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Dedicated to Professor Iulian Coroian on his 60th anniversary

AN INTRODUCTION TO THE THEORY OF PARALLEL COMMUNICATING GRAMMAR SYSTEMS

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Abstract. In our days, a compiler to be efficient must be quick and a method which appear more often in this way is parallel calculus, method materialized through concurrent languages and a compilers for this. From here come the necessity to develop a formal model for parallel calculus. The results obtained are parallel communicating grammar systems, noted from now GCCP.

From point of view of formal languages, the study base on Chomsky grammar. The GCCP will be compute in this way: will be put together a lot of Chomsky grammar (same type or different type) which will work simultaneous and will communicate between them in a way well defined. The grammars generate words and will communicate through transmitions of words from one to another grammar. Of course, in that moment appear many questions: when must a grammar to deliver a word to anoher grammar? The word delivered is the whole word generated from source grammar or only a part of it? Where is put this in the current word of the destination grammar? What will do the grammars after delivering or receiving a word?. Starting from this questions we can imagine a lot of GCCP systems.

From now, we will consider communicating systems with request: sometimes the some grammars introduce the request symbols which in the next step will be replaced by the generated words of grammars until then, the grammars been identificate by requested symbols.

For this systems there is a lot of problems which must to be investigate: capacity of generation; properties of closure; syncronisation; syntactical analysis.

In general we will talk about the last class of problems.

Now, we present the main types of GCCP:

•central - which only a grammar request words from others;

•uncentral - when many grammars can request words from others;

•back to axiom - after delivering of word, the grammar which made this thing backs to axiom (initial symbol);

•without back to axiom - after delivering of word, the grammar goes on its work from it was before delivering without back to its initial symbol;

•syncronized - without waiting times when come requested symbols;

•unsyncronized - with waiting times when come requested symbols;

Definition 1: A GCCP system of degree n ($n \in N^*$) is a n-tuples $\gamma=(G_1,G_2,\ldots,G_n)$ where G_i is a Chomsky grammar, $G_i=(N_i,\Sigma_i,P_i,S_i)$, $i=1..n$, $\Sigma_i \subseteq \Sigma_1$, $i=2..n$ and there is a set K of special symbols,request symbols ,

$$K \subseteq \{Q_1,Q_2,\ldots,Q_m\}\ (m \in N^*)\ K \subseteq \bigcup_{i=1}^{n} N_i$$

used in derivation in this way:

For $(x_1,x_2,\ldots,x_n)$, $(y_1,y_2,\ldots,y_n)$ with $x_i,y_i \in (N_i \cup \Sigma_i)^*$, $i=1..n$ we write $(x_1,x_2,\ldots,x_n) \Rightarrow (y_1,y_2,\ldots,y_n)$ if appear one of this cases:

i) if $|x_i|_K=0$, $i=1..n$ and $(\forall)$ $i=1..n$ we have $x_i \Rightarrow y_i$ in G_i or $x_i \in \Sigma_i^*$ and $X_i=x_i$;

ii)if $(\exists)$ $i \in \{1,2,\ldots n\}$ we have that $|x_i|_K \neq 0$, then for each i (from this kind) we write $x_i=Z_1Q_{i1}\ Z_2Q_{i2}\ldots Z_tQ_{it}\ Z_{t+1}$, $t \geq 1$, $|Z_j|_K=0$, $j=1..t+1$

a) if $|x_{ij}|_K=0$,$(\forall)$ $j=1..t$ then

$y_i= Z_1x_{i1}\ Z_2x_{i2}\ldots Z_tx_{it}\ Z_{t+1}$ and $y_{ij}=S_{ij}$, $j=1..t$;

b) if for one $j=1..t$ we have that $|x_{ij}|_K \neq 0$ then $y_i=x_i$;

c) $(\forall)$ $i=1..n$ for which y_i wasn't defined like upper, we put $y_i=x_i$;

In words, an n-tuples $(x_1,x_2,\ldots,x_n)$ directly generate $(y_1,y_2,\ldots,y_n)$ if in the same time we have't a request symbol in $x_1,x_2,\ldots,x_n$ and we have a derivation on components $x_i \Rightarrow y_i$, $i=1..n$. If we have a request symbol we make an communication step like that:

- each Q_{ij} from x_i is replaced by x_{ij}, if x_{ij} don't have an request symbol;
- after communication , the delivery word x_{ij} replaces the request symbol Q_{ij} and the grammar G_{ij} go back to work started from S_{ij}.

