

Bul. Ştiinț. Univ. Baia Mare, Ser. B, Matematică-Informatică, Vol. XVIII(2002), Nr. 2, 303 - 310 (in Romanian). This paper is a continuation of the previous communication which concerned the optimization theory for non-determined models and was also submitted to the journal "Buletinul Institutului Politehnic din Baia Mare", Vol. 18, No. 2, 2002.

OPTIMAL FEEDBACKS ON INACCURATE MEASUREMENTS FOR NON-DETERMINED MODELS

Abstract. The synthesis problem of optimal controls of feedback type for non-determined models and inaccurate measurements of the dynamic system current states is considered. The algorithm of operating optimal controller which is able to calculate values of optimal feedback during each particular control process in real time is described. MSC: 49N30, 93C41

Keywords: optimal feedback, non-determined model, inaccurate measurements, discret control

Introduction

The classical theory of optimal feedback is based on determined mathematical models of control systems and assumptions that during the control process accurate values of current states are accessible. The use of non-determined mathematical models and the supposition that current states of a control system are measured with a bounded accuracy enrich the arsenal of feedbacks which can be realized. Under the assumption that during a control process an accurate value of a current state is accessible three types of optimal feedbacks may be built [2,3]. Inaccurate measurements produce new types of optimal feedbacks. In the paper the simplest one is considered. This type may be interpreted as the generalization of a feedback constructed from the accurate values of the current states [3].

3.1 Problem statement. Open-loop control

Let $T = [t_*, t^*]$ be a control interval, $h = (t^* - t_*)/N$ be a quantization period; N is a natural number, $T_h = \{t_*, t_* + h, \dots, t^* - h\}$. A scalar function $u(t)$, $t \in T$, is said to be a discrete control if it takes the form $u(t) = u(t_* + kh)$, $t \in [t_* + kh, t_* + (k+1)h]$, $k = 0, N-1$.

In the class of discrete controls consider a linear optimal control problem

$$\begin{aligned} &c'x(t^*) \rightarrow \max, \quad \dot{x} = A(t)x + b(t)u + w, \quad x(t_*) = x_0 \in X_*, \\ &x(t^*) \in X^* = \{x \in R^n : g_* \leq Hx \leq g^*\}, \quad |u(t)| \leq 1, \quad t \in T, \end{aligned} \quad (1.1)$$

where $x = x(t)$ is an n -vector of a state of the control system at moment t ; $u = u(t)$ is a value of a scalar (one-dimensional) control; $w = w(t) \in R^n$, $t \in T$, is a piecewise continuous disturbance satisfying the condition $\|w(t)\| \leq \omega$, $t \in T$, ω is a given value; x_0 is an initial state whose value is not known exactly and a priori information on it is exhausted by the inclusion in a given bounded set $X_* = \{x \in R^n : \|x - x_0\| \leq \chi\}$ to be said a prior distribution of initial state; x_* is a given n -vector, χ is a given scalar, $\|x\| = \max |x_i|$, $i = 1, n$; $A(t)$, $t \in T$, is a piecewise continuous $n \times n$ -matrix function, characterizing dynamical features of the control object, $b(t)$, $t \in T$, is a piecewise continuous n -vector-function, characterizing an input unit of the system; $H \in R^{m \times n}$ is a given matrix, characterizing output signal; $g_*, g^* \in R^m$ are given vectors of the output signal constraints.

Multivalued function

Let us introduce the notion of multivalued function. In differential inclusions the solution set $X(t, u_t(\cdot)) = \{x(t|t_*, x_0, u_t(\cdot), w_t(\cdot)), x_0 \in X_*, w_t(\cdot) \in W_t(\cdot)\}, t \in T$, (1.2) is said to be a trajectory of non-determined system (1.1), corresponding to the control $u_t(\cdot) = (u(s), s \in T^t)$, $T^t = [t_*, t]$, where $u_t(\cdot) = (w(s), s \in T^t)$, $W_t(\cdot) = \{w(s) : \|w(s)\| \leq \omega, s \in T^t\}$; $x(t|t_*, x_0, u_t(\cdot), w_t(\cdot))$, $t \in T$, is an individual trajectory of determined system (1.1), corresponding to the initial state $x(t_*) = x_0$, control $u_t(\cdot)$ and disturbance $w_t(\cdot)$. A set $X(t, u_t(\cdot))$ is said to be a priori distribution set of the current state $x(t)$ of system (1.1) at moment t .

