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REDUCTIONS OF n-SEMIGROUPS WITH RIGHT UNIT TO k-SEMIGROUPS

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Abstract. In this paper two procedures for reducing polyadic semigroups with right unit are given: k-reduce with respect to given elements, respectively associated k-semigroup of an n-semigroup. The two structures are isomorphic under certain conditions.

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1. Preliminary notions

An *n-groupoid* is a set A, endowed with an n-ary operation $f : A^n \to A$.

We use the following notation: The sequence $x_i, x_{i+1}, ..., x_j$ is denoted by x_i^j. For $j < i$, x_i^j is the empty symbol; if x appears k times, $k \in \mathbb{N}^*$, then the sequence $x, x, ..., x$ will be denoted by $\overset{(k)}{x}$. For $k \leq 0$, the symbol $\overset{(k)}{x}$ is empty. If the sequence x_i^j appears k times, then $x_i^j x_i^j ... x_i^j$ is denoted by $\overset{(k)}{x_i^j}$ too.

An n-groupoid (A, f) is called *n-semigroup* if for any $i \in \{2, ..., n\}$ and all $x_1^{2n-1} \in A$ the following associativity law holds

$$f\left(f(x_1^n), x_{n+1}^{2n-1}\right) = f\left(x_1^{i-1}, f\left(x_i^{i+n-1}\right), x_{i+n}^{2n-1}\right). \tag{1.1}$$

Given $s \in \mathbb{N}^*$, an $s(n-1)+1$–ary operation on the n-semigroup (A, f), the *long product*, is defined as

$$f_{(s)}\left(x_1^{s(n-1)+1}\right) = f(f(...f(f(x_1^n), x_{n+1}^{2n-1}), ...), x_{(s-1)(n-1)+2}^{s(n-1)+1})$$

If the arity is unimportant, $f_{(s)}$ is denoted by $f_{(\cdot)}$.

An $(n-1)$–ad u_1^{n-1} of elements of A is called *right unit*, respectively *left unit*, if for all $x \in A$ we have

$$f\left(x, u_1^{n-1}\right) = x, \tag{1.2}$$

respectively

$$f\,(a_1^{n-1}, x) = x. \tag{1.3}$$

In an $n-$semigroup (A, f) an element $e \in A$ is called a *neutral element* if $f(\overset{(i-1)}{e}, a, \overset{(n-i)}{e}) = a$ for all $a \in A$ and $i = 1, 2, ..., n$.

An $n-$semigroup (A, f) is called i-cancellative, $i \in \{1, 2, ..., n\}$, if for all $x_j \in A$, $j \in \{1, 2, ..., n\} \setminus \{i\}$, $a, b \in A$, the following implication holds

$$f(x_1^{i-1}, a, x_{i+1}^n) = f(x_1^{i-1}, b, x_{i+1}^n) \Rightarrow a = b. \tag{1.4}$$

An $n-$semigroup (A, f) is called *$n-$group* ([3]) if for any $i \in \{1, 2, \ldots, n\}$ and all $a_1^n \in A$, the equation

$$f\,(a_1^{i-1}, x, a_{i+1}^n) = a_i \tag{1.5}$$

has a unique solution in A.

In an n-group (A, f), the unique solution of the equation

$$f(\overset{(n-1)}{a}, x) = a \tag{1.6}$$

is called the *querelement* of a and it is denoted by $\overline{a}$.

Notice that if (A, f) is an n-group, then for any $a \in A$, the sequence $\overset{(i-1)}{a}\ \overline{a}\ \overset{(n-i-1)}{a}$ is a right and a left unit for any $i \in \{1, 2, ..., n-1\}$.

