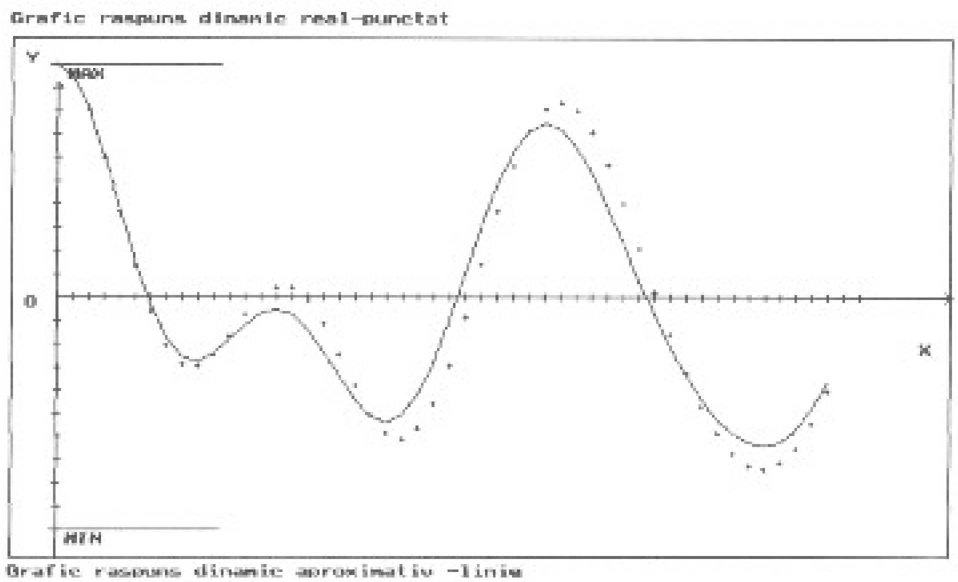
1[th] component. The maxim value for the displacement is:= 1.15136mm



2[th] component. The maxim value for the displacement is:= 1.85123mm



3^{th} component. The maxim value for the displacement is:− 1.00000mm



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