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Dedicated to Professor Ion PĂVĂLOIU on his 60^{th} anniversary

ON SIMPLE n-SEMIGROUPS

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Abstract. The following paper is concerned with various theoretic aspects of the i-simple and simple n-semigroups. Many of the results are generalization of known theorems in the theory of binary semigroups, but it is interesting to mention that in contrast with the binary case where exists simple semigroups which are not i-simple, for $n \geq 3$, is necessary that to be every simple n-semigroup 2-simple and (n-1)-simple.

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1. Preliminary notions and results

All over we will use the following abbreviated notation

$$x_i^j = \begin{cases} \text{sequence } x_i, x_{i+1}, \dots, x_j & \text{for } i<j \\ x_i & \text{for } i=j \\ \text{empty sequence} & \text{for } i>j \end{cases}$$

If $x_1 = x_2 = \dots = x_k = x$, then x_1^k will denoted by $\overset{(k)}{x}$. For $k \leq 0$, $\overset{(k)}{x}$ is empty.

1.1. Definition [2]. A set A together with an n-ary operation $(\ )_\circ : A^n \to A$ is called *n-semigroup* if for any $i \in \{2, 3, \dots, n\}$ and all $x_1^{2n-1} \in A$, the following associativity laws hold:

$$\left((x_1^n)_\circ, x_{n+1}^{2n-1}\right)_\circ = \left(x_1^{i-1}, (x_i^{i+n-1})_\circ, x_{i+n}^{2n-1}\right)_\circ$$

The unit in a semigroup allows several generalizations: we cite here two useful ones:

- an element $e \in A$ is called *i-unit (unit)* if for all $x \in A$ we have

$$\left(\overset{(i-1)}{e}, x, \overset{(n-i)}{e}\right)_\circ = x \quad \text{(for any } i \in \{1,2,\dots,n\}\text{ ;}$$

- an (n-1)-ad u_1^{n-1} of elements of A is called *right (left) unit* if for all $x \in A$ we have

$$(x, u_1^{n-1})_\circ = x \quad \text{(or } (u_1^{n-1}, x)_\circ = x \text{ respectively).}$$

The element $z \in A$ of an n-semigroup is said *i-zero (zero)* element if for all $x_1^n \in A$ we have

$$(x_1^{i-1}, z, x_{i+1}^n)_\circ = z \quad \text{(for any } i \in \{1,2,\dots,n\}\text{)}$$

Obvious, if zero element exists, then it is unique.
Recursivly, one may also define

$$x^{[0]} = x \ ; \ x^{[1]} = \left(\overset{(n)}{x}\right)_\circ \ ; \ x^{[k]} = \left(x^{[k-1]}, \overset{(n-1)}{x}\right)_\circ$$

An n-semigroup $\left(A, (\ )_\circ\right)$ is called *surjective* if $A^{[1]} = A$.

1.2. Definitions [7]. Let $\left(A, (\ )_\circ\right)$ be an n-semigroup and $i \in \{1,2,\dots,n\}$.

A subset $I \subseteq A$ is said to be an *i-ideal (ideal)* of A if $(\overset{(i-1)}{A}, I, \overset{(n-i)}{A})_\circ \subseteq I$ (if I is an i-ideal of A for all $i \in \{1,\dots,n\}$).

By convention, $(\overset{(0)}{A}, I, \overset{(n-1)}{A})_\circ = (I, \overset{(n-1)}{A})_\circ$, $(\overset{(n-1)}{A}, I, \overset{(0)}{A})_\circ = (\overset{(n-1)}{A}, I)_\circ$

and $(\overset{(0)}{A}, I \overset{(0)}{A})_\circ = I$.

The smalest i-ideal (ideal) of A, containing an element $a \in A$, called *the principal i-ideal (ideal)* generated by a, will be denoted $(a)_i$ (respectively *(a)*). Constructively, the principal i-ideal generated by a, is given by

$$(a)_i = \bigcup_{k=0}^{\infty} X_k , \quad \text{when} \quad X_0 = \{a\} ; X_{k+1} = (\overset{(i-1)}{A}, X_k, \overset{(n-i)}{A})_\circ$$

If $(A, (\ )_\circ)$ is an surjective n-semigroup it is easy to see that only a finite number of the terms which appear are actually distinct, respectively $\frac{n-1}{(n-1, i-1)}$.

