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A relation of Knopoff- De Hoop type in Thermoelasticity of dipolar bodies with voids

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Abstract. The aim of our study is to derive a relation of Knopoff-De Hoop type for displacement vector fields within context of thermoelastic dipolar bodies with voids. Then, as a consequnece, an explicit expression of the body loadings equivalent to the dislocation, is obtained.

1. Introduction

The theory of thermo-microstretch elastic solids was first elaborated by Eringen, [4] and, in short, this is a theory of elasticity with microstructure that include intrinsic rotations and microstructural expansions and contractions.

This theory can be useful in the applications which deal with porous materials as geological materials, solid packed granular materials and many others.

On the other hand, materials which operate at elevated temperatures will invariably be subjected to heat flow at some time during normal use. Such heat flow will involve a non-linear temperature distribution which will inevitably give rise to thermal stresses. For these reasons, the development, design and selection of materails for high temperature applications requires a great deal of care. The role of the pertinent material properties and other variables which can affect the magnitude of thermal stress must be considered.

The present paper must be considered as a first step to a better understanding of microstretch and thermal stress in the study of materials.

The reciprocity and representation relations that appear in our study constitute a powerful theoretical tools in the assessment of the theory of seismic-source mechanism, in the studies connected with seismic wave propagation.

Also, we think that this paper is a first step to understanding the application of microstretch mechanism to earthquate problems.

2. Basic equations

For convenience the notations and therminology chosen are almost identical to those of [6], [7]. Our paper is concerned with an anisotropic and homogeneous material.

Let the body occupy, at time $t = 0$, a properly regular region B of the Euclidian three-dimensional space, bounded by the piece-wise smooth surface ∂B. We refer the motion of the body to a fix system of rectangular Cartesian axes $Ox_i, i = 1, 2, 3$. Throughout this work the Einstein summation convention over repeated indices. The subcript j after a comma indicates partial differentiation with respect to the spatial argument x_j. All Latin subscripts are understood to range over the integers $(1, 2, 3)$, while the Greek indices have the range $1, 2$. A superposed dot denotes the derivative with respect to the t-time variable.

For clarity in presentation, the regularity hypothesis on the considered function will be omitted and, also, the spatial argument and the time argument of a function will be omitted when there is no likehood of confusion.

On these grounds, the field equations in the dynamic theory of thermoelasticity of dipolar bodies with voids, are, (see also [6], [7])

- the equations of motion

$$\begin{aligned} (\tau_{ij} + \eta_{ij})_{,j} + \varrho F_i &= \varrho \ddot{u}_i, \\ \mu_{ijk,i} + \eta_{jk} + \varrho G_{jk} &= I_{kr} \ddot{\varphi}_{jr}; \end{aligned} \tag{1}$$

- the balance of the equilibrated forces

$$\lambda_{i,i} - s + \varrho L = \varrho \kappa \ddot{\sigma}; \tag{2}$$

- the energy equation

$$\varrho T_0 \dot{\eta} = q_{i,i} + \varrho r. \tag{3}$$

- the constitutive equations

$$\begin{aligned} \tau_{ij} &= C_{ijmn} \varepsilon_{mn} + G_{mnij} \gamma_{mn} + F_{mnrij} \chi_{mnr} + a_{ij} \sigma + d_{ijk} \sigma_{,k} - \alpha_{ij} \theta, \\ \eta_{ij} &= G_{ijmn} \varepsilon_{mn} + B_{ijmn} \gamma_{mn} + D_{ijmnr} \chi_{mnr} + b_{ij} \sigma + e_{ijk} \sigma_{,k} - \beta_{ij} \theta, \\ \mu_{ijk} &= F_{ijkmn} \varepsilon_{mn} + D_{mnijk} \gamma_{mn} + A_{ijkmnr} \chi_{mnr} + c_{ijk} \sigma + f_{ijkm} \sigma_{,m} - \omega_{ijk} \theta, \\ h_i &= d_{mni} \varepsilon_{mn} + e_{mni} \gamma_{mn} + f_{mnri} \chi_{mnr} + d_i \sigma - a_i \theta + P_{ij} \sigma_{,j}, \\ s &= -a_{ij} \varepsilon_{ij} - b_{ij} \gamma_{ij} - c_{ijk} \chi_{ijk} - \xi \sigma - d_i \sigma_{,i} + m\theta, \\ S &= S_0 + \alpha_{ij} \varepsilon_{ij} + \beta_{ij} \gamma_{ij} + \omega_{ijk} \chi_{ijk} + m\sigma + a_i \sigma_{,i} + a\theta, \\ q_i &= k_{ij} \theta_{,j}; \end{aligned} \tag{4}$$

