Buletinul Ştiinţific al Universităţii din Baia Mare
Seria B, Matematică-Informatică, vol.XIII (1997), 39-44

ON THE DYNAMICS OF A SPACECRAFT IN THE PHOTOGRAVITATIONAL FIELD OF THE SUN

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Abstract. The idea that the light pressure should influence the motion of illuminated particles has been known since J. Kepler (1571-1630); then, this idea explained the fact that the comets' tails were oriented in an opposite direction to the direction of the Sun. In the early twenties, P.N.Lebedev (1866-1921), E.F.Nichols (1869-1924) and A.W.Hull (1880-1966) found that the pressure of the light on a reflecting surface of 1 km^2 was about $8 \cdot 10^{-6}$ N/m^2, [3]. This pressure affects the orbital behaviour of artificial and natural celestial bodies with a relatively large area-to-mass ratio.

The aim of this paper is to study such a perturbed motion for a spacecraft moving under the gravitational attraction and light pressure of the Sun. A simulation of a heliocentric trajectory is made using the photogravitational model of C. Popovici [5].

1. It is well-known that in the classical two-body problem the relative motion of one body takes place in a plane and its differential equations are:

$$(1) \qquad \frac{d^2\vec{r}}{dt^2} + \frac{\mu}{r^3}\vec{r} = 0$$

where $\mu = G(M + m)$ is the gravitational parameter. In this paper we consider a perturbed two-body problem, governed by the following equations:

(2) $$\frac{d^2\bar{r}}{dt^2} + \frac{\mu}{r^3}\bar{r} = \bar{F},$$

where $\bar{F}$ is the perturbed force, the explicit form of $\bar{F}$ will be given in the next section.

According to the planar character of the motion, different kinds of coordinates can be used. For example, if $\bar{F}$ is a radial force (thus the motion is planar), we rewrite (2) in polar coordinates (r, u):

(3) $$\begin{cases} \frac{d^2r}{dt^2} - r\left(\frac{du}{dt}\right)^2 = F_r \\ \frac{1}{r}\frac{d}{dt}\left(r^2\frac{du}{dt}\right) = F_n \end{cases}$$

(note that in the case of a central force, $F_n = 0$).

2. Among the problems that lead to differential equations of the type shown above, we have chosen to deal with the motion of a spacecraft moving under the gravitational attraction and light pressure of the Sun. Outstanding results on this problem were obtained by C. Popovici (1878-1954), professor at the Iassy University, Romania, [1], [4].

C. Popovici studies (3) by taking:

(4) $$\bar{F} = -\frac{\mu}{r^2}(1 - \varepsilon\dot{r})\frac{\bar{r}}{r}$$

where $\varepsilon > 0$. His theory can be considered a post-newtonian theory because at that time it was meant to be "Newton's law rectified by a repulsive force". Taking the pressure of the light for the repulsive force, C. Popovici applied his theoretical studies to the motion of comets. But his results could become of present interest in the case of navigation with "solar sails".

In the paper [2], the equations of motion in the case of the corrected Newtonian force given by Popovici have the following cartesian form:

$$(5)\qquad \frac{d^2x}{dt^2} = -\left(k + l\frac{dr}{dt}\right)\frac{x}{r^3}, \qquad \frac{d^2y}{dt^2} = -\left(k + l\frac{dr}{dt}\right)\frac{y}{r^3}$$

where $k = A - R$, $l = R/c > 0$ and $r^2 = x^2 + y^2$, A representing the Newtonian attraction of the central body (the Sun, for example) and R being the force due to the light pressure; both forces are estimated at unit distance.

The attraction of the illuminated body is given then by

$$(6)\qquad F = -\frac{k}{r^2},$$

where $k = A - R$. This force will be a repulsive one for bodies like "solar sails" or particles of a comet's tail; this happens because the force due to the light pressure exceeds the gravitational attraction.