The following theorem showes that these various notions of i-ideals are not independent.

1.3. Theorem [7]. *Let* $(A, (\ )_\circ)$ *be a surjective n-semigroup. If the great common divisor of (i-1) and (n-1) divides that of (j-1) and (n-1), then* $(a)_j \subset (a)_i$ *for each* $a \in A$, *and* $(a)_i$ *is a j-ideal of A.*

1.4. Corollary [7]. *Let* $(A, (\ )_\circ)$ *be a surjective n-semigroup. The following statements hold:*

1^0. Every i-ideal of A is a j-ideal if and only if $(i-1, n-1) = (j-1, n-1)$, *In this case* $(a)_i = (a)_j$ *for each* $a \in A$.

2^0. If n is a prime number, then every i-ideal is also j-ideal for all $i, j = 2, 3, \ldots, n-2$;

3^0. If $i = 2, \ldots, n-1$, *then every i-ideal of A is an* $(n-i+1)$ *-ideal and conversely;*

4^0. Every i-ideal $i = 2, 3, \ldots, n-2$ *is contained in some 2-ideal (and hence is some (n-1)-ideal). Moreover, every 2-ideal is an i-ideal for each* $i = 2, 3, \ldots, n-1$.

1.5. Definition. An n-semigroup $(A, (\ )_\circ)$ is called *n-group* if for any $i \in \{1, 2, ..., n\}$ and all $a_1^n \in A$, the equation $(a_1^{i-1}, x, a_{i+1}^n)_\circ = a_i$ has a unique solution in *A*. In an n-group the unique solution of the equation $(\overset{(n-1)}{a}, x)_\circ = a$ is called the querelement of *a* and it is denoted by $\bar{a}$.

Reductions and extensions play an important role in the theory of n-ary structure. In the sequel we briefely present some of the mainly definitions and theorems.

1.6. Definition. Let $(A, (\ )_\circ)$ be an n-semigroup and $u_1^{n-2} \in A$ arbitrary, fixed elements of *A*. The algebraic structure (A,*) where $x * y = (x, u_1^{n-2}, y)_\circ$ is an semigroup called *the binary reduce of A with respect to the elements* u_1^{n-2}, it is denotes $red_{u_1^{n-2}}(A, (\ )_\circ)$.

1.7. Definition. Let $(A, \cdot)$ be an semigroup, $c \in A$, and $\alpha \in End(A, \cdot)$. The structure $(A, (\ )_\circ)$ where n-ary operation $(\ )_\circ : A^n \to A$ is defined by

$$(x_1^n)_\circ = x_1 \cdot \alpha(x_2) \cdot ... \cdot \alpha^{n-1}(x_n) \cdot c$$

is called the *n-ary extension of the semigroup A with respect to the endomorphism α and the element* $c \in A$; it is denoted by $ext_{\alpha,c}(A, \cdot)$.

In [6] is proved following important results:

1.8. Proposition. *If (A,·) is a semigroup,* $c \in A$ *and* $\alpha \in End(A, \cdot)$ *have the property*

$$\alpha^n(x) \cdot \alpha(c) = c \cdot \alpha(x), \quad \forall\, x \in A,$$

then $ext_{\alpha,c}(A, \cdot)$ *is an n-semigroup.*

1.8. Theorem. [6]. *Let* u_1^{n-1} *be a right unit in the n-semigroup* $(A,(\ )_\circ)$. *If* $\alpha: A \to A$; $\alpha(x) = \left(u_{n-1}, x\, u_1^{n-2}\right)_\circ$ *and* $c = u_{n-1}^{[1]}$, *then* α *is an endomorphism of their binary reduce,* $red_{u_1^{n-2}}(A,(\ )_\circ)$ *and*

$$ext_{\alpha,c}(red_{u_1^{n-2}}(A,(\ )_\circ)) = (A,(\ )_\circ).$$

2. i-Simple and simple n-semigroups

Let $(A,(\ )_\circ)$ be an n-semigroup without zero.