A discrete control $u(t)$, $t \in T$, is said to be an accessible (guaranteed) open-loop control if $|w(t)| \leq 1$, $t \in T$ and $X(t^*, u_t(\cdot)) \subset X^*$. The quality of the accessible control is evaluated by the functional $J(u) = \min c'x$, $x \in X(t^*, u_t(\cdot))$. An accessible control $u^0(t)$, $t \in T$, is said to be an optimal (guaranteed) open-loop control if $J(u^0) = \max J(u)$.

Describe the approach to the program optimization of system (1.1). An individual trajectory $x(t|t_*, x_0, u_t(\cdot), w_t(\cdot))$, $t \in T$, of system (1.1) satisfies the i -th terminal constraint if the inequalities

$$g_{*i} \leq h_{(i)}^t x(t^*|t_*, x_0, u_t(\cdot), w_t(\cdot)) \leq g_i^*, \quad i \in I, \quad (1.3)$$

are fulfilled. Here $h_{(i)}$ is the i -th row of the matrix H .

Up to the beginning of the control process the true value of the state x_0 and disturbance, $w(t)$, $t \in T$, to come are unknown.

The only available information is $x_0 \in X_*$, $\|w(t)\| \leq \omega$, $t \in T$. Conditions (1.3) are fulfilled under the fixed $u_t(\cdot)$ for all probable initial states $x_0 \in X_*$, disturbances $w_t(\cdot) \in W_t(\cdot)$ and $i \in I = \{1, 2, \dots, m\}$ if and only if no feasible set $\{x(t^*)\} = T$ and $\{x(t^*)\} = \{x(t^*|t_*, x_0, u_t(\cdot), w_t(\cdot))\} = \Delta$, Δ is a union function $\bigcup_{t \in T} \Delta(t) = \Delta$, $\Delta(t+k-1) \subset \Delta(t)$, $\Delta(t) = \Delta(t+k)$ and $\Delta(t) = \Delta(t+k)$ solution of horizon algorithm, considering $\bar{g}_{*i} \leq \int h_{(i)}^t F(t^*, s)b(s)u(s)ds \leq \bar{g}_i^*$, $i \in I$, $u(s)$ to satisfy (1.4)

$$(1.4) \quad \begin{aligned} & \mathcal{F}(t^*, s) = \int_{t^*}^s F(t^*, s)b(s)u(s)ds = \mathcal{F}(t^*, t^*) + \int_{t^*}^s F(t^*, s)du(s) \\ & \mathcal{F}(t^*, t^*) \geq 0, \quad \mathcal{F}(t^*, t^*) \leq 0, \quad \mathcal{F}(t^*, t^*) = 0, \quad \mathcal{F}(t^*, t^*) = 0 \end{aligned}$$

Here state function $x(t)$ is given by (1.1) and the control $u(t)$ is defined by (1.2).

$$\begin{aligned} F(t, s) &= F(t)F^{-1}(s), \quad \dot{F} = A(t)F, F(t_*) = E; \\ g_{*i} &= g_{*i} - \alpha_{*i}, \quad \alpha_{*i} = h'_{(i)}F(t^*, t_*)x_* - \chi\|h'_{(i)}F(t^*, t_*)\|^* + \omega \int_T^{t^*}\|h'_{(i)}F(t^*, s)\|^*ds; \\ g_i^* &= g_i^* - \alpha_i^*, \quad \alpha_i^* = h'_{(i)}F(t^*, t_*)x_* + \chi\|h'_{(i)}F(t^*, t_*)\|^* + \omega \int_T^{t^*}\|h'_{(i)}F(t^*, s)\|^*ds; \|z\|^* = \sum_{i=1}^n|\tilde{z}_i|, \end{aligned}$$