The modification operated by C. Popovici on (6) consists in adding the term $-R\frac{dr}{dt}(cr^2)^{-1}$, representing the force due to the finite light speed. Here c denotes the light speed and $\frac{dr}{dt}$ radial component of the speed of the attracted body. The force considered by C. Popovici will be then:

$$\varphi = -\frac{A}{r^2} + \frac{R}{r^2} - \frac{R\dot r}{cr^2}.$$

Denoting $\varepsilon = \frac{R}{kc}$, $k = A - R$, $k \neq 0$, one can easily find the expression (4) of the force.

3. Numerical integration of the equations of motion.

For the integration of (5) there is an analytical solution, presented in [2] and [5]. Still, in order to obtain quicker other needed quantities (*e.g.* variation of orbital energy, components of the velocity, etc.) and for the numerical simulations we used in this paper a numerical algorithm.

We rewrite (5) under the following standard form:

$$(7)\qquad \begin{cases} \dfrac{dy_1}{dt} = -\left(k + l\dfrac{dr}{dt}\right)\dfrac{y_3}{r^3} \\ \dfrac{dy_2}{dt} = -\left(k + l\dfrac{dr}{dt}\right)\dfrac{y_4}{r^3} \\ \dfrac{dy_3}{dt} = y_1 \\ \dfrac{dy_4}{dt} = y_2 \end{cases}$$

with

$$(8)\qquad y_1 = \frac{dx}{dt},\ y_2 = \frac{dy}{dt},\ y_3 = x,\ y_4 = y$$

Having the initial conditions (Cauchy problem):

$$(9)\qquad y_1(0) = \left(\frac{dx}{dt}\right)_0,\ y_2(0) = \left(\frac{dy}{dt}\right)_0,\ y_3(0) = x_0,\ y_4(0) = y_0$$

we can obtain a numerical solution of (5).

We also note that the system (7) admits the area's first integral:

$$(10)\qquad y_1\ y_4 - y_2\ y_3 = const.$$

The variation of the total orbital energy obeys the following, the law:

$$(11)\qquad \frac{dE}{dt} = -l\left(\frac{r}{r}\right)^2$$

where

$$E = \frac{v^2}{2} - \frac{k}{r}$$

(12) $$r^2 = y_3^2 + y_4^2$$

$$v^2 = y_1^2 + y_2^2$$

$$\dot{r} = \frac{y_1 y_3 + y_2 y_4}{r}.$$

4. Solar sails in the photogravitational field of the Sun.

In order to investigate the movement of a "solar sail" in the photogravitational field of the Sun, we need the following constants:

$c = 3 \cdot 10^8\ m/s$ (light speed)

$$A = G(M_\odot + m) \cong GM_\odot = 1.32712438 \cdot 10^{20}\ m^3\ s^{-2} =$$

$$= 2.95915418 \cdot 10^{-4}\ (a.u.)^3\ day^{-2}$$

$L_\odot = 3.85 \cdot 10^{26}\ W.$

In calculations we also need the following

$$\frac{L_\odot}{2\pi c} = 2.03116121 \times 10^{17}\ kg\ m^2\ s^{-1}$$

$$= 1.01355668 \times 10^{18}\ kg\ (a.u.)^2\ day^{-1}$$

$$\frac{L_\odot}{2\pi c^2} = 5.85379398 \times 10^{13}\ kg\ (a.u.)^{-1},$$

$$R = \frac{L_\odot}{2\pi c \sigma},$$

where $\sigma = \frac{m}{a}$ represents the mass-to-area ratio (usually measured in kg/m^2) and the symbol index "⊙" is used for the Sun.

Application: We consider the following problem: under what conditions a spacecraft with solar sails can survey the Sun from a heliosyncronous orbit? (That means that the distance to the Sun is $r_0 = 0.02\ a.u.$ and its period of revolution is $T = 25$ days.)

Using the theory exposed until now, we found the solution

$$k = 13.16 \times 10^{19}\ m^3 s^{-2}$$

$$\sigma = 0.014\ kg\ m^{-2}.$$

Remark: Other possible orbits of spacecrafts with solar sails will be given in future papers.

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Received 30.09.1997

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