2.1. Definition. The n-semigroup $(A,(\ )_\circ)$ is called *simple (i-simple; i = 1,2, ..., n)* if it possesses no ideals (i-ideals) except for itself.

2.2 Remarks. a) Every i-simple n-semigroup is simple;

b) Every i-simple (simple) n-semigroup; $i = 1,2,\dots,n$ is surjective, because the subset $A^{[1]}$ is i-ideal (ideal) in *A*, which according to i-simplicity (simplicity) implies $A^{[1]} = A$.

2.3. Theorem. *a) The n-semigroup* $(A,(\ )_\circ)$ *is 1-simple (n-simple) if and only if*

$$(x, \overset{(n-1)}{A})_\circ = A\,; \qquad ((\overset{(n-1)}{A}, x)_\circ = A)\,; \quad \forall\, x \in A \qquad (1)$$

b) The n-semigroup A is i-simple, $i = 2,3,\dots,n-1$, *if and only if*

$$\bigcup_{k=1}^{\infty}\Big(\overset{\overline{(k(i-1))}}{A}, x, \overset{\overline{(k(n-i))}}{A}\Big)_\circ \bigcup \Big((\overset{(n-1)}{A}, x)_\circ, \overset{(n-1)}{A}\Big)_\circ = A\,, \quad \forall\, x \in A \qquad (2)$$

where $\overline{(k(i-1))}$, $\overline{k(n-i)}$ *respectively denote* $k(i-1)$, $k(n-i)$ *reduced modulo n-1;*

c) The n-semigroup $(A, (\ )_\circ)$ is simple if and only if

$$\bigcup_{i=2}^{n-1}\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ \cup \left(\left(\overset{(n-1)}{A}, x\right)_\circ, \overset{(n-1)}{A}\right)_\circ = A, \quad \forall\ x \in A \qquad (3)$$

Proof. a) If A is 1-simple n-semigroup, then because $\left(x, \overset{(n-1)}{A}\right)_\circ$ is 1-ideal of A, it resultes that $\left(x, \overset{(n-1)}{A}\right)_\circ = A, \quad \forall\ x \in A$. Conversely, if (1) holds and I is an 1-ideal of A, then for all $x \in I$ we have $A = \left(x, \overset{(n-1)}{A}\right)_\circ \subseteq I \subseteq A$, therefore $I = A$ and it follows that A is 1-simple n-semigroup.

b) If A is i-simple, then A is surjective and the principal i-ideal generated by any $x \in A$ coincides with A, which according to his gived constructively way, implies the relation (2).

The converse statement is true: if (2) holds, then $A = A^{[1]} \cup A^{[2]} \subseteq A^{[1]} \cup A^{[1]} = A^{[1]}$ and it results $A^{[1]} = A$, that is A is surjective.

If I is an i-ideal in A, then for all $x \in I$ we have $\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ \subseteq I$ and by surjectivity $\left(\overset{(\overline{k(i-1)})}{A}, x, \overset{(\overline{k(n-i)})}{A}\right)_\circ = \left(\overset{k(i-1)}{A}, x, \overset{k(n-i)}{A}\right)_\circ \subseteq I$ too.

In the same time, by the corollary $1.4.3^0$, I is an $(n-i+1)$ -ideal too, so that we have

$$\left(\left(\overset{(n-1)}{A}, x\right)_\circ, \overset{(n-1)}{A}\right)_\circ = \left(\overset{(n-i)}{A}\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ, \overset{(i-1)}{A}\right)_\circ \subseteq \left(\overset{(n-i)}{A}, I, \overset{(i-1)}{A}\right)_\circ \subseteq I$$

By (2) we have $A = (x)_i \subseteq I \subseteq A$, therefore $I = A$ and A is i-simple.

c) If A is simple, then A is surjective. Let $f: A \to \mathcal{P}(A)$ be the mapping

$$f(x) = \bigcup_{i=2}^{n-1} \Big(\overset{(i-1)}{A}, x, \overset{(n-i)}{A} \Big)_\circ \cup \Big(\overset{(n-1)}{A}, x, \overset{(n-1)}{A} \Big)_\circ .$$