Evaluating in a similar manner the quality of the accessible control

$$\begin{aligned} J(u) &= c'F(t^*, t_*)x_* - \chi\|c'F(t^*, t_*)\|^* - \omega \int_T^{t^*}\|c'F(t^*, s)\|^* + \\ &+ \int_T^{t^*}c'F(t^*, s)b(s)u(s)ds \end{aligned}$$

one can easily reduce the problem of calculating the optimal control $u^0(t)$, $t \in T$ to the determined problem whose dynamic form is

$$c'x(t^*) \rightarrow \max, \quad \dot{x} = A(t)x + b(t)u, \quad x(t_*) = 0, \quad (1.5)$$

$$\bar{g}_* \leq Hx(t^*) \leq \bar{g}^*, |u(t)| \leq 1, t \in T,$$

where $\bar{g}_* = (\bar{g}_{*i}, i \in I)$; $\bar{g}^* = (\bar{g}_i^*, i \in I)$.

Thus the problem for the program optimization of the non-determined dynamic system with an unknown initial state and disturbance is reduced to the investigation of the determined linear optimal control problem for the determined dynamic system by the approach [1].

To obtain the numerical solution of problem (1.1) one can use a fast dual algorithm from [1]. In [1] there is also an algorithm for constructing the positional solution of the problems of type (1.5) in case $T = \mathbb{R}$ and its modification for solving the problems of type (1.5) in case $T = \mathbb{Z}$.

2 Optimal feedbacks constructed from the current measurements

Optimal programs constructed at the previous section are useful to begin the control possesses. In the majority of applications of the control theory preference is given to the controls of feedback type.

Let us suppose that in the control posses at every current moment $\tau \in T_h$ only the inaccurate measurement $y(\tau)$ of the current state $x(\tau)$ is in our disposal: $\|y(\tau) - x(\tau)\| \leq \xi$.

where ξ measurement accuracy, $\xi > 0$. Thus, it is obvious, that the current state $x(\tau)$ belongs to the set $X_c(\tau, u_r(\cdot)|\tau, y(\tau)) = \{x \in R^n : \|y(\tau) - x\| \leq \xi\}$.

The set $X_c(\tau, u_r(\cdot)|\tau, y(\tau))$ is said to be sensory distribution of the current state. Let us consider, that the accessible technical resources allow to process at every current moment $\tau \in T_h$ only a prior information and current measurement. Thus feedback takes form $u(\tau, y(\tau))$. The set $X(\tau, u_r(\cdot)|\tau, y(\tau)) = X_c(\tau, u_r(\cdot)|\tau, y(\tau)) \cap X(\tau, u_r(\cdot))$ including all states $x(\tau)$ of system (1.1) to be compatible with the current measurement $y(\tau)$ and the prior distribution $X(\tau, u_r(\cdot))$ is said to be primary distribution of the state $x(\tau)$. Imbed problem (1.1) into a family of problems

$$\begin{aligned} c'x(t^*) \rightarrow \max_{\tau} \quad & \dot{x} = A(t)x + b(t)u + w, \quad x(\tau) \in X(\tau, u_r(\cdot)|\tau, v), \\ & x(t^*) \in X^*, \quad |u(t)| \leq 1, \quad \|w(t)\| \leq \omega, \quad t \in T(\tau) = [\tau, t^*], \end{aligned} \quad (2.1)$$

depending on a scalar $\tau \in T_h$ and an n -vector v . Let $u^0(t|\tau, v)$, $t \in T(\tau)$, be an optimal open-loop control of problem (2.1) for the position (τ, v) , Y_τ be a set of all vectors $v \in R^n$ for which problem (2.1) has an open-loop solution for a fixed moment τ .

A function

$$u^0(\tau, v) = u^0(\tau|\tau, v), \quad v \in Y_\tau, \quad \tau \in T_h, \quad (2.2)$$

is said to be an optimal (disconnected, discrete) control of feedback type (constructed from inaccurate measurements of current states) for problem (1.1). The optimal control system is said to be system (1.1) closed by feedback (2.2):

$$\dot{x} = A(t)x + b(t)u^0(t, y) + w, \quad x(t_*) \in X^*, \quad \|w(t)\| \leq \omega, \quad t \in T. \quad (2.3)$$

The trajectory of nonlinear (non-determined) system (2.3) is thought to be the trajectory of the linear (non-determined) system $\dot{x} = A(t)x + b(t)u + w$, $x(t_*) \in X^*$, $\|w(t)\| \leq \omega$, $t \in T$, with the control $u = u(t) = u^0(\tau, y(\tau))$, $t \in [\tau, \tau + h]$, $\tau = t_* + kh$, $k = \overline{0, N+1}$.

