Bul. Ştiinţ. Univ. Baia Mare, Ser. B,
Matematică-Informatică, Vol. XV(1999), Nr. 1-2, 127-130

Dedicated to Professor Ion PĂVĂLOIU on his 60th anniversary

REGULARITY OF BERNSTEIN-ROGOSINSKI-TYPE MEANS OF DOUBLE FOURIER SERIES

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Abstract

Regularity of Bernstein-Rogosinsky-type means for Fourier series of continuous functions in two variables is investigated. Here partial sums of double Fourier series are generated by right polygons. Results obtained are negative if the number of sides of mentioned polygons is both odd or even.

Let $T^2 = (-\pi, \pi]^2$, a function $f \in C(T^2)$ be 2π-periodic with respect to every variable,

$$f(x_1, x_2) \sim \sum_{(k_1,k_2)} c_{k_1,k_2} e^{i(k_1x_1+k_2x_2)} \qquad (1)$$

be the Fourier series for f, c_{k_1,k_2} are Fourier coefficients for f in the trigonometric system $e^{i(k_1x_1+k_2x_2)}$.

Let W_0 be some domain from R^2 containing the origin inside itself, nW_0 be homothetic transformation of W_0 with the coefficient of homothety n

Then

$$S_n(f; W_0; x_1, x_2) = \sum_{(k_1,k_2)\in nW_0} c_{k_1,k_2} e^{i(k_1x_1+k_2x_2)}$$

are partial sums of (1) corresponding to W_0.

The means

$$R_n(f; x_1, x_2) = \int_{R^2} S_n\left(f; W_0; x_1 - \frac{\gamma u}{n}; x_2 - \frac{\gamma v}{n}\right) d\mu(u, v) \qquad (2)$$

were first introduced in this most general form and investigated in different directions by R.M. Trigub [1]. These means are known as Bernstein-Rogosinski-type means. Here $n \in N, \gamma \in R, \mu$ is finite and normalized Borelian measure on R^2. Generally speaking (2) depend on numerical parameter γ, a choice of a measure μ and a shape of W_0. In discrete case when the measure α_k ($\sum_k \alpha_k = 1$) is concentrated at the points (u_k, v_k) (1) can be reduced to

$$R_n(f; x_1, x_2) = \sum_k \alpha_k S_n\left(f; W_0; x_1 - \frac{\gamma u_k}{n}, x_2 - \frac{\gamma v_k}{n}\right). \qquad (3)$$

The classical one-dimensional Bernstein and Rogosinski means are the special cases of (3) with uniformly disrtibuted measure (1/2 at every point) at two points (different for these means) (see [1] for references). The principal directions of investigation of (2) are first to obtain the conditions of regularity and then to investigate the approximation properties of (2) for different distributions of measure and for different shapes of W_0.

We will concentrate here our attention on regularity of (2) (more exactly (3)) that is on convergence of (3) to corresponding function generating (3) for discrete distributions of the measure. We will restrict ourselves to the standard and natural case where W_0 be a right polygon inscribed into a unit circle.

This problem is solved for some special cases of W_0 and uniform (as a rule) distribution of measure in the situations:

1) W_0 is a square and a measure is concentrated at the vertices of W_0 [1],

2) W_0 is a right triangle and complex measure is concentrated at 6 points namely at the union of vertices of W_0 and points obtained from these ones by rotation about the origin by $\frac{\pi}{6}$ and for the same W_0 and nonhomogeneous measure distributed over the union of mentioned 6 points and the origin [2],

3) W_0 is a right hexagon and nonhomogeneous measure is distributed at the union of points obtained from vertices of W_0 by rotation by $\frac{\pi}{6}$ and the origin [3].

The main result of the paper is formulated in the theorem below

Theorem 1 *Let W_0 is a right polygon with n sides ($n \geq 3$) which is inscribed into a unit circle. A measure is homogeneously distributed at points of this unit circle. Means $R_n(f; x_1, x_2)$ can not be regular. In other words there is no real γ and there is no such the distribution of measure such that R_n are regular.*

The proof of this theorem is different for three different situations:

1) n is odd,

2) n is even and $\frac{n}{2}$ is even,

3) n is even and $\frac{n}{2}$ is odd.

This prof uses the necessary condition of regularity for (2) in $C(T^2)$ obtained by R.M. Trigub [1] (if W_0 is measurable in Jordan sense in $C(T^2)$ and

dW_0 is the boundary of W_0)

$$\sum_k \alpha_k e^{-i\gamma(xu_k+yv_k)} = 0$$

for every $(x, y) \in dW_0$.

The general scheme of proof is very like to one in the cases of [2] and [3].

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Received: 30.09.1999