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ANALYSIS OF VERTICAL SPREAD STRATEGIES AND SUGGESTION OF OPTIMAL ALGORITHM FOR PRACTICAL INVESTMENT

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Abstract. Vertical Bull Spread strategy and Vertical Bear Spread strategy are analysed in this paper. Profit functions are described for all possible ways of their creation. Conditions are provided under which particular methods are equivalent in terms of profit function. Based on these theoretical principles optimal algorithm for use of Vertical Spread strategies in practical investment is designed.

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1.Introduction

Vertical Spread strategy is strategy created by combination of long and short position of options of the same type, that means either options type Call or type Put for the same underlying asset with the same expiration time T, but different pricies at expiration. We shall consider the Europen type of options, that means options that can be executed only on the expiration day.

Vertical Bull Spread strategy is used especially if mild increase of expiration price of underlying asset is expected. In case that mild decrease in expiration price of underlying asset is assumed, it is reasonable to use Vertical Bear Spread strategy. The advantage is that during strategy creation it is possible to determine intervals where the strategy will produce profit and where loss. It is also possible to determine the level of profit and loss, while as we show later, both values are limited.

The aim of this paper is to find analytical expressions of profit functions for all particular ways of strategy creation, analyse them and based on these theoretical principles to design optimal algorithm for practical investment.

2.Profit function for Vertical Bull Spread strategy and its analysis

I. Firstly, let us create Vertical Bull Spread strategy by buing n Call options (long position) with lower expiration price x_1 and premium p_{1N} per option along with selling n Call options (short position) with higher expiration price x_2 and premium p_{2P} per option, while still considering option for the same underlying asset and same expiration time T. In the whole paper we assume that $x_1 < x_2$. Let us signify the strategy as VBuCC.

By buing n Call options we have gained the right (not obligation) to buy n pcs of given underlying assent for expiration price x_1 per piece at time T. We have paid premium np_{1N} for that right. It is obvious that if the market price S of this underlying asset at the time T is lower than expiration price x_1, we shall not realise the option right but in opposite case we shall.

Profit function in that case is:

$$P_1(S) = \begin{cases} -np_{1N} & S < x_1 \\ n(S - x_1 - p_{1N}) & S \geq x_1. \end{cases}$$

By selling n Call options we have gained premium np_{2P} and we are obliged to sell n pieces of underlying asset for expiration price x_2 per piece at time T, provided the buyer would like to buy the asset, that is to say, if $S > x_2$ at time T.

Profit function of this transaction is sd follows:

$$P_2(S) = \begin{cases} np_{2P} & \text{if} \quad S < x_2 \\ -n(S - x_2 - p_{2P}) & \text{if} \quad S \geq x_2. \end{cases}$$

Profit function of the whole Vertical Bull Spread strategy which was created by means of Call type option is defined as sum of profit functions of particular transactions. That is

$$P_I(S) = \begin{cases} -n(p_{1N} - p_{2P}) & S < x_1 \\ n(S - x_1 - p_{1N} + p_{2P}) & x_1 \leq S < x_2 \\ n(x_2 - x_1 - p_{1N} + p_{2P}) & S \geq x_2. \end{cases}$$

Graph of the total profit function of Vertical Bull Spread strategy is in this case of the following shape:

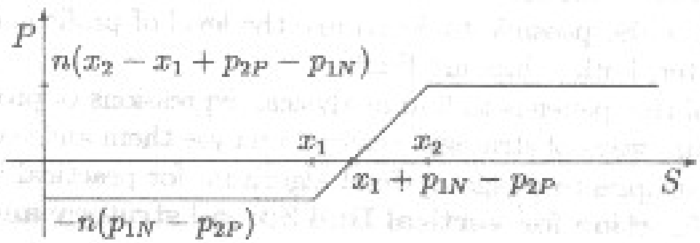

Figure 1: Profit function $P(S)$ for Vertical Bull Spread strategy

Analysing profit function we obtain following statments:

1. Break-event point is $x_1+p_{1N}-p_{2P}$. If $S > x_1+p_{1N}-p_{2P}$, then strategy produces profit. Maximum of profit $P_{\max} = n(x_2 - x_1 - p_{1N} + p_{2P})$ is obtained for $S \geq x_2$.