- the kinetic relations

$$\begin{aligned} \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \gamma_{ij} = u_{j,i} - \varphi_{ij}, \\ \chi_{ijk} = \varphi_{jk,i}, \quad \theta = T - T_0, \quad \sigma = \varphi - \varphi_0. \end{aligned} \tag{5}$$

In the above equations we have used the following notations: ϱ-the constant mass density; S-the specific entropy; T_0-the constant absolute temperature of the body in its reference state; I_{ij}-coefficients of microinertia; κ-the equilibrated inertia; u_i-the components of displacement vector; φ_{jk}-the components of dipolar displacement tensor; φ-the volume distribution function which in the reference state is φ_0; σ-the change in volume fraction measured from the reference state; θ-the temperature variation measured from the reference temperature T_0; ε_{ij}, γ_{ij}, χ_{ijk}-kinematic characteristics of the strain; τ_{ij}, η_{ij}, μ_{ijk}-the components of the stress tensors; λ_i-the components of the equlibrated stress vector; q_i-the components of the heat flux vector; F_i-the components of the body forces; G_{jk}-the components of the dipolar body forces; r-the heat supply per unit time; s-the intrinsic equilibrated force; L-the extrinsic equilibrated body force; C_{ijmn}, $B_{ijmn}, \dots$, k_{ij}-the characteristic functions of the material, and they obey the symmetry relations

$$\begin{gathered} C_{ijmn} = C_{mnij} = C_{jimn}, \quad B_{ijmn} = B_{mnij}, \\ A_{ijkmnr} = A_{mnrijk}, \quad F_{ijkmn} = F_{ijknm}, \quad G_{ijmn} = G_{ijnm}, \\ a_{ij} = a_{ji}, \quad d_{ijk} = d_{jik}, \quad P_{ij} = P_{ji}, \quad k_{ij} = k_{ji}. \end{gathered} \tag{6}$$

The entropy inequality implies

$$k_{ij}\theta_{,i}\theta_{,j} \geq 0. \tag{7}$$

To the system of field equations (1)-(5) we adjoin the following initial conditions

$$\begin{gathered} u_i(x,0) = u_i^0(x), \quad \dot{u}_i(x,0) = u_i^1(x), \\ \varphi_{jk}(x,0) = \varphi_{jk}^0(x), \quad \dot{\varphi}_{jk}(x,0) = \varphi_{jk}^1(x), \\ \theta(x,0) = \theta^0(x), \quad \sigma(x,0) = \sigma^0(x), \quad \dot{\sigma}(x,0) = \sigma^1(x), \quad x \in \bar{B}, \end{gathered} \tag{8}$$

and the following prescribed boundary conditions

$$\begin{gathered} u_i = \bar{u}_i \text{ on } \partial B_1 \times [0,t_0), \ (\tau_{ij} + \eta_{ij})n_j = \bar{t}_i \text{ on } \partial B_1^c \times [0,t_0), \\ \varphi_{jk} = \bar{\varphi}_{jk} \text{ on } \partial B_2 \times [0,t_0), \ \mu_{ijk}n_i = \bar{\mu}_{jk} \text{ on } \partial B_2^c \times [0,t_0), \\ \sigma = \bar{\sigma} \text{ on } \partial B_3 \times [0,t_0), \ \ \lambda_i n_i = \bar{h} \text{ on } \partial B_3^c \times [0,t_0), \\ \theta = \bar{\theta} \text{ on } \partial B_4 \times [0,t_0), \ \ q_i n_i = \bar{q} \text{ on } \partial B_4^c \times [0,t_0), \end{gathered} \tag{9}$$

where ∂B_1, ∂B_2, ∂B_3 and ∂B_4 with respective complements ∂B_1^c, ∂B_2^c, ∂B_3^c and ∂B_4^c are subsets of ∂B, such that