In an n-semigroup (A, f), one may also define recursively

$$x^{[0]} = x;\, x^{[1]} = f(\overset{(n)}{x});\, x^{[k]} = f(x^{[k-1]}, \overset{(n-1)}{x}). \tag{1.7}$$

If (A, f) is an n-group then we can speak about $a^{[k]}$ for all $k \in \mathbb{Z}$. So $a^{[-1]} = \overline{a}$ and for $k < 1, a^{[k]}$ solves the equation

$$f(x, a^{[-k-1]}, \overset{(n-2)}{a}) = a. \tag{1.8}$$

Example 1. Let $\mathbb{Z}_{n-1}$, $n > 2$ be the set of integers modulo $n - 1$. Define on $\mathbb{Z}_{n-1}$ the $k-$ary operation as follows

$$(a_1^k)_* = r, \text{ where } r \equiv a_1 + \cdots + a_k (\text{mod}\, n-1).$$

If k is natural number such that $(k-1)s = n-1$; $k \geq 2$, then $(\mathbb{Z}_{n-1}, ()_*)$ is a commutative $k-$group, having $k-1$ neutral elements, namely: $0, s, 2s, \ldots, (k-2)s$. For $k = 2$, $(\mathbb{Z}_{n-1}, ()_*)$ is the well know cyclic binary group generated by 1. For $k > 2$, this $k-$group is generated by two elements, 0 and 1.

Analogous to the universal algebras ([13]), the notions of subalgebra, congruence, homomorphism can be defined in a similar manner.

2. Reductions and extensions of n-semigroups with right unit.

Reductions and extensions play an important role in the theory of n-ary structures. Here we consider constructions of certain types, Hosszù ([5])and Post ([12]), which are given for n-groups. Extensions of Hosszù type constructions for reducing an n-semigroups to a binary one are proposed by Zupnik ([14]). These results are extended in [6],[7],[10]. Post type theorems for n-semigroups are proposed in [1],[2]. The special case of n-semigroups with right (or left) unit is investigated in [6] and [9].

Here we propose a method for reducing n-semigroups to k-semigroups (in the sense of Post) and give necessary and sufficient conditions for which the two approaches are isomorphic.

Throughout this paper we set $n-1 = s(k-1)$, $k \in \mathbb{N}$, $k \geq 2$.

Let (A, f) be an n-groupoid and let $u_1^{s-1} \in A$ be arbitrary fixed elements. With the operation $g : A^k \to A$ defined as

$$g(x_1^k) = f(x_1, u_1^{s-1}, x_2, u_1^{s-1}, ..., u_1^{s-1}, x_k), \tag{2.1}$$

the pair (A, g) is called *the k-reduce of* (A, f) *relative to* u_1^{s-1}. It is denoted by $red^k_{u_1^{s-1}}(A, f)$.

In [10] we prove that if (A, f) is an n-semigroup, then $red^k_{u_1^{s-1}}(A, f)$ is a k-semigroup ton. If (A, f) is an n-group then $red^k_{u_1^{s-1}}(A, f)$ is a k-group and it is isomorphic with $red^k_{v_1^{s-1}}(A, f)$ for each $v_1^{s-1} \in A$. In particular, if (A, f) is an n-group and $k = 2$, one denotes the binary reduce $red^2_{(c, ..., c)}(A, f)$ by $red_c(A, f)$.

The n-ary extension of a k-semigroup, with $(k-1) \mid (n-1)$, is defined in [10] and [4], in the following way:

Let (A, g) be a k-groupoid, let $c_1^{k-1} \in A$ be arbitrary fixed elements of A and let $\alpha \in End(A, g)$ be an endomorphism of (A, g). The operation $f : A^n \to A$ given by

$$f(x_1^n) = g_{(s+1)}(x_1, \alpha(x_2), \alpha^2(x_3), ..., \alpha^{n-1}(x_n), c_1^{k-1}) \tag{2.2}$$

defines *the n-ary extension of the k-semigroup* (A, g) *relative to the endomorphism* α *and the elements* $c_1^{k-1} \in A$, denoted by $ext^n_{\alpha, c_1^{k-1}}(A, g) = (A, f)$.

