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DETERMINATION OF SOME PARAMETERS OF THE TOTAL SOLAR ECLIPSE OF 11 AUGUST 1999 BY INVERSE INTERPOLATION SPLINES

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Abstract: The technique of inverse interpolation spline functions is applied for the determination of the instants corresponding to different phases of solar eclipses. Using this method, we compute the visibility conditions for the 11 August 1999 solar eclipse in Baia-Mare.

1 Introduction

The total solar eclipses are important astronomical phenomena which, for a given site, occur with a periodicity of about 350 years. However, there are exceptions; for instance, during the second half of this century, both the TSE of 15 February 1961 and that of 11 August 1999 are visible as total in Romania. The next TSE visible from our country will occur in 2136.

The maximum of the eclipse occurs when the axis of the lunar shadow cone is at minimum distance from the Earth's centre. On 11 August 1999, this maximum will be on the Romanian territory.

This paper proposes a method for the determination of the instants corresponding to different phases of the eclipse, based on inverse interpolation splines.

2 Inverse interpolation quadratic splines

Consider the function $f : I \to \mathbf{R}$, where $I \in \mathbf{R}$, and the partition Δ given by the points

$$\Delta := \{a = x_0 < x_1 < ... < x_n = b\},$$

where $[a,b] \subseteq I$. Suppose that f is differentiable on $[a,b]$, and $f'(x) \neq 0$, $\forall x \in [a,b]$; denote by $F = f([a,b])$ the image of $[a,b]$ through f.

From the above hypotheses it follows that $f:[a,b]\to F$ is bijective, hence there exists $f^{-1}:F\to[a,b]$. Since f is differentiable on $[a,b]$, f^{-1} will be differentiable on F, and its derivative in a point $f_i=f(x_i)$, $f_i\in F$, $i=\overline{0,n}$, has the form

$$[f^{-1}(f_i)]'=1/f'(x_i),\quad i=\overline{0,n}.$$

Supposing that the equation

$$f(x)=0$$

has a root $\bar{x}\in[a,b]$, then from $f(\bar{x})=0$ it results $f^{-1}(0)=\bar{x}$, while from $f'(x)\neq 0$, $\forall x\in[a,b]$ it results the uniqueness of $\bar{x}$. The algorithm for the solution of the equation $f(x)=0$ was given by Iancu (1983). Our application assumes that f is tabularly given, that is, we know the values $f_i=f(x_i)$, $f_i\in F$, $i=\overline{0,n}$, and choose $f(x_0)=m_0\simeq(f(x_1)-f(x_0))/(x_1-x_0)$ for which it is convenient to use the inverse interpolation quadratic splines. So we construct a function of the form

$$H_i(y)=a_iy^2+b_iy+c_i,$$

whose coefficients are uniquely determined by the conditions

$$\begin{aligned}H_i(f_i)&=x_i,\\ H_i'(f_i)&=D_i',\\ H_i(f_{i+1})&=x_{i+1},\end{aligned}$$

where

$$D_0'=1/f'(x_0),\quad D_i'=H_{i-1}'(f_i),\quad i=\overline{1,n}.$$

These conditions are equivalent to

$$\begin{aligned}a_if_i^2+b_if_i+c_i&=x_i,\\ 2a_if_i+b_i&=D_i',\\ a_if_{i+1}^2+b_if_{i+1}+c_i&=x_{i+1},\quad i=\overline{0,n-1},\end{aligned}$$

which provide uniquely the coefficients a_i, b_i, c_i, $i=\overline{0,n-1}$, as follows:

$$\begin{aligned}a_i&=\tfrac{1-D_i'[x_i,x_{i+1};f]}{(x_{i+1}-x_i)[x_i,x_{i+1};f]^2},\\ b_i&=D_i'-2a_if_i,\\ c_i&=x_i-a_if_i^2-f_iD_i',\end{aligned}$$

where $[x_i, x_{i+1}; f]$ stands for the divided difference of first order of f with the first node x_i, $i = \overline{0, n-1}$.