The subset *f(x)* is an ideal in *A* because for all $j = 1, \ldots, n$ we have

$$\Big(\overset{(j-1)}{A}, f(x), \overset{(n-j)}{A} \Big)_\circ = \bigcup_{i=2}^{n-1} \Big(\overset{(i+j-2)}{A}, x, \overset{(2n-j-i)}{A} \Big)_\circ \cup \Big(\overset{(n+j-2)}{A}, x, \overset{2n-j-1}{A} \Big)_\circ$$

and every term of this reunion is contained in *f(x)*. Indeed

$$\Big(\overset{(i+j-2)}{A}, x, \overset{(2n-i-j)}{A} \Big)_\circ = \begin{cases} \Big(\overset{(i+j-2)}{A}, x, \overset{(n+1-i-j)}{A} \Big)_\circ \subseteq f(x) & for\ 2 < i+j \le n \\ \Big(\Big(\overset{(n-1)}{A}, x \Big)_\circ, \overset{(n-1)}{A} \Big)_\circ \subseteq f(x) & for\ i+j = n+1 \\ \Big(\overset{(i+j-n-1)}{A}, x, \overset{(n-(i+j-n))}{A} \Big)_\circ \subseteq f(x) & for\ i+j \ge n+2 \end{cases}$$

and

$$\Big(\overset{(n+j-2)}{A}, x, \overset{(2n-j-1)}{A} \Big)_\circ = \begin{cases} \Big(\Big(\overset{(n-1)}{A}, x \Big)_\circ, \overset{(n-1)}{A} \Big)_\circ \subseteq f(x) & for\ j = 1\ or\ j = n \\ \Big(\overset{(j-1)}{A}, x, \overset{(n-j)}{A} \Big)_\circ \subseteq f(x) & for\ j = 2, \ldots, n-1 \end{cases}$$

Because *A* is simple n-semigroup it resultes that $f(x) = A$ for all $x \in A$, which gives us (3).

Conversely, if (3) holds and *I* is an ideal of *A*, then for $\forall\ x \in I$ and every $i = 2, \ldots, n-1$ we have

$$\Big(\overset{(i-1)}{A}, x, \overset{(n-i)}{A} \Big)_\circ \subseteq I \ \text{ and } \ \Big(\Big(\overset{(n-1)}{A}, x \Big)_\circ \overset{(n-1)}{A} \Big)_\circ \subseteq \Big(I, \overset{(n-1)}{A} \Big)_\circ \subseteq I ,$$

which proves that $A \subseteq I$, hence $I = A$, that is *A* is simple n-semigroup.

For $n \ge 3$, we have more that the binary semigroups the following statements:

2.4. Theorem. *a) Every i-simple n-semigroup* $(i=2,\dots,n-1)$ *is* $(n-i+1)-$ *simple;*

b) Every i-simple n-semigroup $(i=3,\dots,n-2)$ *is also 2-simple n-semigroup (therefore (n-1)-simple too);*

c) If A is an 1-simple (n-simple) n-semigroup, then A is an i-simple n-semigroup for $i=2,\dots,n-1$ *;*

d) Every simple n-semigroup is 2-simple ((n-1)-simple).

Proof. The statements a) and b) comes immediately from the properties 1.3 and 1.4 of ideals of a surjective n-semigroup.

c) If A is an 1-simple n-semigroup, then for all $x \in A$ the subset

$\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ$; $i=1,\dots,n-1$ is an 1-ideal since

$$\left(\overset{(i-1)}{A}, x \overset{(n-1)}{A}\right)_\circ = \left(\overset{(i-1)}{A}, x, A^{[1]}, \overset{(n-i-1)}{A}\right)_\circ = \left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ .$$

As A posseses no 1-ideals except itself, it follows that $\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ = A$

and because the i-ideal generated by x, $(x)_i$ includes $\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ$, we have

$(x)_i = A$. That proves that A is i-simple for $i=2,\dots,n-1$.

