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A MODALITY OF PAYMENT FOR A GOOD WITH LONG UTILITY

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Abstract. In this paper is presented a modality of payment for a good with long utility by combination of an anuity with a demise insurance agreement and it is established for a concrete example the percentage for the calculus of the comutation numbers i.e. the sum for which the insurance agreement is perfected to coincide, at a moment of time, with the accomplished payments actualizated at the same moment of time.

There is the situations when saving a big sum of money for bying a good with long utility is an enough hard problem; so, it is interesting to establish an efficient method of payment for this kind of good. By example, a house for an young family.

Let introduce some denotices.

Let be A a lonely older person, who possess a home which value is W m.u. (monetary unities).

Let be, also, B an young family, without home, which engaged oneself to give an **initial sum** S_1 m.u. at the moment of time $t = 0$ (when the agreement is perfected) and a **rent** from S m.u. at the end of every year, during n years (an anuity whole and posticipate) to the person A, following that in the moment of the demise of the person A, the family B obtain the house.

Let be x the age of the husband from the family and let be y the age of the person A in the moment when the agreement (sale-buy) is perfected (t = 0).

At this time, the family B concludes an insurance agreement for the person A with an insurance institution.

Through this contract, the family B is obliged to pay **an installment** of P m.u. at the begining of every year from the moment t = 0 (of the concluding of the agreement), during n years if in this time both the husband of the family B and the person A are in life.

The insurance institution is obliged to pay to the family B **the sum** of S_2 m.u. in the moment of the person A demise anytime when the moment of concluding the insurance agreement.

With the other words, we can write that:

$$P\cdot {}_{/n}a_x \cdot {}_{/n}a_y = S_2\cdot A_y \tag{1}$$

if we take into consideration that the life durations for the person A and the family B are independents.

We presumed that the family B have **an anual income** of V m.u. from which it must be covered **the expense for the consumer goods** of C m.u., **the rent** of S m.u. and **the installment** of P m.u.

$$V = C + S + P \tag{2}$$

We presume that the sum S_1 m.u. (the advance) was already save up in an previous period (or, by example, the sum S_1 represent the sum which remain to the family B after the wedding).

So, we have the following intuitive interpretation on the time axis for the system of the payments which must be effect: initial sum, rent, insurance payments:

the moment of the time	0	1	2 ...	n-1	n
the sum	S_1+P	S+P	S+P ...	S+P	S

The advance (the initial sum) S_1 plus the actualized value of the anual rents (the rent) must be equal, at the moment zero, with the home value W; so we can write:

$$S_1 + S\cdot\frac{1-v^n}{i} = W \tag{3}$$

The annual unitary interest i is negociated between the both parts A and B, being equal with the mean of the devaluation of the monetary unity for n years.

The initial sum S_1 is also negociated between the both parts being a certain part from the home value W.

Actualisated, at the moment t = n, the family B paied the sum denote by S_3:

$$S_3 = S_1 \cdot u^n + P \cdot u \cdot \frac{u^n - 1}{i} + S \cdot \frac{u^n - 1}{i} \qquad (4)$$

where u = 1+i.

Let impose the condition that the sum S_2, which will be received by the family B at the demise moment of the person A, be equal with the sum S_3, gived by (4); with the other words, using the relations (1) and (4) to have:

$$P \cdot \frac{/n^{a}x \cdot /n^{a}y}{A_y} = S_1 \cdot u^n + p \cdot u \cdot \frac{u^n - 1}{i} + S \cdot \frac{u^n - 1}{i} \qquad (5)$$

or, replacing with the commutation numbers,

$$P \cdot \frac{(N_x - N_{x+n}) \cdot (N_y - N_{y+n})}{D_x \cdot M_y} = S_1 \cdot u^n + p \cdot u \cdot \frac{u^n - 1}{i} + S \cdot \frac{u^n - 1}{i} \qquad (6)$$

The percentage 100j which we calculate the commutation numbers is negociate between the family B and the institution which concludes the insurance for the person A.

We want, as follows, to determine the annual unitary interest j i.e. to take place the relation (6).

Let suppose, by an example, that:

- the advance S_1 represents 40% from the house value W, so that

$$S_1 = \frac{2}{5} W \qquad (7)$$

- the annual income V represents 10% from W and that:

C represents 60% from V

S represents 30% from V

P represents 10% from V

from where follows the relations:

$$V = \frac{1}{10} \cdot W \qquad (8)$$

$$S = \frac{3}{10} \cdot V \text{ or } S = \frac{3}{100} \cdot W \qquad (9)$$

$$P = \frac{1}{10} \cdot V \text{ or } P = \frac{1}{100} \cdot W \qquad (10)$$

$$C = \frac{6}{10} \cdot V \text{ or } C = \frac{6}{100} \cdot W \qquad (11)$$

The equivalence relation (6) can be written, after the reducing with

W, as follows:

$$\frac{(N_x-N_{x+n})\cdot(N_y-N_{y+n})}{D_x\cdot M_y} = 40\cdot u^n + (u+3)\cdot\frac{u^n-1}{i} \qquad (12)$$

Let consider the following numerical data:

n = 10 years

x = 25 years

y = 60 years

i = 0,10

j = ?

Computing the right member of the relation (12) we obtain:

$$\frac{(N_x-N_{x+n})\cdot(N_y-N_{y+n})}{D_x\cdot M_y} = 169,09314$$

We compute the left member of the relation (12) for different values of j:

- for j = 0,20

$$\frac{(N_x-N_{x+n})\cdot(N_y-N_{y+n})}{D_x\cdot M_y} = 168,40646$$

- for j = 0,25

$$\frac{(N_x-N_{x+n})\cdot(N_y-N_{y+n})}{D_x\cdot M_y} = 204,23407$$

By conclusion,

for j = 0,20 ⟶ 168,40646

for j = ? ⟶ 169,09314

for j = 0,25 ⟶ 204,23407

Using the linear interpolation we can write:

$$\frac{j-0,20}{0,25-0,20} = \frac{0,68668}{35,82761}$$

and we obtain

$$j = 0,2009583,$$

which give a percentage from 2% with which will be calculated the commutation numbers.

The conclusion is: if the family B make an isurance agreement for the person A, the family economies are moved from an period in the other period of the life, but in fact the family does not expend anything, finaly going in the possession of the home and of the sums which was paied belong the years.

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