Proposition 2.1.([10]) *Let* (A, g) *be a k-semigroup and* $\alpha \in End(A, g)$. *If the elements* $c_1^{k-1} \in A$ *exist such that*

$$g(\alpha^n(x), \alpha(c_1), ..., \alpha(c_{k-1})) = g(c_1^{k-1}, \alpha(x)), \text{ for all } x \in A. \tag{2.3}$$

then the n-ary operation defined by (2.2) induces a structure of n-semigroup on $ext^n_{\alpha, c_1^{k-1}}(A, g)$.

Theorem 2.2.([10], *generalization of Zupnik's Theorem* [14]). *Let* u_1^{n-1} *be a right unit in the n-semigroup* (A, f). *Put* $(A, g) = red^k_{u_1^{s-1}}(A, f)$ *and let* $\alpha \in End(A, g)$ *be given by*

$$\alpha(x) = f(u_s^{n-1}, x, u_1^{s-1}). \tag{2.4}$$

Then the k-reduce (A, g) *is k-semigroup with right unit* $c_1^* c_2^{k-1}$, *where*

$$c_1 = f_{(n+1-s)}(\overset{(n)}{u_s^{n-1}}, u_1^s); \tag{2.5}$$

$$c_i = f(u_s^{n-1}, u_{(i-1)s+1}^{is}); \quad i = \overline{2, k-1} \tag{2.6}$$

$$c_1^* = f(u_s^{n-1}, u_1^s), \tag{2.7}$$

and

$$ext^n_{\alpha, c_1^{k-1}}(red^k_{u_1^{s-1}}(A, f)) = (A, f). \tag{2.8}$$

The corresponding result for k-groups is:

Theorem 2.3.([7]) *Let* (A, g) *be a k-group,* $\alpha \in End(A, g)$, *and let* $c_1^{k-1} \in A$ *be fixed elements. If* $(A, f) = ext^n_{\alpha, c_1^{k-1}}(A, g)$ *defined as (2.2), then* (A, f) *is an n-group if and only if* $\alpha \in Aut(A, g)$ *and (2.3) holds.*

From Theorem 2.2 and Theorem 2.3 we can draw out a corollary that generalizes Hosszú's Theorem [5]:

Corollary 2.4.([7]) *An n-groupoid* (A, f) *is an n-group if and only if* $f : A^n \to A$ *is of the form (2.2), where* (A, g) *is a k-group,* α *is an automorphism of* (A, g) *such that there exist the elements* $c_1^{k-1} \in A$ *and the relation (2.3) holds.*

Remark 1. *The conditions* $\alpha(c_i) = c_i; i = \overline{2, k-1}$ *from the similar generalization of Hosszú's Theorem, which had been studied in* [4], *are not necessary for* $k > 2$.

As an counterexample see [7].

Remark 2. *To reduce n-groups to binary ones the condition* $\alpha(c_1) = c_1$ *is necessary.*

Indeed, because u_1^{n-1} is a right unit in (A, f), then $u_{n-1} u_1^{n-2}$ is a right unit in (A, f). From Theorem 2.3 we have $\alpha : A \to A$; $\alpha(x) = f(u_{n-1}, x, u_1^{n-2})$; $\alpha \in Aut(A, \cdot)$. Since $c_1 = f(\overset{(n)}{u_{n-1}}) = u_{n-1}^{[1]}$:

$$\alpha(c_1) = f(u_{n-1}, u_{n-1}^{[1]}, u_1^{n-2}) = f(u_{n-1}^{[1]}, u_{n-1}, u_1^{n-2}) = u_{n-1}^{[1]} = c_1$$

and condition (2.3) $\alpha^n(x) \cdot \alpha(c_1) = c_1 \cdot \alpha(x)$, implies $\alpha^{n-1}(x) = c_1 \cdot x \cdot c_1^{-1}$.