Building of optimal feedback (2.2) is said to be the synthesis of the optimal system. To realize the optimal synthesis in an explicit form is impossible. Following [3], describe the method of the optimal feedback realization in real time. This process is based on the analysis how the optimal feedback is used in the real control process.

Suppose, that optimal feedback (2.2) has been built. Denote by $x(t_*) = x_0^*$ an initial state by $w^*(t)$, $t \in T$, a disturbance and by $y^*(t)$, $t \in T_h$, an inaccurate measurements realized in some particular control process.

They generate the individual trajectory $x^*(t)$, $t \in T$, of closed system (2.3) to satisfy the identity

$$\text{identity condition } \dot{x}^*(t) = A(t)x^*(t) + b(t)u^0(t, y^*(t)) + w^*(t), \quad t \in T. \quad (2.4)$$

From (2.4) it is obvious that in control process feedback (2.2) is not used completely. Its values $u^*(t) = u^0(t, y^*(t))$, $t \in T_h$ are necessary only along the isolated sequence $y^*(t)$, $t \in T_h$. Moreover, for each current moment $\tau \in T$ the value $u^*(\tau) = u^0(\tau, y^*(\tau))$ isn't to be known beforehand; it is enough to know how to calculate it at moment τ , when the

measurement $y^*(\tau)$ is obtained, as the functions $u^*(t)$, $t \in T_h$, $u^*(t)$, $t \in T$, are identical due to the discrete control definition. The function $u^*(t)$, $t \in T_h$ is said to be a realization of optimal feedback in a particular process.

We'll say, that it is possible to construct $u^*(t)$, $t \in T$ (the realization of optimal feedback (2.2)) in real time if at every current position $(\tau, y^*(\tau))$ time $s(\tau)$ for the calculation of the value $u^*(\tau) = u^0(\tau, y^*(\tau))$ doesn't exceed h . A device able to calculate the value $u^*(\tau) = u^0(\tau, y^*(\tau))$ in real time is said to be the optimal controller. The signal $u^*(\tau)$ is given to the input of system (2.3) from moment $\tau + s(\tau)$. Up to this moment the control $u^*(t)$, $t \in [\tau - h, \tau + s(\tau)]$ is thought to be equal $u^0(\tau - h, \tau + h, y^*(\tau - h))$.

Thus the problem of the synthesis of optimal system in the new statement is reduced to constructing an algorithm of operating the optimal controller.

Suppose that an algorithm of the optimal controller has been created. Up to the beginning of the control process the optimal controller carries out the following preliminary operations.

The set of possible results of the initial state measurements $Y(t_*) = \{y \in R^{n+1} : \|y - z\| \leq \xi, z \in X_*\}$ is covered with sufficiently dense finite ε -grid $Y^*(t_*)$. Before the beginning of the control process the optimal controller calculates optimal open-loop solutions of problem (2.1) for nodes $y_i \in Y^*(t_*)$, $i = 1, l$. The time for this operations isn't limited. Let $u^0(t|t_*, y_i)$, $t \in T$, be an open-loop control of problem (2.1) for the node y_i .

The control process begins at moment t_* , when the controller receives the signal $y(t_*)$ about the value of the initial state x_0 of system (1.1).

The optimal controller seeks the closest to $y(t_*)$ node y_h of the grid $Y^*(t_*)$ and corrects the optimal open-loop control $u^0(t|t_*, y_h)$, $t \in T$, to the control $u^0(t|t_*, y(t_*))$, $t \in T$ by the dual method [1].

