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A MATHEMATICAL MODEL FOR A DECISION PROBLEM IN MEDICINE

Luciana NEAMŢIU

Abstract. The paper presents a mathematical model and a solution for a medical decision problem about prioritising interventions for a number of individual surgical patients.

The necessary amount of money for all needed intervention exceed the alloted budget. The problem is how to choose the patients for surgery. The list of demands contains for each patient: the healts state, the life expectency, the cost of surgery, the job, the family condition and if the patient is guilty for the disease.

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1. Introduction

An important problem in health economics is to decide how to spend a limited budget. Usualy pharmacoeconomics uses cost-effectiveness analysis, cost benefit analysis, cost minimization analysis or cost-utility analysis.

As an example we have a budget of 7.000 m.u. (monetary units) and the following cases:

Case 1: Mr.Harper, a 68 year old retired tool-maker with two grown up children, has prostate cancer and his consultant has suggested surgery Mr. Harper will live for 10 years in a health state of 0,9. With surgery, Mr. Harper will live for 13 years, but due to problems of incontinence and impotence associated with surgery, his health state will be rated as 0,6. Cost of surgery: 3.200 m.u.

Case 2: Mrs. Patel, aged 58, cares for her 2 year old grand-daughter works and also looks after her husband and two grown-up sons who live with her. She will live for 20 additional years whether or not she receives a hip replacement. The hip replacement will, however, improve her quality of life by a factor of 0,2 for those 20 years. Cost of surgery: 4.100 m.u.

Case 3: Mrs. Hargreaves, aged 69, a non-smoking widow with no children, will die without cardiac surgery. With surgery, she will live an additional ten years in a health state rated as 0,7 of perfect health. Cost of surgery: 6.900 m.u.

Case 4: Marta, a 25 year old secretary, has a tattoo on her neck with a flower saying. "Dean" in the middle. This is the name of an abusive boyfriend whom Marta has escaped by moving town. She says the tattoo is psychologically damaging and has affected her employment opportunities and she rates her health state as 0,8. She says her health state would improve to full health if the tattoo were removed. Marta is expected to live until age 79. Cost of surgery: 3.100 m.u.

Case 5: Javinder, a 7 year old with cystic fibrosis, is expected to live for another 6 months without a heart and lung transplant in a health state rated at 0,3 of perfect health. With a transplant, he can expect to live for 10 years in a health state rated as 0,7. Cost of surgery: 7.000 m.u.

Case 6: Patrick, a 35 year old anesthetist, had a vasectomy 8 years ago during his first marriage and has no children. He has now remarried and he and his wife, aged 37, are extremely keen to have children. He wants a vasectomy reversal operation. His health state, currently rated as perfect, is unlikely to be affected by whether or not he has the operation although his doctor considers that his wife may be at risk of depression if she is unable to have children. Patrick's life expectancy is 75 years. Cost of surgery: 900 m.u.

Case 7: Mr. Moss, a divorced manual worker aged 48 who has smoked 40 cigarettes a day for the last 28 years but has now stopped, has one teenaged child at home and another whom he is supporting in her college studies. He has already had one severe myocardial infarction and needs a coronary artery bypass graft (CABG). He is likely to die without surgery will live for 10 years with a health state rated as 0,8 of perfect health. Cost of surgery: 5.300 m.u. We have to decide which of these interventions to fund. First we consider a cost-utility approach.

If we calculate the QALYs gained (QALYs gained = QALYs with surgery - QALYs without surgery, where QALYs = health state × live years), we obtain

Name of case	Rank	QALYs gained	Cost of intervention	Cost per QALY gained
Mr Harper	7	-1.2	3200 u.m.	-2666 u.m. / QALY
Patrick	6	0	900 u.m.	–
Marta	1	15.8	3100 u.m.	196
Mr Moss	2	8	5300 u.m.	662
Mrs Hargreaves	3	7	6900 u.m.	986
Javinder	4	6.85	7000	1022
Mrs Patel	5	4	4100	1025

A cost-utility analysis seeks to maximise the overall number of QALY gained and minimise the cost per QALY gained. This table would definitely counsel against Mr Harper (prostate cancer), as he would not only result in a loss of QALYs but would cost money into bargain. The table would also suggest no benefit in treating Patrick (vasectomy), since no QALYs can be gained but again costs would be incurred. The best value would appear to be offered by Marta (tattoo removal), who would gain 15.8 QALYs at a very reasonable cost, while Mrs Patel (hip replacement) would seem to offer the highest cost per QALY gained.

This analysis does not take into account issues such as the role of patients in supporting others, the contribution that individuals may have made to their problems by their own lifestyle (e.g. by smoking or agreeing to have a tattoo). We are not satisfied for the cost-utility solution - paying the surgery for the tattooed person. She was not obliged to make her the tattoo and in other distribution of the founds a life could be saved. We propose to generalize the problem, to attach a mathematical model to solve and to analyze the solution for the particular case presented above.

2. Mathematical model

We have a sum S of money which can be spent for a number of surgery. We also have n demands for surgery for patients P_1, $P_2, \ldots, P_n$. The problem is how to choose the patient to be operated. For each patient P_i we know:

- age t_i,
- cost of the surgery c_i,
- the modification of the quality of life after the surgery q_i,
- if the surgery is vital $v_i = 1$, or not $v_i = 0$,
- if the patient supports persons who has not income other then the patient's salary or pension and there number,
- if the patient looks after others members of the family and there number,
- if the patient P_i is guilty for the disease,
- if the surgery modifyes other person quality of life.

Knowing all these data we'll give a mathematical model for the problem introducing 3 variables same type as quality of life:

- social coefficient s,
- guilty coefficient g, and
- responsibility for other persons coefficient r.