Therefore α^{n-1} is an inner automorphism of bigroup $(A, \cdot)$. We find again Hosszú's Theorem ([5]):

Theorem 2.5.([5]) *An n-groupoid (A, f) is an n-group if and only if there exists the group $(A, \cdot)$ and $\alpha \in Aut(A, \cdot)$ such that α^{n-1} is an inner automorphism of $(A, \cdot)$ and there exists a $c \in A$ with $\alpha^{n-1}(x) = c \cdot x \cdot c^{-1}$; $\alpha(c) = c$, and*

$$(A, f) = ext^n_{\alpha; c}(A, \cdot).$$

Proposition 2.6.([7]) *If (A, f) is an $n-$group, $a \in A$, then there exist*

the $k-$group $(A, g) = red^k_{u_1^{s-1}}(A, f)$ where $u_1^{s-1} = \overset{(t-1)}{a}\, \bar{a}\, \overset{(s-t-1)}{a}$; $t = \overline{1, s}$,

the automorphism $\alpha \in Aut(A, g)$, where $\alpha(x) = f(\overset{(n-s)}{a}, x, \overset{(t-1)}{a}, \bar{a}, \overset{(s-t-1)}{a})$ and elements $c_1 = a^{[n-s]}$; $c_2 = \ldots = c_{k-1} = a^{[1]}$; such that

$$ext^n_{\alpha, c_1^{k-1}}(red^k_{u_1^{s-1}}(A, f)) = (A, f).$$

Post ([12]) showed the existence of a "covering" group and of an associated group for any n-groups:

Theorem 2.7.([13]) *For any n-group (A, f) there exists a group $(A^*, *)$ and a normal subgroup A_0 of A^* such that:*

1) A is a coset of A^* with respect to A_0;

2) A^*/A_0 is a cyclic group of order $n-1$, generated by the coset A;

3) the n-ary operation f coincides on A with the repeated applying of the binary product $*$.

The subgroup $(A_0, *)$ is called the associated group of n-group A; Post showed that all covering groups $(A^*, *)$ are isomorphic, as well as the associated groups of an n-group.

Theorem 2.8.([6], [9]) *For any n-semigroup (A, f) with a right unit u_1^{n-1} there exists a semigroup $(S, *)$ with unit and a subsemigroup A_0 of S such that:*

1) A is a coset of S with respect to A_0;

2) the n-ary operation f is obtained by applying of the binary product $*$ repeatedly;

3) the binary reduce $red_{u_1^{n-2}}(A, f) \simeq A_0$ if and only if $u_{n-1}u_1^{n-2}$ is also left unit in (A, f).

$(S, *)$ is called the covering semigroup, with $(A_0, *)$ the associated semigroup of n-semigroup A.

In [11] we prove that for n-semigroups with a right unit u_1^{n-1} such that $u_{n-1}u_1^{n-2}$ is a left unit, the two procedures of reduction (by constructing of "Post cover" and associated semigroup respectively binary reduced) lead to naturally equivalent functors. This result generalizes our pervious theorem for n-groups ([8]).

3. Main result.

In the sequel we use an adequate generalization of Theorem 2.8. to show that the results obtained in the case of the binary reduced are preserved for the k-reduced of an n-semigroup, provided $k-1$ divides $n-1$.

Theorem 3.1. *For any n-semigroup (A, f) with a right unit u_1^{n-1} there exists a k-semigroup (S, h) with right unit and a sub-k-semigroup A_* of S such that:*

1) A is a coset of S with respect to A_s;

2) the n-ary operation f coincides with the long product $h_{(s)}$;

3) the k- reduced of (A, f) relative to u_1^{s-1} is isomorphic to A_s,

$$red^k_{u_1^{s-1}}(A, f) \simeq A_s,$$

if and only if $u_s^{n-1}u_1^{s-1}$ is also left unit in (A, f).

(S, h) is called the covering k-semigroup, with (A_s, h) the associated k-semigroup of the n-semigroup A.