d) Let A be an simple n-semigroup. Suppose that A is not 2-simple, let I be an 2-ideal of A, $I \neq A$. Because for all $x \in I$ we have $\left(A, x, \overset{(n-2)}{A}\right)_\circ \subseteq I$ and A is surjective it is easy to see that

$$\left(\overset{(2)}{A}, x, \overset{(n-3)}{A}\right)_\circ = \left(A\left(A, x, \overset{(n-2)}{A}\right)_\circ \overset{(n-2)}{A}\right)_\circ \subseteq \left(A, I, \overset{(n-2)}{A}\right)_\circ \subseteq I$$

and similarly $\left(\overset{(i-1)}{A}, x, \overset{(n-i)}{A}\right)_\circ \subseteq I$ for every $i=2,\dots,n-1$. From $\left(\overset{(n-2)}{A}, x, A\right)_\circ \subseteq I$

we have $\left(\left(\overset{(n-1)}{A},x\right)_\circ \overset{(n-1)}{A}\right)_\circ \subseteq \left(A\left(\overset{(n-2)}{A},x,A\right)_\circ \overset{(n-2)}{A}\right)_\circ \subseteq \left(A,I,\overset{(n-2)}{A}\right)_\circ \subseteq I$.

Thus, by theorem 2.3.c) $A = \bigcup_{i=2}^{n-1}\left(\overset{(i-1)}{A},x,\overset{(n-i)}{A}\right)_\circ \cup \left(\left(\overset{(n-1)}{A},x\right)_\circ \overset{(n-1)}{A}\right)_\circ \subseteq I \subseteq A$

and therefore $I = A$.

2.5. Theorem. *If the n-semigroup* $(A,(\;)_\circ)$ *is 1-simple and n-simple, then A is an n-group.*

Proof. Let $a_1,\dots,a_{n-1},a \in A$. Because $(A,(\;)_\circ)$ is 1-simple by a previous result we have $\left(x,\overset{(n-1)}{A}\right)_\circ = A$, $\forall\, x \in A$, hence there exist $a_{12},\dots,a_{1n} \in A$ so that $(a_1,a_{12}^{1n})_\circ = a$. At the same time there exist $a_{22},\dots,a_{2n} \in A$ so that $(a_2,a_{22}^{2n})_\circ = a_{12}$, hence $(a_1(a_2,a_{22}^{2n})_\circ a_{13}^{1n})_\circ = a$. Continuing this reasoning there is $a_{i2},\dots,a_{in} \in A$; $i = 3,\dots,n-1$, so that

$$(a_1(a_2,\dots,(a_{n-1},a_{n-1,2}^{n-1,n})_\circ\dots)_\circ a_{23}^{2n})_\circ a_{13}^{1n})_\circ = a$$

Therefore, for each set of elements $a, a_1^{n-1} \in A$, a solution exists for the equation $(a_1^{n-1},x)_\circ = a$. In the same way, because A is an n-simple n-semigroup, a solution exists for equation $(x,a_1^{n-1})_\circ = a$ and also for $(a_1^{i-1},x,a_i^{n-1}) = a$, $i = 2,\dots,n-1$. The proof is very similarly. By the theorem 1.4 [5], $(A,(\;)_\circ)$ is an n-group.

2.6. Corollary. *An n-semigroup is an n-group if and only if* $\left(\overset{(n-1)}{A},x\right)_\circ = A$ *and* $\left(x,\overset{(n-1)}{A}\right)_\circ = A$; $\forall\, x \in A$. We rediscovered so a result of Szász [8].

2.7. Remark. There exists a simple n-semigroup, with not 1-simple and n-simple, but he is 2-simple.

For example, let $(A,(\ )_\circ)$ be the 3-semigroup defined as $A=\{(x,y)\in\mathbb{R}^2;\ x,y>0\}$

and $(\ )_\circ : A^3\to A:\quad ((x_1,y_1),(x_2,y_2),(x_3,y_3))_\circ=(x_1x_2x_3, y_1x_2x_3+y_2x_3+y_3)$.

He is 2-simple, hence simple 3-semigroup, because for each $(a,b),(c,d)\in A$,

there is $(x_1,y_1)(x_3,y_3)\in A$ so that $((x_1,y_1),(c,d),(x_3,y_3))_\circ=(a,b)$ since

$\begin{cases} x_1cx_3=a \\ y_1cx_3+dx_3+y_3=b \end{cases}$. Indeed, for all $y_1>0$ and $x_1>\dfrac{a(d+cy_1)}{bc}$ we have

$x_3=\dfrac{a}{x_1c}>0$ and $y_3=b-\dfrac{a(d+cy_1)}{x_1c}>0$.