Then it sets $u^*(t) = u^0(t)$, $t \in [t_*, t_* + s(t_*)]$, $u^*(t) = u^0(t_*|t_*, y(t_*))$, $t \in [t_* + s(t_*), t_* + h + s(t_* + h)]$.

Suppose that the optimal controller has been operating on the interval $[\tau, \tau + s(\tau)]$, having produced the control signals $u^*(t_*)$, $u^*(t_* + h)$, ..., $u^*(\tau)$. Denote by $u^*(t)$, $t \in [\tau, \tau + h]$ a disturbance realized up to moment $\tau + h$, by $y^*(\tau + h)$ a value of the measurement of the current state $x^*(\tau + h)$. To calculate the current value $u^*(\tau + h)$ of the control according to definition (2.2) the optimal controller has to know the open-loop solution $u^0(t|\tau + h, y^*(\tau + h))$, $t \in T(\tau + h)$ to problem (1.1) for the position $(\tau + h, y^*(\tau + h))$.

By the assumption the optimal controller has already calculated the value of $u^*(\tau) = u^0(\tau|t_*, y^*(\tau)) = u^0(\tau|\tau, y^*(\tau))$ at previous moment $\tau \in T_h$ having solved the problem

$$c'x(t^*) \rightarrow \max, \quad \dot{x} = A(t)x + b(t)u, \quad x(\tau) = 0, \quad (2.5)$$

$(A + B^T)(Q, B - VV^T)$ for the given initial condition $x(\tau) = 0$ and the final time $T = (\tau + h) + s(\tau)$. In addition $c'(t^*) \geq \bar{g}_*(\tau) \leq Hx(t^*) \leq \bar{g}^*(\tau)$, $|u(t)| \leq 1$, $t \in T(\tau)$.

Let's consider the case when $\tau + h$ from (2.5) is taken as the next node of time at the present position and $\tau + h + s(\tau)$ from (2.2) is taken as the next node of time at the present position. Then we have to solve the problem (2.5) under the condition $x(\tau) = 0$ and $x(\tau + h) = 0$ and the final time $T = (\tau + h) + s(\tau) + s(\tau + h)$.

where $\tilde{g}_{*i}(\tau) = g_{*i} + \alpha_{*i}(\tau)$, $\tilde{g}_i^*(\tau) = g_i^* - \alpha_i^*(\tau)$, and notice from (2.5) that $\alpha_{*i}(\tau)$ is determined by condition (2.5) and $\alpha_i^*(\tau)$ is determined by condition (2.5) and $\alpha_i^*(\tau)$ is determined by condition (2.5).

Let $\alpha_{*i}(\tau) = \lambda_i(-1) - \omega \int_0^\tau \|h'_{(i)} F(t^*, s)\|^* ds$, $\alpha_i^*(\tau) = \lambda_i(1) + \omega \int_0^\tau \|h'_{(i)} F(t^*, s)\|^* ds$, $\lambda_i(-1)$ and $\lambda_i(1)$ are the eigenvalues of the matrix A at the point t^* , ω is a positive number, $\|h'_{(i)} F(t^*, s)\|^*$ is given in (2.5) and $\|\cdot\|^*$ is the biggest off-diagonal element of the matrix $F(t^*, s)$. Then we have the following estimate: $\lambda_i(\mu) = \max[\mu[h'_{(i)} F(t^*, t_s)x + \int_{t_s}^{t^*} h'_{(i)} F(t^*, s)w(s)ds] + h'_{(i)} \varphi_u(\tau)]$, $\mu = 1, -1$. (2.6)

Denote the right-hand side of the equality (2.6) by $\xi_*(\tau)$ and the left-hand side by $\xi^*(\tau)$. Then we have the inequality $|\xi_*(\tau) - \xi^*(\tau)| \leq \chi$, where $\chi = \omega \int_0^\tau \|h'_{(i)} F(t^*, s)\|^* ds$. This implies that $|\xi_*(\tau) - \xi^*(\tau)| \leq \chi$, $\|x - x_*\| \leq \chi$, $\|w(t)\| \leq \omega$, $t \in T^*$.