Proof. Let ρ be the equivalence relation on $\bigcup_{i=1}^{n-1} A^i$, defined as follows

$$(a_1^i)\rho(b_1^i) \text{ if and only if } f(u_i^{n-1}, a_1^i) = f(u_i^{n-1}, b_1^i). \tag{3.1}$$

We notice that $(a_1^i)\rho(b_1^i)$ if and only if $f(x_1^{n-i}, a_1^i) = f(x_1^{n-i}, b_1^i)$, for all $x_1^{n-i} \in A$; $i = \overline{1, n-1}$. By $\langle a_1^i \rangle$ we denote the class of equivalence with the representant a_1^i.

Let $\langle a_{11}^{1i_1} \rangle, \langle a_{21}^{2i_2} \rangle, ..., \langle a_{k1}^{ki_k} \rangle$ be k equivalence classes from $S = \left(\bigcup_{i=1}^{n-1} A^i \right) / \rho$, and $r \equiv i_1 + i_2 + ... + i_k (\text{mod } n-1)$, $1 \le r \le n-1$. The k-ary operation $h : S^k \to S$ is defined as

$$h(\langle a_{11}^{1i_1} \rangle, \langle a_{21}^{2i_2} \rangle, ..., \langle a_{k1}^{ki_k} \rangle) = \langle c_1^{r-1}, f_{(\cdot)}(c_r^{i_1+...+i_k}) \rangle, \tag{3.2}$$

where $c_1^{i_1+...+i_k} = a_{11}...a_{1i_1}a_{21}...a_{2i_2}...a_{k1}...a_{ki_k}$; i.e. a concatenation.

The pair (S, h) is a k-semigroup with a right unit.

Let $A_i; i = \overline{1, n-1}$ be the subset consisting of classes of sequences with i components, $A_i = \{\langle a_1^i \rangle ; a_1, ..., a_i \in A\}$.

The subsets $A_{ms} = \{< a_1^{ms} >; a_1, \ldots, a_{ms} \in A\}$ consisting of the classes of sequences with ms components, $m = 1, 2, ..., k-1$, are sub-k-semigroups (A_{ms}, h) and $< u_1^s >< u_{s+1}^{2s} > ... < u_{n-s}^{n-1} >$ is a right unit of the sub-k-semigroup (A_s, h) and $h(A_i, \overset{(k-1)}{A_s}) = A_i$.

For all $x \in A$ we have $f(u_1^{n-1}, x) = f(u_1^{n-1}, f(u_1^{n-1}, x))$. That implies $< x >=< f(u_1^{n-1}, x) >$, hence $< x >= h(< u_1^s >, \ldots, < u_{n-s}^{n-1} >, < x >)$ and for any $< a_1^i >\in S$ we have $< a_1^i >= h(< u_1^s >, \ldots, < u_{n-s}^{n-1} >, < a_1^i >)$ too.

Let $(A, g) = red^k_{u_1^{s-1}}(A, f)$, where g is defined in (2.1), and $\alpha : A \to A_s$,

$\alpha(x) =< u_1^{s-1}, x >$. Because

$$\begin{aligned}
\alpha(g(x_1^k)) &= \langle u_1^{s-1}, g(x_1^k) \rangle \\
&= \langle u_1^{s-1}, f(x_1, u_1^{s-1}, x_2, u_1^{s-1}, \ldots, u_1^{s-1}, x_k) \rangle \\
&= h(\langle u_1^{s-1}, x_1 \rangle, \langle u_1^{s-1}, x_2 \rangle, \ldots, \langle u_1^{s-1}, x_k \rangle) \\
&= h(\alpha(x_1), \alpha(x_2), \ldots, \alpha(x_k)),
\end{aligned}$$

α is an homomorphism of k-semigroups.