2. If $S \leq x_1 + p_{1N} - p_{2P}$, then strategy produces loss. Loss reaches its maximum $P_{\min} = -n(p_{1N} - p_{2P})$ if $S \leq x_1$ and it is equal to difference between accepted and paid premium.

3. Only expiration price x_1 and both premiums have direct influence on break-event point. We see that p_{2P} decreases it so it has positive influence.

II. Now let us create Vertical Bull Spread strategy by means of Put option by buying n Put options (long position) with lower expiration price x_1 and premium per $\overline{p_{1N}}$ option along with selling n Put options (short position) with higher expiration price x_2 and premium $\overline{p_{2P}}$ per option.

We again consider options upon the same underlying asset and with the same expiration time T.

Strategy is signified as VBuPP.

In this case profit function has following formula

$$P_{II}(S) = \begin{cases} -n(x_2 - x_1 + \overline{p_{1N}} - \overline{p_{2P}}) & \text{if} \quad S < x_1 \\ n(S - x_2 + \overline{p_{2P}} - \overline{p_{1N}}) & \text{if} \quad x_1 \leq S < x_2 \\ n(\overline{p_{2P}} - \overline{p_{1N}}) & \text{if} \quad S \geq x_2. \end{cases}$$

These statments can be obtained analysing it:

1.

4. The break-event point is $x_2 + \overline{p_{1N}} - \overline{p_{2P}}$, directly dependent on expiration price x_2 and both premiums, while the premium $\overline{p_{2P}}$ has positive influence. (decreases break-event point.)

5. If $S > x_2 + \overline{p_{1N}} - \overline{p_{2P}}$, then strategy produces profit, with maximum $P_{\max} = n(\overline{p_{2P}} - \overline{p_{1N}})$ for $S \geq x_2$.

6. If $S \leq x_2 + \overline{p_{1N}} - \overline{p_{2P}}$, then strategy produces loss. The loss reaches its maximum $P_{\min} = -n(x_2 - x_1 + \overline{p_{1N}} - \overline{p_{2P}})$ for $S \leq x_1$.

Definition 1 *Two strategies are equivalent only if their profit functions are identical.*

Proposition 2 *Strategy VBuCC is equivalent with strategy VBuPP just when*

$$x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} = 0.$$

Proof It follows from definition that stategies are equivalent only if

$$\begin{aligned} -n(p_{1N} - p_{2P}) &= -n(x_2 - x_1 + \overline{p_{1N}} - \overline{p_{2P}}) \\ n(S - x_1 - p_{1N} + p_{2P}) &= n(S - x_2 + \overline{p_{2P}} - \overline{p_{1N}}) \\ n(x_2 - x_1 + p_{2P} - p_{1N}) &= n(\overline{p_{2P}} - \overline{p_{1N}}), \end{aligned}$$

by simple modification we obtain that it is only if

$$x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} = 0,$$

what is condition in proposition 1. As such, proposition 1 is proved.

Proposition 3 *If* $x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} > 0$, *then strategy VBuCC is more profitable than VBuPP strategy.*

Proof From assumption of Proposition 2 we can state that $x_1 + p_{1N} - p_{2P} < x_2 + \overline{p_{1N}} - \overline{p_{2P}}$, so strategy VBuCC has lower break-event point (it is profitable sooner). We can also easily state that

$$n(x_2 - x_1 + p_{2P} - p_{1N}) > n(\overline{p_{2P}} - \overline{p_{1N}})$$

and

$$-n(x_2 - x_1 + \overline{p_{1N}} - \overline{p_{2P}}) < -n(p_{1N} - p_{2P}),$$

and so, Proposition 2 is proved.

Proposition 3 can be proved analogously.

Proposition 4 *If* $x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} < 0$, *then strategy VBuPP is more profitable than VBuCC strategy.*

3.Optimal algorithm for use of Vertical Bull Spread strategy in practical investment

When investing using Vertical Bull Spread strategy, we proceed as follows:

1. If $x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} = 0$,, then VBuCC and VBuPPP strategies have the same profit function. In this case we invest using VBuPP strategy, because we do not need any financial funds for its creation.
2. If $x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} > 0$, then we invest using VbuCC strategy.
3. If $x_2 - x_1 + \overline{p_{1N}} - p_{1N} + p_{2P} - \overline{p_{2P}} < 0$, then we invest using VbuPP strategy.