The guilty g and the responsibility r are measured on a scale from 0 to 1. For the social coefficient we have other 2 "subjective" variables (supporting grade z, and attendance grade a, which are also measured from 0 to 10:

0 if the patient has no one to look after,

10 if the patient has no other income and can't to fend for oneself).

If the patient P_i supports m_i persons with responsibility grade $z_{i1}, ..., z_{im_i}$ and looks after n_i persons with responsibility grade $a_{i1}, ..., a_{in_i}$ then

$$s_i = \sum_{k=1}^{m_i} z_{ik} + \sum_{k=1}^{n_i} a_{ik}.$$

Let us introduce the variables x_i, $i \in \{1, ..., n\}$, which can be only 0 or 1 :

- $x_i := 1$ means that the pacient P_i is chosen for surgery ;
- $x_i := 0$ means that the pacient P_i is not chosen for surgery.

Let the functions:

$$f_1 : \{0,1\}^n \to \mathbf{R}, \; f_1(x) = \sum_{i=1}^{n} q_i x_i, \text{ for each } x = (x_1, ..., x_n) \in \{0,1\}^n,$$

$$f_2 : \{0,1\}^n \to \mathbf{R}, \; f_2(x) = \sum_{i=1}^{n} s_i x_i, \text{ for each } x = (x_1, ..., x_n) \in \{0,1\}^n,$$

$$f_3 : \{0,1\}^n \to \mathbf{R}, \; f_3(x) = \sum_{i=1}^{n} u_i x_i, \text{ for each } x = (x_1, ..., x_n) \in \{0,1\}^n,$$

$$f_4 : \{0,1\}^n \to \mathbf{R}, \; f_4(x) = \sum_{i=1}^{n} r_i x_i, \text{ for each } x = (x_1, ..., x_n) \in \{0,1\}^n,$$

$$f_5 : \{0,1\}^n \to \mathbf{R}, \; f_5(x) = \sum_{i=1}^{n} x_i, \text{ for each } x = (x_1, ..., x_n) \in \{0,1\}^n,$$

$$f_6 : \{0,1\}^n \to \mathbf{R}, \; f_6(x) = \sum_{i=1}^{n} g_i x_i, \text{ for each } x = (x_1, ..., x_n) \in \{0,1\}^n.$$

Then, the problem comes back to the following multicriteria programming problem

$$(P)\quad \begin{cases} \sum_{i=1}^{n} q_i x_i \to \max \\ \sum_{i=1}^{n} s_i x_i \to \max \\ \sum_{i=1}^{n} v_i x_i \to \max \\ \sum_{i=1}^{n} r_i x_i \to \max \\ \sum_{i=1}^{n} x_i \to \max \\ \sum_{i=1}^{n} g_i x_i \to \min \\ \text{with conditions} \\ \sum_{i=1}^{n} c_i x_i \leq S, \\ x_i \in \{1, 0\}, \text{ for each } i \in \{1, ..., n\}. \end{cases}$$

Using the pounds method and having the pounds $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ with $\lambda_i > 0$, for $i \in \{1, ..., 6\}$ and $\sum_{i=1}^{6} \lambda_i = 1$ any solution of the problem

$$(Q)\quad \begin{cases} g(x) \to \max \\ x \in X \end{cases}$$

where $g : \{0,1\}^n \to \mathbf{R}$, is given by

$$g(x) = \sum_{j=1}^{5} \lambda_j f_j(x) - \lambda_6 f_6(x), \text{ for all } x \in \{0,1\}^n,$$

and

$$X = \{x = (x_1, ..., x_n) \in \{0,1\}^n \mid \sum_{i=1}^{n} c_i x_i \leq S\},$$

can be considered as a solution of the multicriteria problem.

3. Example

We reconsider the initial example and we'll solve the problem using our method. We have:

$$c_1 = 3.200,\ v_1 = 0,\ q_1 = -1,2,\ g_1 = 0,\ r_1 = 0,\ s_1 = 0.$$

$$c_2 = 4.100,\ v_2 = 0,\ q_2 = 4,\ g_2 = 0,\ r_2 = 0,\ s_2 = 5.$$

$$c_3 = 6.900,\ v_3 = 1,\ q_3 = 7,\ g_3 = 0,\ r_3 = 0,\ s_3 = 0.$$

$$c_4 = 3.100,\ v_4 = 0,\ q_4 = 15,8,\ g_4 = 1,\ r_4 = 0,\ s_4 = 0.$$

$$c_5 = 7.000,\ v_5 = 1,\ q_5 = 6,85,\ g_5 = 0,\ r_5 = 0,\ s_5 = 0.$$

$$c_6 = 900,\ v_6 = 0,\ q_6 = 0,\ g_6 = 1,\ r_6 = 0,20,\ s_6 = 0.$$

$$c_7 = 5.300,\ v_7 = 1,\ q_7 = 8,\ g_7 = 1,\ r_7 = 0,\ s_7 = 15.$$

Taking

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 2\lambda_5 = 2\lambda_6 = \frac{1}{8},$$

function g is given by

$$g(x_1, x_2, ..., x_7) = \frac{1}{8}(-0,4x_1 + 15x_2 + 18x_3 + 32,6x_4 + 17,7x_5 + 1,2x_6 + 40x_7).$$

The optimal solution of problem (Q) is

$$x = (0,0,0,0,0,1,1).$$

Hence Mr Moss and Patrick are chosen for surgery.

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Ion Chiricuţă Oncological Institute,
Str. Republicii 34-36, RO-3400
Cluj-Napoca,
luciana@onco.codec.ro