If the requested symbols are not satisfied to an communication step, then will be satisfied to next communication step (the requested words can't have the request symbols).

Definition 2 : Let $x_i, y_i \in (N_i \cup \Sigma_i)$, $i=1..n$ then if $l \in N$

1. $(x_1,x_2,\ldots,x_n) \overset{l}{\underset{\gamma}{\Rightarrow}} (y_1,y_2,\ldots,y_n)$ if there is the direct derivations

$$(x_1^0,x_2^0,\ldots,x_n^0) \Rightarrow (x_1^1,x_2^1,\ldots,x_n^1),$$
$$(x_1^1,x_2^1,\ldots,x_n^1) \Rightarrow (x_1^2,x_2^2,\ldots,x_n^2),$$

....

$(x_1^{l-1},x_2^{l-1},\ldots,x_n^{l-1}) \Rightarrow (x_1^l,x_2^l,\ldots,x_n^l)$, where $x_i^0=x_i$, $i=1..n$ and $x_i^l=y_i$, $i=1..n$

2. $(x_1,x_2,\ldots,x_n) \overset{+}{\underset{\gamma}{\Rightarrow}} (y_1,y_2,\ldots,y_n)$, if $(\exists)$ $l>0$ that

$$(x_1,x_2,\ldots,x_n) \overset{l}{\underset{\gamma}{\Rightarrow}} (y_1,y_2,\ldots,y_n)$$

3. $(x_1,x_2,\ldots,x_n) \overset{*}{\underset{\gamma}{\Rightarrow}} (y_1,y_2,\ldots,y_n)$, if $y_i=x_i$, $i=1..n$ or

$$(x_1,x_2,\ldots,x_n) \overset{+}{\underset{\gamma}{\Rightarrow}} (y_1,y_2,\ldots,y_n)$$

Definition 3 : The generated language by γ is:

$$L(\gamma)=\{x \in \Sigma_1^* \mid (S_1,S_2,\ldots,S_n) \overset{*}{\underset{\gamma}{\Rightarrow}} (x,x_2,\ldots,x_n), \text{ where } x_i \in (N_i \cup \Sigma_i)^*, i=2..n$$

A derivation is a repeted rewriting and steps of comunicating, started from $(S_1,S_2,\ldots,S_n)$.

G_1 is the **main grammar** of system γ and the other grammar are the **secundary grammar.**

The upper definition is for systems with back to axiom.

Definition 4 : A GCCP system without back to axiom is defined in the same way, except only that the point ii)a) is without $y_{ij}=S_{ij}$, $j=1..t$. So at this type of systems, after communication of the sequence x_j in x_i the grammar G_{ij} don't come back at S_{ij}, it goes on to process the sequence x_{ij}. If we add the restriction $K \cap N_i=\varphi$, $i=2..n$ then the GCCP system is centralized.

Let $\gamma=(G_1,G_2,\ldots,G_n)$ a GCCP system of degree n. If x_i, $y_i \in (N_i \cup \Sigma_i)^*$, $i=1..n$ and in γ we have the derivation $(x_1,x_2,\ldots,x_n) \Rightarrow (y_1,y_2,\ldots,y_n)$ and the derivation on components

are $x_i \overset{left}{\underset{G_i}{\Rightarrow}} y_i$, then derivation from γ is left derivation and we note:

$$(x_1,x_2,\ldots,x_n) \overset{left}{\underset{\gamma}{\Rightarrow}} (y_1,y_2,\ldots,y_n) \quad \text{or} \quad (x_1,x_2,\ldots,x_n) \underset{left}{\Rightarrow} (y_1,y_2,\ldots,y_n)$$

Theorem 1: In a GCCP system for any derivation there is a equivalent left derivation.