$$\begin{gathered} \partial B_1 \cup \partial B_1^c = \partial B_2 \cup \partial B_2^c = \partial B_3 \cup \partial B_3^c = \partial B_4 \cup \partial B_4^c = \partial B, \\ \partial B_1 \cap \partial B_1^c = \partial B_2 \cap \partial B_2^c = \partial B_3 \cap \partial B_3^c = \partial B_4 \cap \partial B_4^c = \emptyset, \end{gathered}$$

n_i are the components of the unit outward normal to ∂B, t_0 is some instant that may be infinite, u_i^0, u_i^1, φ_{jk}^0, φ_{jk}^1, θ^0, σ^0, σ^1, $\bar{u}_i$, $\bar{t}_i$, $\bar{\varphi}_{jk}$, $\bar{\mu}_{jk}$, $\bar{\sigma}$, $\bar{\theta}$, $\bar{h}$ and $\bar{q}$ are prescribed functions in their domains.

By a solution of the mixed initial boundary value problem of the theory of thermoelasticity of bodies with voids in the cylinder $\Omega_0 = B \times [0,t_0)$ we mean an ordered array $(u_i, \ \varphi_{jk}, \ \sigma, \ \theta)$ which satisfies the equations (1), (2) and (3) for all $(x,t) \in \Omega_0$, the boundary conditions (9) and the intial conditions (8).

3. Main results

Let u and v be functions defined on $\bar{B}\times[0,\infty)$, and continuous on $[0,\infty)$ with respect to the t for each $x\in\bar{B}$. We denote by $u*v$ the convolution of u and v, that is

$$[u*v](x,t)=\int_0^t u(x,t-\tau)v(x,\tau)d\tau. \tag{10}$$

Let us introduce the notations

$$\begin{aligned} g(t)=t,\ h(t)=1,\\ f_i=\varrho g*F_i+\varrho\left[tu_i^1(x)+u_i^0(x)\right],\\ g_{jk}=\varrho g*G_{jk}+I_{kr}\left[t\varphi_{jr}^1(x)+\varphi_{jr}^0(x)\right],\\ l=\varrho g*L+\varrho\kappa\left[t\sigma^1(x)+\sigma^0(x)\right],\\ w=\varrho h*r+\varrho T_0S_0. \end{aligned} \tag{11}$$

Following the same procedure used by Iesan in [5], it is easy to prove the folowing result, that enables us to give an alternative formulation of the initial-boundary value problem in which the initial data are incorporated into the field of equations.

Theorem 1. *The functions* , u_i, φ_{jk}, σ, θ, τ_{ij}, η_{ij}, μ_{ijk} and q_i *satisfy the equations* (1), (2), (3) *and the initial conditions* (8) *if and only if*

$$\begin{aligned} g*(\tau_{ji}+\eta_{ji})_{,j}+f_i=\varrho u_i,\\ g*(\mu_{ijk,i}+\eta_{jk})+g_{jk}=I_{kr}\varphi_{jr},\\ g*(\lambda_{i,i}-s)+l=\varrho\kappa\sigma,\\ h*q_{i,i}+w=\varrho T_0S. \end{aligned} \tag{12}$$

In our following estimations, we use the above formulation of the problem. We wish to find the behavior of the considered medium when embedded in B there is a discontinuity surface Σ for the displacements, the dipolar displacements, the stretch and the temperature. The sides of Σ are denoted Σ^- and Σ^+. Let ν_i be the components of the unit normal vector of Σ, directed from (-) to (+) side. Then on Σ we have the conditions

$$\begin{aligned} u_i^+-u_i^-=U_i,\ \ (\tau_{ji}^++\eta_{ji}^+)\nu_j=(\tau_{ji}^-+\eta_{ji}^-)\nu_j,\\ \varphi_{jk}^+-\varphi_{jk}^-=\Phi_{jk},\ \ \mu_{ijk}^+\nu_k=\mu_{ijk}^-\nu_k,\\ \sigma^+-\sigma^-=\Psi,\ \ \lambda_j^+\nu_j=\lambda_j^-\nu_j,\\ \theta^+-\theta^-=\Theta,\ \ q_j^+\nu_j=q_j^-\nu_j, \end{aligned} \tag{13}$$

where f^+ and f^- are the limits of $f(x)$ as x approaches a point on the side (+) or (-) of Σ, respectively, and U_i, Φ_{ij}, Ψ and Θ are prescribed functions. In this way, we consider the equations (12) in $B\setminus\Sigma$.

