Bul. Stiinţ. Univ. Baia Mare, Ser. B,
Matematică-Informatică, Vol. XIV(1998), Nr. 2, 191-196

First-order equilibria for an abstract economy

Anton Mureşan

1991 AMS Subject Classification: 90A14.

Key words: game, equilibrium, first-order locally consistent equlibria, first-order locally consistent economic equilibria

Abstract

In this paper we give a result concerning the existence of first-order equilibrium for an abstract economy.

1 Introduction

In a market economy the prices are determined by supply and demand. When the supply of commodities from producers is equal to the demand for commodities by consumers we say that is a market equilibrium.

In a large market economy the number of prices determined is enormous.

Aside from the practical difficulty of computing and communicating all those prices, how can we even be sure that it is possible to find prices that will equate supply and demand in all markets at once? Mathematicians will recognise the problem as one of proving the existence of a solution to a set of nonlinear equations.

A game in strategic form is specified by a list of strategy spaces and preferences over strategy vectors for each player. A variation of the notion of a non-cooperative game is that of an abstract economy.

A first-order locally consistent equilibrium of an abstract economy is a configuration of strategies at which the first-order condition for pay-off maximization is simultaneously satisfied for all players.

In [1], D'Agata have provided a general existence result of a first-order locally consistent equilibrium for an abstract economy and he have employed his existence result to prove the existence of a first-order locally consistent equilibrium for an economic application in finite-dimensional spaces.

In [4], we gave a result of existence of first-order locally consistent equilibrium for an abstract economy.

2 The main result

We prove the existence of a first-order locally consistent economic equilibrium in a model of monopolistic competition similar to the Bonanno and Zeeman's one.

We consider a monopolistic competitive market with n price-making firms, $i \in I$, $I = \{1, 2, \dots, n\}$. The cost function of firm i is $C_i(q_i) = c_i q_i$, where q_i is the level of output of firm i anc c_i is a positive number. We assume that the firm i chooses any price in the interval $[c_i, P_i] = J_i$. Set $J = \prod_{i \in I} J_i$ and $J_{-i} = \prod_{j \neq i} J_j$. The price set by firm i is denoted by p_i. Denote by p_{-i} the $(n-1)$-dimensional vector whose elements are the prices set by all firms except the i-th one. Set $p = (p_i, p_{-i})$. The function $D_i : J \to \mathbf{R}$ is the demand function of firm i, and it is indicated by $D_i(p)$. The true profits of firm are

$\Pi_i(p) = D_i(p)(p_i - c_i)$.

We suppose that:

A1. For every $i \in I$, funtcion D_i is continuous on J, and the derivative $\partial D_i/\partial p_i : J \to \mathbf{R}$ exists and is continuous.

A2. For every $p_{-i} \in J_{-i}$, if $D_i(p_i', p_{-i}) = 0$, $p_i' \in J_i \setminus \{P_i\}$, then $(\partial D_i/\partial p_i)(p_i', p_{-i}) \leq 0$ and $D_i(p_i'', p_{-i}) = 0$ for every $p_i'' \geq p_i'$.

Here it is possible that for every price in J_i, firm i's market demand is zero.

Remark 1. We shall assume that firms maximize their conjectural profit function calculated by taking into account the linear approximation of their demand function. Given the **status quo** $p^0 \in J$, the conjectural demand of firm i is

$$\Delta_i(p_i, p^0) := D_i(p^0) + (\partial D_i/\partial p_i)(p^0)(p_i - p_i^0)$$

and conjectural profit is

$$\Pi_i^*(p_i, p^0) := \Delta_i(p_i, p^0)(p_i - c_i).$$

Definition 1. A first-order locally consistent economic equilibrium is a vector $p^* \in J$ such that for every $i \in I$ we have

$$\Pi_i^*(p_i^*, p^*) \geq \Pi_i^*(p_i, p^*) \text{ for every } p_i \in J_i. \tag{1}$$

Definition 1 means that at equilibrium firms are maximizing their conjectural profit function. It is easily seen that if p^* is a first-order locally consistent economic equilibrium then:

i) $\Delta_i(p_i^*, p^*) = D_i(p^*)$, and

ii) $(\partial \Delta_i/\partial p_i)(p^*) = (\partial D_i/\partial p_i)(p^*)$.

The condition i) means that at equilibrium the conjectural demand must be equal to the true demand.

The condition ii) means that at equilibrium the slope of the true demand function is equal to the slope of the conjectural demand.

We have

Theorem 1. *Under A1 and A2 there exists a first-order locally consistent economic equilibrium.*

Proof. By setting $X_i = J_i$ and $x_i = p_i$, $i \in I$, the industry we are considering reduces to the game Γ considered in [4]. Under A1 and A2 the game Γ has clearly a first-order locally consistent equilibrium $x^* = (x_i^*)_{i \in I}$. Set $p_i^* = x_i^*$, $i \in I$. Thus, to prove Theorem 1 it is sufficient to prove that if $(p_i^*)_{i \in I}$ is a first-order locally consistent equilibrium then it satisfies condition (1) in Definition 1. We have to consider three possible cases:

a) $p_i^* = P_i$, b) $p_i^* = c_i$, c) $p_i^* \in J_i \setminus \partial J_i$, $i \in I$.

Case a) $p_i^* = P_i$. Assumption A2 ensures that $D_i(p^*) = (\partial D_i/\partial p_i)(p^*) = 0$. It follows that $\Delta_i(p_i, p^*) = 0$, $p_i \in J_i$. Therefore $H_i^*(p_i^*, p^*) = H_i^*(p_i, p^*) = 0, \quad p_i \in J_i$.

