Buletinul Ştiinţific al Universităţii din Baia Mare
Seria B, Matematică-Informatică, vol.XIII(1997), 57-65

LOCAL NUMERICAL STABILITY OF THE COLLOCATION METHODS FOR VOLTERRA INTEGRAL EQUATIONS

I. DANCIU

1 Introduction

In the papers [2] and [3] we have presented a method for the construction of an approximation to the solution of the following nonlinear Volterra integral equation of the second kind:

$$y(t) = f(t) + \int_0^t K(t, s, y(s))\,ds, \quad t \in I := [0, T], \tag{1.1}$$

with the given functions $f : I \to R$ and $K : S \times R \to R$ with $S := \{(t, s) : 0 \le s \le t \le T\}$), which are supposed to be sufficiently smooth so that the integral equation (1.1) would have a unique solution $y \in C^{\alpha}(I)$, with $\alpha \in N$

In order to describe this method let $\Pi_N : 0 = t_0 < t_1 < \dots < t_N = T$ $\left(\text{with } t_n = t_n^{(N)}\right)$ be a quasi uniform mesh for the given interval I, and set:

$$\sigma_0 := [t_0, t_1], \; \sigma_n := (t_n, t_{n+1}], \; n = 1, 2, \dots, N-1,$$

$$h_n := t_{n+1} - t_n, \; h = \max_{(n)} h_n, \; n = 0, 1, 2, \dots, N-1,$$

$$Z_N := \{t_n : n = 1, \dots, N-1\}, \; \overline{Z_N} = Z_N \cup \{T\}$$

Moreover, let P_k denote the space of (real) polynomials of degree not exceeding k. We then define, for given integers m and d with $m \ge 1$ and $d \ge -1$,

$$S^{(d)}_{m+d}(Z_N) := \{u : u(t)|_{t \in \sigma_n} = u_n(t) \in P_{m+d}, \; n = 0, \dots, N-1,$$

$$u^{(j)}_{n-1}(t_n) = u^{(j)}_n(t_n) \text{ for } j = 0, 1, \dots, d \text{ and } t_n \in Z_N\},$$