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ON CONGRUENCES ON n-SEMIGROUPS AND ON THEIR BINARY REDUCES

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Abstract. Let $(A,()_\circ)$ be an n-semigroup $n \geq 3$ with right unit $u_1^{n-1}$and ρ an equivalence relation on the set A .

1. ρ is a congruence relation on the n-semigroup $(A,()_\circ)$ iff for all $a, b \in A$ and for every sequence c_1^{n-1} over A the following statements hold:
$a\rho b \Longrightarrow (c_1, a ,c_2^{n-1})_\circ \rho (c_1, b ,c_2^{n-1})_\circ$ and $(a ,c_1^{n-1})_\circ \rho (b ,c_2^{n-1})_\circ$;

2. moreover, if $u_{n-1}u_1^{n-2}$is left unit in A, ρ is a congruence relation of the n-semigroup $(A,()_\circ)$ iff for all $a, b \in A$ and for every sequence c_1^{n-1} over A $a\rho b \Longrightarrow (c_1, a, c_2^{n-1})_\circ \rho\ (c_1, b, c_2^{n-1})_\circ$.

If ρ is a congruence relation of the n-semigroup (n-group) $(A,()_\circ)$ then ρ is a congruence on binary reduce $red_{\ u_1^{n-2}}(A,()_\circ)(red_c A; c \in A)$. Moreover if $u_1^{n-1}$is a right unit in $(A,()_\circ)$ and for all $a, b \in A$:

$a\rho b \Longrightarrow (u_{n-1}, a, u_1^{n-2})_\circ \rho (u_{n-1}, b, u_1^{n-2})_\circ ((c, a, \bar{c}, \overset{n-3}{c})_\circ \rho (c, b, \bar{c}, \overset{n-3}{c})_\circ)$,
then the converse statement is also true.

This results generalize and improve the result of Usan [8],[9] for n-groups.

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