Taking into account the fact that

$$H_i'(f_{i+1}) = D_{i+1}', \quad i = \overline{0, n-1}, \quad D_0' = 1/m_0,$$

one obtains the recurrence formula for the calculation of the values D_i', $i \geq 1$:

$$D_{i+1}' = 2/[x_i, x_{i+1}; f] - D_i', \quad i = \overline{0, n-1}.$$

The above relations determine uniquely the function $H(y)$. If the equation $f(x) = 0$ has a root $\bar{x}$ $(f(x) = 0)$ in the interval $(x_j, x_{j+1}) \in [a, b]$, then an approximation of this root can be obtained with the formula

$$H_j(0) = c_j \approx \bar{x}.$$

3 Numerical application

There exist many mathematical models for the prediction of the solar eclipses. The most known such model is that of Bessel. For every solar eclipse, there are computed the so-called Besselian elements, which are tabularly given in astronomical almanachs (see e.g. Seidelmann 1992). The elements of the TSE of 11 August 1999 were calculated by NASA (Espenak and Anderson 1997).

To calculate the local visibility circumstances, we have resorted to the program SOLARECL.BAS, in which we have introduced the geographic coordinates of Baia Mare: $\varphi = 47°40'$ North, $\lambda = 23°35'$ East, $H = 400$ m above sea level. The calculations were performed between the instants UT (initial) $= 8^h$ andUT (final) $= 14^h$, with a step of 0.1 hours. For every moment $(\mathrm{UT})_i$, Table 1 lists:

- the position angle P, having the vertex in the centre of the solar disk, measured from the Northern point of the disk Eastwards;

- the angle V, having the vertex in the centre of the solar disk, measured from the highest position of the Sun towards the zenith;

- the distance p between the observation site and the centre of the umbral circle on the Earth's surface;

- the differences $L_e - p$ and $L_i - p$, where L_e and L_i are the radii of the penumbral and umbral circles, respectively;