Proof: In a grammar, for any derivation there is a equivalent left derivation. We know the definitions of derivation on components and the definitions of left derivation, so the conclusion of the theorem is obvious.

Definition 5: Two GCCP systems γ_1 and γ_2 we say that are equivalent if they generate the same language, that is $L(\gamma_1)=L(\gamma_2)$.

Definition 6: A system γ is ambiguous, if $(\exists)$ $w \in L(\gamma)$ which admits two distinctly relation of derivation:

$$(S_1,S_2,\ldots,S_n) \underset{left}{\Rightarrow} (w,x_2,\ldots,x_n), \text{where } x_i \in (N_i \cup \Sigma_i)^*,\ i=2..n.$$

Examples of GCCP systems:

1) $\gamma_1=(G_1,G_2)$

$G_1=(\{S_1,S_2,Q_2\},\{a,b,c\},P_1,S_1)$, where

$P_1: S_1\to aS_1|aQ_2$

$S_2\to aS_2|c$

$G_2=(\{S_2\},\{b\},\{S_2\to bS_2\},S_2)$, $K=\{Q_2\}$

A terminal derivation has the follow form

$$(S_1,S_2)\Rightarrow(a\,S_1,\,bS_2)\overset{*}{\Rightarrow}(a^{K_1}S_1,\,b^{K_1}S_2)\Rightarrow(a^{K_1+1}Q_2,\,b^{K_1+1}S_2)\Rightarrow(a^{K_1+1}b^{K_1+1}S_2,S_2)\Rightarrow(a^{K_1+1}b^{K_1+1}aS_1,bS_2)\Rightarrow\ldots\Rightarrow(a^{K_1+1}b^{K_1+1}\ldots a^{K_{n+1}+1}b^{K_{n+1}+1}S_2,S_2)\Rightarrow(a^{K_1+1}b^{K_1+1}\ldots a^{K_{n+1}+1}b^{K_{n+1}+1}c,bS_2)$$

with $n\geq 0$, $K_i\geq 0$, $i=1,n+1$. So

$$L(\gamma_1)=\{a^{K_1}b^{K_1}\ldots a^{K_r}b^{K_r}c\mid r\geq 1,\ k_i\geq 1,\ i=1..r\}$$

Observation 1: This two grammar from γ_1 are regular, but $L(\gamma_1)$ is not regular.

2) $\gamma_2=(G_1,G_2,G_3)$

$G_1=(\{S_1,S_2,S_3,Q_2,Q_3\},\{a,b,c\},P_1,S_1)$

P_1: $S_1\to aS_1|a^2Q_2$

$S_2\to b^2Q_3$

$S_3\to c$

$G_2=(\{S_2\},\{b\},\{S_2\to bS_2\},S_2)$

$G_3=(\{S_3\},\{c\},\{S_3\to cS_3\},S_3)$ and $K=\{Q_2,Q_3\}$

The terminal derivation in γ_2 is

$$(S_1,S_2,S_3)\Rightarrow(aS_1,bS_2,cS_3)\overset{*}{\Rightarrow}(a^nS_1,b^nS_2,c^nS_3)\Rightarrow(a^{n+2}Q_2,b^{n+1}S_2,c^{n+1}S_3)$$
$$\Rightarrow(a^{n+2}b^{n+1}S_2,S_2,c^{n+1}S_3)\Rightarrow(a^{n+2}b^{n+3}Q_3,bS_2,c^{n+2}S_3)\Rightarrow(a^{n+3}b^{n+3}c^{n+2}S_3,bS_2,S_3)$$
$$\Rightarrow(a^{n+3}b^{n+3}c^{n+3},b^2S_2,cS_3),\ n\geq 0$$

$$L(\gamma_2)=\{a^mb^mc^m\mid m\geq 3\}$$

Observation 2: The grammars G_1,G_2,G_3 from γ_2 are regulars, but the language $L(\gamma_2)$ is not regular, but is context-free language.

