

COLLOCATOIN METHOD OF SOLVING NONLINEAR SINGULAR
INTEGRAL EQUATIONS GIVEN ON CLOSED SMOOTH CONTOUR

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Let Γ be a closed smooth contour [1, p.14] bounding a simple connected region F^+ of the complex plane C containing the point $z = 0$. In the Banach space of functions $H_\beta(\Gamma)$ [1, c.173] satisfying on Γ the Hölder condition with the exponent $\beta(0 < \beta < 1)$ consider a nonlinear singular integral equation (SIE) of the form

$$A(\varphi) \equiv \Phi[t; \varphi(t); S_r h(t, \tau; \varphi(\tau))] = f(t), \quad (1)$$

where $\Phi[t; u; v](t \in \Gamma; |u|, |v| < \infty)$, $h(t, \tau; u)(t, \tau \in \Gamma, |u| < \infty)$ and $f(t)$ are known continuous functions of their arguments, the singular integral

$$S_r h(t, \tau; \varphi(\tau)) \equiv \frac{1}{\pi i} \int_{\Gamma} \frac{h(t, \tau; \varphi(\tau))}{\tau - t} d\tau, t \in \Gamma,$$

is understood in the meaning of Cauchy principal value, and $\varphi(t)(t \in \Gamma)$ is an unknown function.

In this work we propose a computing scheme of collocation method for the equations (1) and using the results of [2, p.75] give a theoretical foundation of this scheme in the Hölder spaces. Note that earlier in the paper of the author [3] the foundation of collocation method for the equation (1) is obtained for the case of Lyapunov's contour Γ [1, p.14]. In present work the following results are obtained: 1) the class of contours Γ is essentially extended; 2) the basis of method is carried out for the case, when the searching solution $\varphi(t)$ of the equation (1) belongs to the space $H_\beta^r(\Gamma)$, $r = 0, 1, \dots$, that is the function $\varphi(t)$ is r -times differentiable and $\varphi^{(r)}(t) \in H_\beta(\Gamma)$; 3) the conditions on functions $\Phi[t; u; v]$ and $h(t, \tau; u)$ and