Thus, the condition (1) in Definition 1 is satisfied.

Case b) $p_i^* = c_i$. Two case can occur:

b1) $(\partial H_i/\partial p_i)(p^*) = 0$;

b2) $(\partial H_i/\partial p_i)(p^*) \neq 0$.

In the case b1) it is not possible that $D_i(p^*) > 0$. In fact, if it is so, one has that $(\partial H_i/\partial p_i)(p^*) = D_i(p^*) > 0$, which is a contradiction. If $D_i(p^*) = 0$ then $H_i^*(p_i^*, p^*) \geq H_i^*(p_i, p^*)$ for $p_i \in J_i$, since $H_i^*(p_i^*, p^*) = 0$ and $H_i^*(p_i, p^*) = ((\partial D_i/\partial p_i)(p^*)(p_i - p_i^*))(p_i - c_i) \leq 0$ because $p_i^* = c_i$ and $(\partial D_i/\partial p_i)(p^*) \leq 0$ from assumption A2.

In the case b2), then by the fact that p^* is a first-order locally consistent equilibrium, it must satisfy the condition $(D_i(p^*) + (\partial D_i/\partial p_i))(p^*)(p_i^* - c_i))(p_i - p_i^*) \leq 0$, $p_i \in N(c_i)$, where $N(c_i)$ is a right neighbourhood of c_i. Because $p_i^* = c_i$ one has $D_i(p^*)(p_i - p_i^*) \leq 0$,

$p_i \in J_j$. This implies that $D_i(p^*) = 0$, and therefore by A2, that $(\partial D_i/\partial p_i)(p^*) \leq 0$ and that $D_i(p_i, p^*_{-i}) = 0$ for every $p_i \in J_i \setminus \{c_i\}$.

We shall prove that $H_i^*(p_i^*, p^*) \geq H_i^*(p_i, p^*)$ for $p_i \in J_i$. In fact, $H_i^*(p_i^*, p^*) = 0$ while $H_i^*(p_i, p^*) = (D_i(p^*) + (\partial D_i(\partial p_i)(p^*)(p_i - p_i^*))(p_i - c_i) = (\partial D_i/\partial p_i)(p^*)(p_i - c_i)^2 \leq 0$ for every $p_i \in \partial_i \setminus \{c_i\}$, from the above argument. Thus, also in this case condition (1) in Definition 1 is satisfied.

Case c) $p_i \in J_i \setminus \partial J_i$. By definition of first order locally consistent equilibrium, one must have $(\partial H_i/\partial p_i)(p^*) = 0$.

Two cases can occur:

c1) $D_i(p^*) > 0$, and

c2) $D_i(p^*) = 0$.

In the case c1) by noticing that $(\partial H_i/\partial p_i)(p^*) = 0$ implies $(\partial D_i/\partial p_i)(p^*) < 0$ and that $(\partial^2 H_i^*/\partial p_i^2)(p_i^*, p^*) = 2(\partial D_i/\partial p_i)(p^*)$ one can conclude that $(\partial H_i/\partial p_i)(p^*) = 0$ implies $(\partial^2 H_i^*/\partial p_i^2)(p_i^*, p^*) < 0$. Thus the condition (1) in Definition 1 is satisfied.

In the case c2) if we prove that $(\partial D_i/\partial p_i)(p^*) = 0$ we have completed the proof because in this case $H_i^*(p_i^*, p^*) = H_i^*(p_i, p^*) = 0$, $p_i \in J_i$. Suppose, on the contrary, that $(\partial D_i/\partial p_i)(p^*) < 0$, then $(\partial H_i/\partial p_i)(p^*)(p_i - p_i^*) = (D_i(p^*) + (\partial D_i/\partial p_i)(p^*)(p_i^* - c_i))(p_i - p_i^*) = (\partial D_i/\partial p_i)(p^*)(p_i^* - c_i)(p_i - p_i^*) > 0$ for $p_i < p_i^*$, contradicting the hypothesis that p^* is a first-order locally consistent equilibrium. Thus, also in this last case the condition (1) in Definition 1 is satisfied.

Remark 2. In [2], Bonanno and Zeeman have provided a general existence result of a first-order locally consistent equilibrium for an abstract game-theoretic, and they employ their existence result to prove the existence of a first-order locally consistent equilibrium in a monopolistic competitive industry with price-making firms.

References

[1] D'Agata, A., *Existence of first-order locally consistent equilibria*, Annales d'économie et de statistique, 43, 1996, pp. 171-179.

[2] Bonanno, G., Zeeman, C.E., *Limited knowledge of demand and oligolopy equilibria*, Journal of Economic Theory, 35, 1985, pp. 276-283.

[3] Border, K.C., *Fixed point theorems with applications to economics and game theory*, Cambridge University Press, 1985.

[4] Mureşan, A.S., *On the equilibria of an abstract economy*, Bull. Appl. & Comp. Math. BAM (to appear).

[5] Oettli, W., Schäger, D., *Generalized vectorial equilibria and generalized monotonicity*, Functional analysis with current applications in science, technology and industry (Aligarh), 1996, pp. 145-154.

Received 8.07.1998

Assoc. Prof.Dr. Anton Mureşan
Department of Mathematics
University of"Babes-Bolyai" Cluj-Napoca
3400 Cluj-Napoca
Romania
E-mail:amuresan@math.ubbclij.ro