Let us consider two different systems of loadings

$$G^{(\alpha)}=\left\{F_i^{(\alpha)},\ G_{jk}^{(\alpha)},\ L^{(\alpha)},\ r^{(\alpha)},\ \bar{u}_i^{(\alpha)},\ \bar{\varphi}_{ij}^{(\alpha)},\right.$$

$$\left.\bar{\sigma}^{(\alpha)},\ \bar{\theta}^{(\alpha)},\ \bar{t}_i^{(\alpha)},\ \bar{\mu}_{jk}^{(\alpha)},\ \bar{h}^{(\alpha)},\ \bar{q}^{(\alpha)},\ U_i,\ \Phi_{ij},\ \Psi,\ \Theta\right\},\quad \alpha=1,2,$$

for the body, and the two corresponding solutions

$$S^{(\alpha)} = \Big\{u_i^{(\alpha)},\ \varphi_{jk}^{(\alpha)},\ \sigma^{(\alpha)},\ \theta^{(\alpha)},\ \varepsilon_{ij}^{(\alpha)},\ \gamma_{ij}^{(\alpha)},$$
$$\tau_{ij}^{(\alpha)},\ \eta_{ij}^{(\alpha)},\ \chi_{ijk}^{(\alpha)},\ \lambda_i^{(\alpha)},\ s^{(\alpha)},\ q_i^{(\alpha)}\Big\},\quad \alpha = 1, 2.$$

By using the notations

$$t_i = (\tau_{ij} + \eta_{ij})n_j,\ \ T_i = \left(\tau_{ij}^{(+)} + \eta_{ij}^{(+)}\right)\nu_j,$$
$$\mu_{jk} = \mu_{ijk}n_i,\ \ M_{jk} = \mu_{ijk}^{(+)}\nu_i,$$
$$\lambda = \lambda_j n_j,\ \ \Lambda = \lambda_j^{(+)}\nu_j,$$
$$q = q_j n_j,\ \ Q = q_j^{(+)}\nu_j. \tag{14}$$

In the next theorem we prove a reciprocity relation of Betti type.

Theorem 2. *If a dipolar thermoelastic body with voids is subjected to two systems of loadings $G^{(\alpha)}$ then between the corresponding solutions $S^{(\alpha)}$ there is the following reciprocity relation*

$$\int_B \left(f_i^{(1)} * u_i^{(2)} + g_{jk}^{(1)} * \varphi_{jk}^{(2)} + l^{(1)} * \sigma^{(2)} - \frac{1}{T_0} g * w^{(1)} * \theta^{(2)}\right) dV +$$
$$+ \int_{\partial B} g * \left(t_i^{(1)} * u_i^{(2)} + \mu_{jk}^{(1)} * \varphi_{jk}^{(2)} + \lambda^{(1)} * \sigma^{(2)} - \frac{1}{T_0} h * q^{(1)} * \theta^{(2)}\right) dA -$$
$$- \int_\Sigma g * \left(T_i^{(1)} * U_i^{(2)} + M_{jk}^{(1)} * \Phi_{jk}^{(2)} + \Lambda^{(1)} * \Psi^{(2)} - \frac{1}{T_0} h * Q^{(1)} * \Theta^{(2)}\right) dA =$$
$$= \int_B \left(f_i^{(2)} * u_i^{(1)} + g_{jk}^{(2)} * \varphi_{jk}^{(1)} + l^{(2)} * \sigma^{(1)} - \frac{1}{T_0} g * w^{(2)} * \theta^{(1)}\right) dV +$$
$$+ \int_{\partial B} g * \left(t_i^{(2)} * u_i^{(1)} + \mu_{jk}^{(2)} * \varphi_{jk}^{(1)} + \lambda^{(2)} * \sigma^{(1)} - \frac{1}{T_0} h * q^{(2)} * \theta^{(1)}\right) dA -$$
$$- \int_\Sigma g * \left(T_i^{(2)} * U_i^{(1)} + M_{jk}^{(2)} * \Phi_{jk}^{(1)} + \Lambda^{(2)} * \Psi^{(1)} - \frac{1}{T_0} h * Q^{(2)} * \Theta^{(1)}\right) dA \tag{15}$$