Table 1: Baia Mare: $\varphi = 47^0 40'$; $\lambda = 23^0 35'$; $H = 400$m

TU	P	V	$A_\odot$	$h_\odot$	p	$L_s - p$	$\lvert L_s - p \rvert$	ε
8.0000	+284.1	319.9	123.1	45.2	1.1759	-0.6371	-1.1685	-1.1989
8.1000	+284.2	319.2	124.8	46.0	1.1346	-0.5958	1.1272	-1.1214
8.2000	+284.2	318.4	126.6	46.8	1.0938	-0.5548	-1.0861	-1.0443
8.3000	+284.3	317.5	128.3	47.6	1.0527	-0.5140	-1.0453	-0.9676
8.4000	+284.3	316.6	130.2	48.4	1.0121	-0.4735	-1.0048	-0.8913
8.5000	+284.4	315.6	132.0	49.2	0.9717	-0.4331	-0.9642	0.8154
8.6000	+284.4	314.6	133.9	49.9	0.9315	-0.3929	-0.9240	-0.7398
8.7000	+284.4	313.5	135.9	50.6	0.8916	-0.3529	-0.8840	-0.6646
8.8000	+284.4	312.3	137.9	51.3	0.8517	-0.3131	-0.8441	0.5898
8.9000	+284.4	311.1	140.0	52.0	0.8120	0.2735	-0.8046	-0.5150
9.0000	-284.4	309.8	142.1	52.6	0.7726	-0.2340	0.7649	-0.4407
9.1000	+284.4	308.5	144.3	53.2	0.7332	-0.1946	-0.7250	-0.3666
9.2000	+284.4	307.0	146.5	53.8	0.6939	0.1554	-0.6863	-0.2927
9.3000	+284.3	305.5	148.8	54.3	0.6548	-0.1163	-0.6472	-0.2191
9.4000	+284.2	303.9	151.2	54.8	0.6158	-0.0773	-0.6082	0.1456
9.5000	+284.1	302.3	153.6	55.3	0.5769	0.0384	-0.5693	-0.0724
9.6000	+284.0	300.5	156.0	55.7	0.5381	+0.0004	0.5305	+0.0007
9.7000	+283.8	298.7	158.5	56.1	0.4994	+0.0391	0.4918	+0.0736
9.8000	+283.6	296.8	161.0	56.5	0.4608	+0.0777	-0.4531	+0.1464
9.9000	+283.4	294.7	163.6	56.8	0.4222	+0.1163	0.4145	+0.2191
10.0000	+283.0	292.6	166.2	57.0	0.3836	+0.1549	0.3760	+0.2917
10.1000	+282.6	290.4	168.8	57.3	0.3451	+0.1933	-0.3375	+0.3642
10.2000	+282.0	287.9	171.5	57.4	0.3067	+0.2318	-0.2991	+0.4366
10.3000	+281.2	285.3	174.2	57.6	0.2683	+0.2702	-0.2607	+0.5089
10.4000	+280.2	282.5	176.9	57.6	0.2300	+0.3085	-0.2224	+0.5811
10.5000	+278.6	278.9	179.6	57.7	0.1918	+0.3467	0.1842	+0.6530
10.6000	+276.3	274.7	182.3	57.6	0.1538	+0.3847	-0.1462	+0.7246
10.7000	+272.4	268.9	185.0	57.6	0.1162	+0.4224	-0.1086	+0.7954
10.8000	+264.8	259.4	187.7	57.5	0.0796	+0.4589	-0.0721	+0.8648
10.9000	+245.3	238.1	190.3	57.3	0.0467	+0.4919	-0.0392	+0.9282
11.0000	+189.9	180.9	193.0	57.1	0.0324	+0.5062	-0.0249	+0.9532
11.1000	+144.8	134.0	195.6	56.9	0.0544	+0.4843	-0.0469	+0.9117
11.2000	+129.9	117.3	198.2	56.6	0.0892	+0.4496	-0.0816	+0.8484
11.3000	+123.5	109.3	200.7	56.2	0.1265	+0.4122	-0.1190	+0.7759
11.4000	+120.1	104.1	203.2	55.8	0.1649	+0.3738	-0.1575	+0.7036
11.5000	+118.0	100.4	205.7	55.4	0.2038	+0.3350	0.1964	+0.6304
11.6000	+116.6	97.4	208.1	55.0	0.2430	+0.2957	-0.2357	+0.5565
11.7000	+115.6	94.9	210.4	54.5	0.2826	+0.2562	-0.2753	+0.4821
11.8000	+114.8	92.7	212.7	53.9	0.3224	+0.2165	0.3151	+0.4073
11.9000	+114.3	90.7	215.0	53.4	0.3624	+0.1765	-0.3551	+0.3320
12.0000	+113.8	88.9	217.2	52.8	0.4026	+0.1363	-0.3954	+0.2564
12.1000	+113.4	87.2	219.3	52.2	0.4430	+0.0959	0.4359	+0.1804
12.2000	+113.1	85.8	221.4	51.5	0.4837	+0.0553	-0.4766	+0.1039
12.3000	+112.9	84.2	223.4	50.8	0.5247	+0.0144	-0.5176	+0.0270
12.4000	+112.7	82.8	225.4	50.1	0.5658	-0.0267	0.5588	-0.0508
12.5000	+112.5	81.6	227.4	49.4	0.6073	-0.0681	-0.6003	0.1280
12.6000	+112.3	80.4	229.2	48.6	0.6489	0.1097	-0.6420	-0.2062
12.7000	+112.2	79.3	231.1	47.8	0.6909	-0.1516	0.6840	-0.2848
12.8000	+112.1	78.2	232.9	47.0	0.7331	-0.1938	0.7262	-0.3639
12.9000	+112.0	77.3	234.6	46.2	0.7756	0.2362	-0.7688	0.4436
13.0000	+111.9	76.3	236.3	45.4	0.8183	-0.2790	-0.8116	-0.5237
13.1000	+111.8	75.5	238.0	44.5	0.8614	-0.3220	-0.8548	-0.6043
13.2000	+111.7	74.7	239.6	43.7	0.9048	0.3653	-0.8982	-0.6855
13.3000	+111.6	73.9	241.2	42.8	0.9486	-0.4090	-0.9420	-0.7673
13.4000	+111.5	73.2	242.7	41.9	0.9926	-0.4529	0.9860	-0.8495
13.5000	+111.5	72.5	244.2	41.0	1.0369	-0.4972	-1.0304	0.9324
13.6000	+111.4	71.9	245.7	40.1	1.0816	-0.5418	-1.0752	-1.0158
13.7000	+111.3	71.3	247.1	39.2	1.1266	-0.5868	-1.1203	-1.0999
13.8000	+111.3	70.8	248.5	38.3	1.1720	-0.6321	-1.1657	-1.1845
13.9000	+111.2	70.3	249.9	37.3	1.2177	-0.6778	-1.2115	1.2698
14.0000	+111.2	69.8	251.3	36.3	1.2638	-0.7238	-1.2577	-1.3557

- the eclipse magnitude, $g = (L_e - p)/(L_e - L_i)$.

At the exterior contacts we have $p = L_e$, whereas at the interior contacts we have $p = |L_i|$. If $|L_i| < p < L_e$, then we have a partial solar eclipse.

