

SOME PROPERTIES OF m - FAMILIES

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1 Introduction

Let m be a positive integer. The notion which give the title of our paper was introduced by Klee [7] and it one defined as follows:

A family \mathcal{A} of $m + 1$ convex subsets of a vector space E will be called an m - family in E if $\bigcap \mathcal{A} = \emptyset$, but each m members of \mathcal{A} have a common point.

From Helly's theorem [6], it immediately follows that, if E is finite-dimensional then in E there is no m - family for $m > \dim E$. The aim of this work is to point out some properties of the m - families. The main result in [2] will receive a new proof (Theorem 6).

If m is a positive integer by $\langle m \rangle$ we denote the set $\{1, 2, \dots, m\}$. For an m - family $\{A_1, A_2, \dots, A_{m+1}\}$ we shall introduce the following notations:

$$A'_i = \bigcap \{A_j : j \in \langle m+1 \rangle \setminus \{i\}\} \text{ for each } i \in \langle m+1 \rangle;$$

$$C = \text{conv}(\bigcup \{A'_i : i \in \langle m+1 \rangle\}) \text{ and } B_i = A_i \cap C \text{ for each } i \in \langle m+1 \rangle.$$