Proof. In view of symmetry relations (6) and with the aid of the constitutive relations (4), we obtain

$$\tau_{ij}^{(1)} * \varepsilon_{ij}^{(2)} + \eta_{ij}^{(1)} * \gamma_{ij}^{(2)} + \mu_{ijk}^{(1)} * \chi_{ijk}^{(2)} + \lambda_i^{(1)} * \sigma_{,i}^{(2)} + s^{(1)} * \sigma^{(2)} - \varrho\theta^{(1)} * S^{(2)} =$$
$$= \tau_{ij}^{(2)} * \varepsilon_{ij}^{(1)} + \eta_{ij}^{(2)} * \gamma_{ij}^{(1)} + \mu_{ijk}^{(2)} * \chi_{ijk}^{(1)} + \lambda_i^{(2)} * \sigma_{,i}^{(1)} + s^{(2)} * \sigma^{(1)} - \varrho\theta^{(2)} * S^{(1)} \tag{16}$$

Based on the identity (16), it is easy to see that

$$I_{\alpha\beta} = I_{\beta\alpha}, \tag{17}$$

where we have used the notation

$$I_{\alpha\beta} = \int_B g * \Big[\tau_{ij}^{(\alpha)} * \varepsilon_{ij}^{(\beta)} + \eta_{ij}^{(\alpha)} * \gamma_{ij}^{(\beta)} + \mu_{ijk}^{(\alpha)} * \chi_{ijk}^{(\beta)} +$$
$$+ \lambda_i^{(\alpha)} * \sigma_{,i}^{(\beta)} + s^{(\alpha)} * \sigma^{(\beta)} - \varrho\theta^{(\alpha)} * S^{(\beta)}\Big] dV$$

With the aid of the equations of motion and the equations (12), we can write

$$
\begin{aligned}
g * \Big[\tau_{ij}^{(\alpha)} * \varepsilon_{ij}^{(\beta)} + \eta_{ij}^{(\alpha)} * \gamma_{ij}^{(\beta)} + \mu_{ijk}^{(\alpha)} * \chi_{ijk}^{(\beta)} + \\
+\lambda_i^{(\alpha)} * \sigma_{,i}^{(\beta)} + s^{(\alpha)} * \sigma^{(\beta)} - \varrho\theta^{(\alpha)} * S^{(\beta)}\Big] = \\
= g * \Big[\left(\tau_{ji}^{(\alpha)} + \eta_{ji}^{(\alpha)}\right) * u_j^{(\beta)} + \mu_{ijk}^{(\alpha)} * \varphi_{jk}^{(\beta)} + \\
+\lambda_i^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} h * q_i^{(\alpha)} * \theta^{(\beta)}\Big]_{,i} + \\
+f_i^{(\alpha)} * u_i^{(\beta)} + g_{jk}^{(\alpha)} * \varphi_{jk}^{(\beta)} + l^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} g * w^{(\alpha)} * \theta^{(\beta)} - \\
-\Big[\varrho u_i^{(\alpha)} * u_i^{(\beta)} + I_{kr}\varphi_{jr}^{(\alpha)} * \varphi_{jk}^{(\beta)} + \varrho\kappa\sigma^{(\alpha)} * \sigma^{(\beta)}\Big] + \\
+\frac{1}{T_0} g * h * k_{ij}\theta_{,i}^{(\alpha)} * \theta_{,j}^{(\beta)}.
\end{aligned}
\tag{18}
$$