Figure 1 uses the data listed in Table 1 to plot $p = p(UT)$. Observe that the curve presents a minimum at $UT = 11^h$, when the distance between Baia Mare and the axis of the

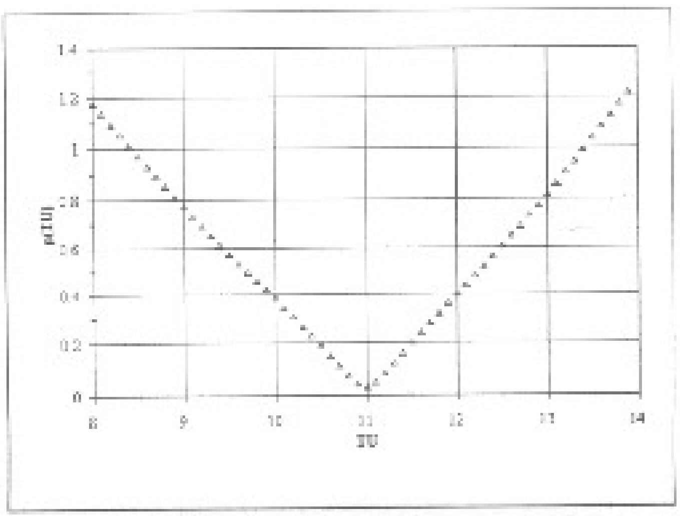

Figure 1: p vs. TU

The curve $(L_e - p)$ as function of UT, plotted in Figure 2, intersects the Ox-axis in two points which correspond to the instants of the first and fourth contacts.

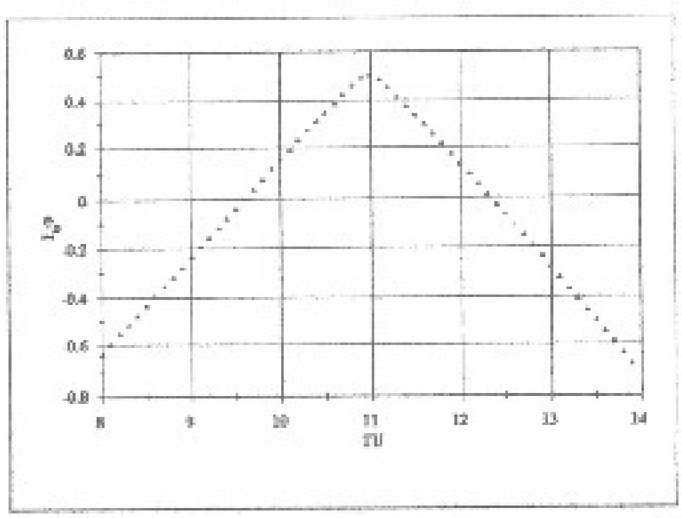

Figure 2: $L_e - p$ vs. TU

For the first contact we have considered the first 30 rows of Table 1, choosing the columns

1 and 7. For the fourth contact we have considered the last 30 rows of Table 1, choosing the same columns.

In a first approximation, $(L_e - p)$ is considered as linearly depending on UT, as follows

$$y = a + bx,$$

where $y = (L_e - p)$, $x = UT$; for $y = 0$ we have $(UT)_1 = 9^h36^m32^s.030$ and $(UT)_4 = 12^h18^m29^s.582$.

In a further approximation, $(L_e - p)$ is considered as quadratically depending on UT, as follows

$$y = a + bx + cx^2,$$

where we have obtained $(UT)_1 = 9^h35^m56^s.460$ and $(UT)_4 = 12^h20^m15^s.304$.

Resorting to inverse interpolation quadratic splines, we have obtained $(UT)_1 = 9^h35^m56^s.614$ and $(UT)_4 = 12^h20^m05^s.979$.

Taking into account the values provided by NASA: $(UT)_1 = 9^h35^m56^s.8$ and $(UT)_4 = 12^h20^m06^s.2$, as well as those computed by us via the three approximations, one obtains the $O - C$ differences below.

Approximation	$(UT)_1^{NASA} - (UT)_1^{comp}$	$(UT)_4^{NASA} - (UT)_4^{comp}$
linear	$-35^s.23$	$+96^s.62$
quadratic	$+5^s.34$	$-9^s.10$
splines	$+0^s.34$	$+0^s.22$

These results illustrate the efficiency of the use of inverse interpolation splines compared with the classical approximation methods.

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