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GENERALIZED T-RECURRENT E-CONNECTIONS

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Let A_n be a space with affine connection. In a coordinate system, we denote by Γ^i_{jk} the components of the affine connection, with $T^i_{jk} = \Gamma^i_{jk} - \Gamma^i_{jk}$ the components of the torsion tensor of connection Γ , and with $T_k = T^i_{ik}$ the components of the torsion vector.

The connection Γ is called E-connection [2] if

$$T_{i,j} = T_{j,i} \tag{1}$$

where comma denotes the covariant derivation with respect to Γ .

A space A_n is of generalized recurrent torsion (generalized T-recurrent) if

$$T^i_{jk,r} = \varphi_r T^i_{jk} + a_r Q^i_{jk} \tag{2}$$

Contracting (2) in i and j we have

$$T_{k,r} = \varphi_r T_k + a_r Q_k \tag{3}$$

where $\phi_k = \phi^i_{ik}$.

From (1) and (3) it follows:

$$\varphi_r T_k - \varphi_k T_r = a_k Q_r - a_r Q_k \tag{4}$$

therefore:

PROPOSITION 1. In a generalized T-recurrent E-connection (4) take place.

If the generalized T-recurrent A_n spaces is with semi-symmetric connection

$$T^i_{jk} = \frac{1}{n-1}(\delta^i_j T_k - \delta^i_k T_j) \tag{5}$$

relation which characterizes the semi-symmetric connections, from (3) and (5) it follows (2) with

$$Q^i_{jk} = \frac{1}{n-1}(\delta^i_j Q_k - \delta^i_k Q_j) \tag{6}$$

and therefore

PROPOSITION 2. The A_n spaces n>1 with semi-symmetric connection and the torsion vector generalized T-recurrent are generalized T-recurrent with Q^i_{jk} given by (6).

Conversely, from (2), (3), (5) it follows (6).

If the generalized T-recurrent A_n spaces are with semi-symmetric E connection, from (1), (2), (3), (4), (5) it follows

$$T^i_{jk,r} - T^i_{jr,k} = \frac{1}{n-1}[T_j(\delta^i_r \varphi_k - \delta^i_k \varphi_r) + Q_j(\delta^i_r a_k + \delta^i_k a_r)] \tag{7}$$

and therefore

PROPOSITION 3. In an A_n space with generalized T-recurrent E connection, relation (7) take place.

In an A_n space with semi-symmetric connection, we have [2]

$$T^i_{jk} T_i = 0 \tag{8}$$

and in an A_n space with generalized T-recurrent semi-symmetric connection [3] we have

$$Q^i_{jk} Q_i = 0 \tag{9}$$

Derivating covariantly (8) and taking (2) and (3) into account [3] we have

$$Q^i_{jk} T_i + T^i_{jk} Q_i = 0 \tag{10}$$

Derivating covariantly (10) and taking (2), (3), (9) and (10) into account we have

$$Q^i_{jk,r} T_i + T^i_{jk} Q_{i,r} = 0 \tag{11}$$

and therefore

PROPOSITION 4. In a generalized T-recurrent A_n space, n>1, with semi-symmetric connection, relation (11) take place.

If the A_n space is with generalized T-recurrent semi-symmetric E-connection, from (10) and (11) it follows

$$Q^i_{jk}T_{r,i} + T^i_{jk,r}Q_i = 0 \quad (12)$$

and we have

PROPOSITION 5. In a generalized T-recurrent A_n space n>1, with semi-symmetric E connection, relation (12) hold good.

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