By integrating in (18) and using the divergence theorem, it results

$$
\begin{aligned}
I_{\alpha\beta} = \int_B \left(f_i^{(\alpha)} * u_i^{(\beta)} + g_{jk}^{(\alpha)} * \varphi_{jk}^{(\beta)} + l^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} g * w^{(\alpha)} * \theta^{(\beta)}\right) dV + \\
+ \int_{\partial B} g * \left(t_i^{(\alpha)} * u_i^{(\beta)} + \mu_{jk}^{(\alpha)} * \varphi_{jk}^{(\beta)} + \lambda^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} h * q^{(\alpha)} * \theta^{(\beta)}\right) dA - \\
- \int_\Sigma g * \left(T_i^{(\alpha)} * U_i^{(\beta)} + M_{jk}^{(\alpha)} * \Phi_{jk}^{(\beta)} + \Lambda^{(\alpha)} * \Psi^{(\beta)} - \frac{1}{T_0} h * Q^{(\alpha)} * \Theta^{(\beta)}\right) dA
\end{aligned}
\tag{19}
$$

Finally, by using (19) into (17) we arrive at the desired result (15). It is easy to see that in the absence of the discontinuities we obtain the generalization to the thermoealsticity of dipolar bodies with voids of the previous results established in classical thermoelastodynamics.

Based on the relation (15), we now calculate the thermomechanical body loadings equivalent to a given dislocation. To this aim, we assume that $u_i^{(2)}$, $\varphi_{jk}^{(2)}$, $\sigma^{(2)}$ and $\theta^{(2)}$ are $C^\infty(B \times [0,\infty))$ functions of $(\mathbf{x}, t)$, $(\mathbf{x} = (x_i))$. When the $u_i^{(2)}$, $\varphi_{jk}^{(2)}$, $\sigma^{(2)}$ and $\theta^{(2)}$ are given then by means of equations (12) the $F_i^{(2)}$, $G_{jk}^{(2)}$, $L^{(2)}$ and $r^{(2)}$ can be calculated. We restricte our considerations in the case when $U_i^{(2)} = \Phi_{jk}^{(2)} = \Psi^{(2)} = \Theta^{(2)}$ and $S^{(1)}$ corresponds to the faulted medium. Then, in the case of identical boundary conditions, we obtain

$$
\begin{aligned}
\int_B \varrho \left(F_i^{(1)} * u_i^{(2)} + G_{jk}^{(1)} * \varphi_{jk}^{(2)} + L^{(1)} * \sigma^{(2)} - \frac{1}{T_0} h * r^{(1)} * \theta^{(2)}\right) dV = \\
= \int_B \varrho \left(F_i^{(2)} * u_i^{(1)} + G_{jk}^{(2)} * \varphi_{jk}^{(1)} + L^{(2)} * \sigma^{(1)} - \frac{1}{T_0} h * r^{(2)} * \theta^{(1)}\right) dV - \\
- \int_\Sigma g * \left(T_i^{(2)} * U_i^{(1)} + M_{jk}^{(2)} * \Phi_{jk}^{(1)} + \Lambda^{(2)} * \Psi^{(1)} - \frac{1}{T_0} h * Q^{(2)} * \Theta^{(1)}\right) dA
\end{aligned}
\tag{20}
$$

In view of (14), we have

$$
\begin{aligned}
T_i^{(2)} &= [(C_{ijmn}+G_{ijmn})e_{mn}+(G_{mnij}+B_{ijmn})\gamma_{mn}+ \\
&+(F_{mnrij}+D_{ijmnr})\chi_{mnr}+(a_{ij}+b_{ij})\sigma+(d_{ijk}+e_{ijk})\sigma_{,k}-(\alpha_{ij}+\beta_{ij})\theta]\nu_j, \\
M_{jk}^{(2)} &= [F_{ijkmn}e_{mn}+D_{mnijk}\gamma_{mn}+A_{ijkmnr}\chi_{mnr}+c_{ijk}\sigma+f_{mijk}\sigma_{,m}-\omega_{ijk}\theta]\nu_i, \\
\Lambda^{(2)} &= [d_{mni}e_{mn}+e_{mni}\gamma_{mn}+f_{mnri}\chi_{mnr}+d_i\sigma-a_i\theta+P_{ij}\sigma_{,j}]\nu_i, \\
\Theta^{(2)} &= k_{ij}\theta_{,i}\nu_j;
\end{aligned}
$$