3) $\gamma_3=(G_1,G_2,G_3)$

$G_1=(\{S_1,S_2,S_3,Q_2,Q_3\},\{a,b,+,-\},P_1,S_1)$

P_1: $S_1\rightarrow a+S_1|a-Q_2$

$S_2\rightarrow b^2-Q_3$

$S_3\rightarrow a$

$G_2=(\{S_2\},\{b,+\},\{S_2\rightarrow b+S_2\},S_2)$

$G_3=(\{S_3\},\{a,b,+\},\{S_3\rightarrow a+b+S_3\},S_3)$ and $K=\{Q_2,Q_3\}$

A terminal derivation in γ_3 has the follow form, where we noted $c=a+b$

$$(S_1,S_2,S_3)\Rightarrow(a+S_1,b+S_2,c+S_3)\overset{*}{\Rightarrow}((a+)^nS_1,(b+)^nS_2,(c+)^nS_3)\Rightarrow$$
$$\Rightarrow((a+)^nS_1,(b+)^nS_2,(c+)^nS_3)\Rightarrow((a+)^{n-1}Q_2,(b+)^{n+1}S_2,(c+)^{n+1}S_3)\Rightarrow$$
$$((a+)^{n+1}(b+)^{n+1}S_2,S_2,(c+)^{n+1}S_3)\Rightarrow((a+)^{n+1}(b+)^{n+1}b^2-Q_3,b+S_2,(c+)^{n+2}S_3)\Rightarrow$$
$$\Rightarrow((a+)^{n-1}(b+)^{n+1}b^2-(c+)^{n+2}S_3,b+S_2,S_3)\Rightarrow$$
$$((a+)^{n+1}(b+)^{n+1}b^2-(c+)^{n+2}a,b+b+S_2,c+S_3).$$

If make $c=a+b$ we obtain:

$(S_1,S_2,S_3)\Rightarrow((a+)^{n-1}(b+)^{n+1}b^2-(a+b+)^{n+2}a,(b+)^2S_2,a+b+S_3)$, $n\geq0$.

So, the language is

$L(\gamma_3)=\{(a+)^m(b+)mb^2-(a+b+)^{m+1}a|m\geq1\}$

Observation 3: In all examples the systems were with back to axiom and centralized.

4) $\gamma=(G_1,G_2)$

$G_1=(\{S_1,S_2,Q_2\},\{a,b\},P_1,S_1)$, where

P_1: $S_1\rightarrow aS_1|aS_2|aQ_2$

$S_2\rightarrow aQ_2|b$

$G_2=(\{S_2\},\{b\},\{S_2\rightarrow bS_2\},S_2)$, $K=\{Q_2\}$

The system γ is ambiguous because there is the word $w=a^2b^3$ from $L(\gamma)$ for that there is two distinctly left derivation

a) $(S_1,S_2) \underset{left}{\Longrightarrow} (aS_1,bS_2) \underset{left}{\Longrightarrow} (a^2Q_2,b^2S_2) \underset{left}{\Longrightarrow} (a^2b^2S_2,S_2) \underset{left}{\Longrightarrow} (a^2b^3,bS_2)$

b) $(S_1,S_2) \underset{left}{\Longrightarrow} (aS_2,bS_2) \underset{left}{\Longrightarrow} (a^2Q_2,b^2S_2) \underset{left}{\Longrightarrow} (a^2b^2S_2,S_2) \underset{left}{\Longrightarrow} (a^2b^3,bS_2)$

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