But $(A,(\ )_\circ)$ is not 1-simple and 3-simple because from the equation

$((1,1),(x_2,y_2),(x_3,y_3))_\circ=(2,1)$ rerspectively $((x_1,y_1),(x_2,y_2),(1,2))_\circ=(1,1)$

we have

$$\begin{cases} x_2x_3=2 \\ y_3=-1-y_2x_3<0 \end{cases},\quad \forall\ x_2,x_3,y_2>0$$

respectively

$$\begin{cases} x_1x_2=1 \\ y_2=-1-y_1x_2<0 \end{cases},\quad \forall\ x_1,x_2,y_1>0\ .$$

We remark that the binary reduce of A to respect $(1,1)\in A$ is simple semigroup because for any $(a,b),(c,d)\in A$ there exists $(x_1,y_1),(x_2,y_2)\in A$ that $(x_1,y_1)*(a,b)*(x_2,y_2)=(c,d)$ such that for $\forall y_1>0$ and

$x_1>\dfrac{c}{ad}(ay_1+a+b+1)$ we have $x_2=\dfrac{c}{ax_1}>0$ and $y_2=d-\dfrac{c}{ax_1}(ay_1+a+b+1)>0$.

We will give in sequel some properties of the binary reduce of an n-semigroup, connecting ideals of an n-semigroup to ideals of its reduce and the correspondence which arises between simple n-semigroup and simplicity of their reduces.

As in the case of (m,n)-rings [4] we have:

2.8. Proposition. *Let* $(A,(\ )_\circ)$ *be an n-semigroup,* $u_1^{n-2} \in A$ *and* $(A,*) = red_{u_1^{n-2}}(A,(\ )_\circ)$. *The following hold:*

1) If $I \subseteq A$ *is a n-ideal of A, then I is a right ideal of* $(A,*)$. *Moreover, if* u_1^{n-1} *is a right unit in A, then the converse statement is also true;*

2) If $I \subseteq A$ *is a n-ideal of A, Then I is a left ideal of* $(A,*)$. *Moreover, if* $u_{n-1}\,u_1^{n-2}$ *is a left unit in A, then the converse is also true.*

2.9. Corollary. *Let* $(A,(\ )_\circ)$ *be an n-semigroup,* $u_1^{n-2} \in A$ *and* $(A,*) = red_{u_1^{n-2}}(A,(\ )_\circ)$.

1) If $(A,*)$ *is a right simple semigroup, then A is 1-simple n-semigroup. Moreover, if* u_1^{n-1} *is a right unit in A, and A is a 1-simple n-semigroup, then* $(A,*)$ *is right simple.*

2) If $(A,*)$ *is a left simple semigroup, then A is n-simple n-semigroup. Moreover, if* $u_{n-1}\,u_1^{n-2}$ *is a left unit in A, then converse is also true.*

2.10. Proposition. *Let* $(A,(\ )_\circ)$ *be an n-semigroup,* $u_1^{n-2} \in A$ *and* $(A,*) = red_{u_1^{n-2}}(A,(\ )_\circ)$. *If* $(A,*)$ *is simple semigroup then* $(A,(\ )_\circ)$ *is 2-simple n-semigroup.*

Proof. If $(A,*)$ is simple semigroup, then $A * A = A$ and $A * x * A = A$ for all $x \in A$. Let I be a 2-ideal of n-semigroup A. By the definitions 1.2 and 1.6, for all

$x \in I$, we have,

$$A = A * x * A = ((A, u_1^{n-2}, x)_\circ, u_1^{n-2}, A)_\circ = (A, u_1^{n-3}, (u_{n-2}, x, u_1^{n-2})_\circ, A)_\circ \subseteq$$

$$\subseteq (A, u_1^{n-3}, I, A)_\circ \ldots \subseteq (A, u_1, I, u_3^{n-2}, A)_\circ = (A, (u_1, I, u_3^{n-2}, A, u_1)_\circ, u_2^{n-2}, A)_\circ \subseteq (A, I, u_2^{n-2}, A)_\circ \subseteq I,$$

So that $A = I$ and $(A, (\ )_\circ)$ is 2-simple.

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