We can derive from the above-prooved propositions that this algorithm is optimal.

4.Profit functions for Vertical Bear Spread strategy and its analysis

Vertical Bear Spread strategy is obtained in similar way as Vertical Bull Spread Strategy, but long position expiration price is higher now than short positon expiration price.

III. Firstly, let us create Vertical Bear Spread strategy by selling n Call options (short position) with expiration price x_1 and premium p_{1P} per option along with buying n pcs. Call options (long position) with expiration price x_2 and premium p_{2N} per option, considering options upon the same underlying asset and the same expiration time T. Again, it is set that $x_1 < x_2$. Strategy is signified as VBeCC. It is possible to prove that profit function for VBeCC strategy is

$$P_{III}(S) = \begin{cases} n(p_{1P} - p_{2N}) & \text{if } S < x_1 \\ -n(S - x_1 + p_{2N} - p_{1P}) & \text{if } x_1 \le S < x_2 \\ -n(x_2 - x_1 + p_{2N} - p_{1P}) & \text{if } S \ge x_2. \end{cases}$$

Graph of the profit function for VBeCC strategy appears as shown above.

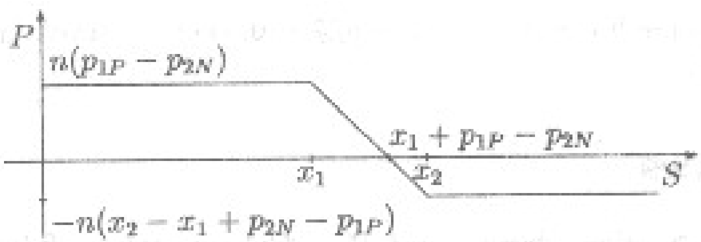

Figure 2: Profit function $P(S)$ for Vertical Bear Spread strategy

IV. Now we shall create Vertical Bear Spread strategy by selling n Put options (short positon) with lower expiration price x_1 and premium $\overline{p_{1P}}$ per option along with buying n Put opitons (long position) with highter expiration price x_2 and premium $\overline{p_{2N}}$ per option while options are upon the same underlying assset and within the same expiration time T. Let us signify the strategy as VBePP. In this case the profit function is:

$$P_{IV}(S) = \begin{cases} n(x_2 - x_1 + \overline{p_{1P}} - \overline{p_{2N}}) & S < x_1 \\ -n(S - x_2 + \overline{p_{2N}} - \overline{p_{1P}}) & x_1 \le S < x_2 \\ n(\overline{p_{1P}} - \overline{p_{2N}}) & S \ge x_2. \end{cases}$$

Analogously, we can prove these propositions:

Proposition 5 *Strategies VBeCC and VBePP are equivalent just when*

$$x_2 - x_1 + \overline{p_{1P}} - p_{1P} + p_{2N} - \overline{p_{2N}} = 0.$$

Proposition 6 *If $x_2 - x_1 + \overline{p_{1P}} - p_{1P} + p_{2N} - \overline{p_{2N}} > 0$, then the VBePP strategy is more profitable than VBeCC strategy.*

Proposition 7 *If $x_2 - x_1 + \overline{p_{1P}} - p_{1P} + p_{2N} - \overline{p_{2N}} < 0$, then VBeCC strategy is more profitable than VBePP strategy.*

5. Optimal algorithm for use of Verical Bear Spread strategy in practical investment

According to above-mentioned knowledge, optimal algorithm for use of Vertical Bear Spread strategy in practical investment is as follows:

1. If $x_2 - x_1 + \overline{p_{1P}} - p_{1P} + p_{2N} - \overline{p_{2N}} = 0$, then VBeCC and VBePP strategies are equivalent in terms of profit (they have the same profit functions). In that case we use the strategy that has lower initial costs, while $PN_{VBeCC} = n(-p_{1P} + p_{2N})$ and $PN_{VBePP} = n(\overline{p_{2N}} - \overline{p_{1P}})$. Provided the initial costs are also the same, then we can use whichever method.
2. If $x_2 - x_1 + \overline{p_{1P}} - p_{1P} + p_{2N} - \overline{p_{2N}} > 0$, then we invest using VbePP strategy.
3. If $x_2 - x_1 + \overline{p_{1P}} - p_{1P} + p_{2N} - \overline{p_{2N}} < 0$, then we invest using VbeCC strategy.

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