Taking into account the definition of the Dirac measure, δ, we can prove the relations of the following type

$$
\begin{aligned}
\psi_i(\xi,t) &= \int_B \psi_i(\mathbf{x},t)\delta(\mathbf{x}-\xi)dV, \\
\psi_{i,j}(\xi,t) &= -\int_B \psi_i(\mathbf{x},t)\delta_{,j}(\mathbf{x}-\xi)dV,
\end{aligned}
\tag{21}
$$

and then the relation (20) can be rewritten as follows

$$
\begin{aligned}
&\int_B \varrho\left[\left(F_i^{(1)}+\mathcal{F}_i\right)*u_i^{(2)}+\left(G_{jk}^{(1)}+\mathcal{G}_{jk}\right)*\varphi_{jk}^{(2)}+\right. \\
&\left.+\left(L^{(1)}+\mathcal{L}\right)*\sigma^{(2)}-\frac{1}{T_0}h*\left(r^{(1)}+\mathcal{R}\right)*\theta^{(2)}\right]dV= \\
&=\int_B \varrho\left(F_i^{(2)}*u_i^{(1)}+G_{jk}^{(2)}*\varphi_{jk}^{(1)}+L^{(2)}*\sigma^{(1)}-\frac{1}{T_0}h*r^{(2)}*\theta^{(1)}\right)dV.
\end{aligned}
\tag{22}
$$

In the above relation we have used the notations

$$
\begin{aligned}
\mathcal{F}_k &= -\frac{1}{\varrho}\int_\Sigma\Big[\left(C_{jirk}+2G_{jirk}+B_{jirk}\right)U_i^{(1)}+ \\
&+\left(F_{jimrk}+D_{rkjim}\right)\Phi_{im}^{(1)}+\left(d_{rji}+e_{rji}\right)\Psi_i^{(1)}\Big]\delta_{,r}(\mathbf{x}-\xi)\nu_j dA_\xi, \\
\mathcal{G}_{lk} &= -\frac{1}{\varrho}\int_\Sigma\Big\{\left[\left(G_{lkji}+B_{jilk}\right)\delta(\mathbf{x}-\xi)-\left(F_{rjilk}+D_{rlkji}\right)\delta_{,r}(\mathbf{x}-\xi)\right]U_i^{(1)}+ \\
&+\left[D_{rjilk}\delta(\mathbf{x}-\xi)-A_{rjimlk}\delta_{,r}(\mathbf{x}-\xi)\right]\Phi_{im}^{(1)}+ \\
&+\left[e_{jlk}\delta(\mathbf{x}-\xi)-f_{rjlk}\delta_{,r}(\mathbf{x}-\xi)\right]\Psi^{(1)}\Big\}\nu_j dA_\xi, \\
\mathcal{L} &= -\frac{1}{\varrho}\int_\Sigma\Big\{\left[\left(a_{ji}+b_{ji}\right)\delta(\mathbf{x}-\xi)-\left(d_{rji}+e_{rji}\right)\delta_{,r}(\mathbf{x}-\xi)\right]U_i^{(1)}+ \\
&+\left[c_{jim}\delta(\mathbf{x}-\xi)-f_{rjim}\delta_{,r}(\mathbf{x}-\xi)\right]\Phi_{im}^{(1)}+ \\
&+\left[d_j\delta(\mathbf{x}-\xi)-P_{rj}\delta_{,r}(\mathbf{x}-\xi)\right]\Psi^{(1)}\Big\}\nu_j dA_\xi, \\
\mathcal{R} &= \frac{1}{\varrho}\int_\Sigma\Big\{T_0\left[\left(\alpha_{ji}+\beta_{ji}\right)\dot{U}_i^{(1)}+\omega_{jim}\dot{\Phi}_{im}^{(1)}+a_j\dot{\Psi}^{(1)}\right]\delta(\mathbf{x}-\xi)- \\
&-k_{jr}\Theta^{(1)}\delta_{,r}(\mathbf{x}-\xi)\Big\}\nu_j dA_\xi.
\end{aligned}
$$

In view of (22) we deduce that the effect of the discontinuities across Σ can be represented by extra external body loads and heat supply. Although these are supposed to act in an unfaulted medium and cannot in any sense represent real forces acting in the real medium, they may nevertheless provide, as pointed out in [1] and [3], a useful theoretical tool and this because if two dislocations have the same equivalent force, they also emit the same radiation.

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