For all $\langle a_1^s \rangle \in A_s$, from

$$f\,(u_s^{n-1} a_1^s) = f_{(2)}(u_s^{n-1}, u_1^{n-1}, a_1^s) = f\,\left(u_s^{n-1}, u_1^{s-1}, f\,(u_s^{n-1}, a_1^s)\right),$$

it follows that

$$\langle a_1^s \rangle = \left\langle u_1^{s-1}, f\ (u_s^{n-1}, a_1^s) \right\rangle = \alpha\ \left(f\ (u_s^{n-1}, a_1^s)\right);$$

therefore α is surjective.

If $\alpha(x) = \alpha(y)$, then

$$\langle a_1^{s-1}, x \rangle = \langle u_1^{s-1}, y \rangle,$$

hence

$$f\,(u_s^{n-1} u_1^{s-1}, x) = f\,(u_s^{n-1}, a_1^{s-1}, y)$$

So $x = y$ if and only if $u_s^{n-1} u_1^{s-1}$ is a left unit of (A, f). Thus α is an isomorphism of k-semigroups (A, g) and (A_s, h). $\square$

Moreover if (A, f) is a cancellative $n-$semigroup having an $n-1-$adic right unit, then its Post type covering $k-$semigroup is unique up to an isomorphism.

The corresponding result for n-groups is:

Theorem 3.2. *For any n-group (A, f) and for any element $a \in A$ there exists a k-group (S, h) and a seminvariant sub-k-group A_s of S such that*

1) A is a coset of S with respect to A_s;

2) S/A_s is a k-group of order $n-1$ generated by the cosets A and A_{n-1};

3) the n-ary operation f in A coincides with the long product $h_{(s)}$;

4) the k- reduced of (A, f) relative to $\overset{(s-1)}{a}, a \in A$ is isomorphic with A_s.

Proof. For $n-$groups any right unit is left unit too. The equivalence relation ρ from the proof of Theorem 3.1 implies $x\rho y$ if and only if $x = y$. The $k-$semigroup (S, h) is an $k-$group because for any $\langle a_1^i \rangle \in S$ there exists the querelement $\overline{\langle a_1^i \rangle}$

$h(\langle a_1^i \rangle, \overset{(k-1)}{A_s}) = h(\overset{(k-1)}{A_s}, \langle a_1^i \rangle)$. By Theorem 2.2, $red^k_{u_s^{n-1}}(A, f)$ is a $k-$group with unit $c_1^s c_2^{k-1}$

$$c_1^s c_2^{k-1} = f(u_s^{n-1}, u_1^s) f(u_s^{n-1}, u_{s+1}^{2s}) \ldots f(u_s^{n-1}, u_{(k-2)s+1}^{n-1})$$

and

$$\alpha(c_i) = \langle u_1^{s-1}, c_i \rangle = \left\langle u_1^{s-1}, f(u_s^{n-1}, u_{(i-1)s+1}^{is}) \right\rangle = \left\langle u_{(i-1)s+1}^{is} \right\rangle, \forall i = 2, ..., k$$

$$\alpha(c_1^s) = \langle u_1^{s-1} c_1^s \rangle = \langle u_1^{s-1}, f(u_s^{n-1}, u_1^s) \rangle = \langle u_1^s \rangle$$

In the special case, for $u_1^{n-1} = \overset{(n-2)}{a} \; \overline{a}$, we have $c_1^* = c_2 = \ldots = c_{k-2} = a$; $c_{k-1} = \overline{a}$ and $red^k_{\substack{(n-1) \\ a}}(A, f) \simeq (A_s, h)$.

The mapping $\beta : S/A_s \rightarrow \mathbb{Z}_{n-1}$; $\beta(A_i) = \begin{cases} i, & \text{if } i \in \{1, 2, \ldots, n-2\} \\ 0 & \text{if } i = n-1 \end{cases}$ is an isomorphism of $k-$ groups, where $(\mathbb{Z}_{n-1}, (\ )_*)$ is the $k-$group defined in Example 1.

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