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THE DOPPER EFFECT ASSOCIATED WITH THE SPACE-TIME METRICS REDUCTION IN THE RADIAL MOVE CASE CANONICAL FORM

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Abstract. This paper presents the reduction to canonical form of the space-time metrics corresponding to a radial move at the speed w. A physics interpretation is also provided.

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1.Notations

In generalized and geometrized spherical coordinates:

$$x^0 = t, \quad x^1 = r, \quad x^2 = \theta, \quad x^3 = \phi \tag{1}$$

with the specifications and notations from the paper *"The aberration of light and space-time metrics reduction in the case of rotation to canonical form"* of the author of this publication the metrics of the quadridimensional space next to a spherical body has the form

$$ds^2 = g_{00}dx^{0^2} + g_{11}dx^{1^2} + g_{22}dx^{2^2} + g_{33}dx^{3^2} \tag{2}$$

Supposing a point $A(r,\theta,\phi)$ next to the body started in the radial direction at the speed w. In metrics (2) we change dx^1 for $dx^1 + wdx^0$. We obtain

$$ds^2 = (g_{00} + g_{11}w^2)dx^{0^2} + g_{11}dx^{1^2} + g_{22}dx^{2^2} + g_{33}dx^{3^2} + 2wg_{11}dx^0dx^1 \tag{3}$$

Noting with

$$\begin{aligned} &v_0 = dx^0,\ v_1 = dx^1,\ v_2 = x^1dx^2, \\ &v_3 = x^1 \sin x^2 dx^3 \end{aligned} \tag{4}$$

metrics (3) will be written as the quadratic form of R^4:

$$ds^2 = a_{00}v_0^2 + a_{11}v_1^2 + a_{22}v_2^2 + a_{33}v_3^2 + 2a_{01}v_0v_1 \tag{5}$$

where

$$a_{00} = g_{00} + g_{11}w^2; \qquad a_{11} = g_{11};$$
$$a_{22} = a_{33} = -1; \qquad a_{01} = wg_{11}. \tag{6}$$

For the Reissner-Nordström metrics

$$g_{00} = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} > 0 \text{ and } g_{11} = -\frac{1}{g_{00}} < 0 \tag{7}$$

The matrix of the quadratic form (5) has the form

$$A = \begin{bmatrix} a_{00} & a_{01} & 0 & 0 \\ a_{01} & a_{11} & 0 & 0 \\ 0 & 0 & a_{22} & 0 \\ 0 & 0 & 0 & a_{33} \end{bmatrix} \tag{8}$$

The reduction of the quadratic form (5) to its canonical form is done through the method of eigenvalues and eigenvectors, becams in this way we can determine an orthonormed basis $B' = \{e_0', e_1', e_2', e_3'\}$ corresponding to an isometry (rotation in R^4).

Solving the characteristic equation

$$p(\lambda) = (-1-\lambda)^2\left[\lambda^2 - (a_{00} + a_{11})\lambda + (a_{00}a_{11} - a_{01}^2)\right] = 0 \tag{9}$$

we obtain the eigenvalues

$$\lambda_0 = \frac{a_{00} + a_{11}\sqrt{\Delta}}{2} > 0; \; \lambda_1 = \frac{a_{00} + a_{11} - \sqrt{\Delta}}{2} < 0; \; \lambda_2 = \lambda_3 = -1 \tag{10}$$

where

$$\Delta = (a_{00} - a_{11})^2 + 4a_{01}^2 > 0. \tag{11}$$

Therefor, metrics (5) becomes

$$ds^2 = \lambda_0 v_0^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 + \lambda_3 v_3^2. \tag{12}$$

After the reduction of metrics (5) to the canonical form (12) we will have the ortonormed basis

$$B' = \{e_0', e_1', e_2', e_3'\} \tag{13}$$

which is obtained by norming the eigenvectors resulting from solving the characteristic system

$$\begin{cases} (a_{00} - \lambda)x_0 & + & a_{01}x_1 & = 0 \\ a_{01}x_0 & + & (a_{11} - \lambda)x_1 & = 0 \\ (-1-\lambda)x_2 & & & = 0 \\ (-1-\lambda)x_3 & & & = 0 \end{cases} \tag{14}$$

by replacing in tours the eigenvalues (10) in this system.

We get

$$\begin{cases} e_0' = \left(\frac{1}{\sqrt{1+\alpha^2}}, \frac{-\alpha}{\sqrt{1+\alpha^2}}, 0, 0\right) \\ e_1' = \left(\frac{\alpha}{\sqrt{1+\alpha^2}}, \frac{1}{\sqrt{1+\alpha^2}}, 0, 0\right) \\ e_2' = (0,0,1,0); \qquad e_3' = (0,0,0,1) \end{cases} \tag{15}$$

$$B_0 = \{e_0 = (1,0,0,0), e_1 = (0,1,0,0,), e_2 = (0,0,1,0), e_3 = (0,0,0,1)\} \tag{16}$$

to basis B' is equal

$$C = \begin{bmatrix} \frac{1}{\sqrt{1+\alpha^2}} & \frac{+\alpha}{\sqrt{1+\alpha^2}} & 0 & 0 \\ \frac{-\alpha}{\sqrt{1+\alpha^2}} & \frac{1}{\sqrt{1+\alpha^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

and

$$\det C = 1 \tag{18}$$

where

$$\alpha = \frac{\sqrt{\Delta} - (a_{00} - a_{11})}{-2a_{01}} \tag{19}$$

From (18) it result that the present transformation is a rotation of the real space with four dimensions. The angular coordinates x^2 and x^3 are not modified.

The rotation angle, γ, is determined from the formulae:

$$\cos\gamma = \frac{1}{\sqrt{1+\alpha^2}}; \quad \sin\gamma = \frac{+\alpha}{\sqrt{1+\alpha^2}}; \quad \text{or} \quad tg\gamma = +\alpha. \tag{20}$$

From relation (20), (19) and (11) it result that

$$\alpha = tg\gamma = \frac{\sqrt{(g_{00}^2 + 1 - w^2)^2 + 4w^2} - (g_{00}^2 + 1 - w^2)}{2w} \tag{21}$$

Taking into (21) $m = 0$, $Q = 0$, we get

$$tg\gamma = \frac{\sqrt{w^4+4}+w^2-2}{2w} = \frac{w}{2}\left(1+\frac{1}{4}w^2-\frac{1}{64}w^6+\frac{1}{516}w^{10}\dots\right) \approx \frac{w}{2} \quad (22)$$

where in the first approximation is the half radial welocity in geometrized units:

$$w = \frac{V_{rad}}{c} \quad (23)$$

The Doppler's formula showst that

$$\frac{\Delta\lambda}{\lambda} = \frac{V_{rad}}{c} = w \quad (24)$$

and

$$\frac{\Delta\lambda}{\lambda} = 2tg\gamma . \quad (25)$$

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