Here $\mu = -1$ for calculating α_{*i} and $\mu = 1$ for α_i^* , $\varphi_u(\tau) = \int_{t_s}^{t^*} F(\tau, s)b(s)u(s)ds$ is a vector representing the known contribution of the control to the measurement $y(\tau)$ at moment $\tau \in T_h$, $\xi_*(\tau) = y(\tau) - \varphi_u(\tau) - \xi$, $\xi^*(\tau) = y(\tau) - \varphi_u(\tau) + \xi$. Let us note that problem (2.6) contains "infinite" number of variables $w(t)$, $t \in T^*$, and finite number of constraints. At this section the function $w(t)$, $t \in T^*$, is considered to be a discrete function with the quantization period h : $w(t) = w(s)$, $t \in [s, s+h]$, $s \in T_h^* = T^* \cap T_h$. An effective algorithm for solving such problems is described in [1].

To solve problem (2.6) it is sufficient to correct the solution of the problem close to problem (2.6) by the method proposed in [1].

To calculate the current value $u^*(\tau+h)$ of the control $u^0(\tau+h, y^*(\tau+h))$ the optimal controller has to know the open-loop solution $u(t|\tau+h, y^*(\tau+h))$ to the problem: $\dot{x}(t^*) \rightarrow \max$, $\dot{x} = A(t)x + b(t)u$, $x(\tau+h) = 0$, $\tilde{g}_*(\tau+h) \leq Hx(t^*) \leq \tilde{g}^*(\tau+h)$, $|u(t)| \leq 1$, $t \in T(\tau+h)$, which is equivalent to the problem

$\dot{x}(t^*) \rightarrow \max$, $\dot{x} = A(t)x + b(t)u$, $x(\tau) = 0$, $\tilde{g}_*(\tau+h) \leq Hx(t^*) \leq \tilde{g}^*(\tau+h)$, $|u(t)| \leq 1$, $t \in T(\tau)$, where $\tilde{g}_*(\tau+h) = g_*(\tau) + \Delta g(\tau+h) + \Delta \alpha_*(\tau+h)$, $\tilde{g}^*(\tau+h) = g^*(\tau) + \Delta g(\tau+h) + \Delta \alpha^*(\tau+h)$, $\Delta g(\tau+h) = [\int_0^\tau H F(t^*, s)b(s)ds]u^*(\tau)$, $\alpha^*(\tau) = (\alpha_i^*(\tau), i \in I)$, $\alpha_{*i}(\tau) = (\alpha_{*i}(\tau), i \in I)$, $\Delta \alpha^*(\tau+h) = \alpha^*(\tau) - \alpha^*(\tau+h)$, $\Delta \alpha_*(\tau+h) = \alpha_*(\tau) - \alpha_*(\tau+h)$.

When h is small terminal constraints (2.7) are close to the terminal constraints of problem (2.5). Therefore, to construct the optimal open-loop control $u^0(t|\tau+h, y(\tau+h))$, $t \in T(\tau+h)$ the optimal controller corrects the control $u^0(t|\tau, y(\tau))$, $t \in T(\tau)$ by the dual method [1].

In a similar manner it is easy to show that problem (2.6) at moment $\tau+h$ is close to problem (2.6) at moment τ when the value h is small. Thus, it is easy to construct

the solution of problem (2.6) at moment $\tau + h$ correcting the solution obtained at the previous stage. When the errors $\xi(t)$, $t \in T_h$ or disturbances $w(t)$, $t \in T$ are too large the measurement $y(\tau + h)$ may markedly differ from the measurement $y(\tau)$. Due to this fact the set $X(\tau, u, (\cdot)|\tau, y(\tau))$ may be markedly different from the set $X(\tau + h, u_{\tau+h}(\cdot)|\tau + h, y(\tau + h))$.

In this case the following three situations are possible:

- 1) the time for correcting the open-loop control $u^0(t|\tau, y(\tau))$, $t \in T(\tau)$ to the control $u^0(t|\tau + h, y(\tau + h))$, $t \in T(\tau + h)$ increases greatly;
- 2) problem (2.6) for moment $\tau + h$ with the $X(\tau + h, u_{\tau+h}(\cdot)|\tau + h, y(\tau + h))$ has no accessible control $u(t, \tau + h, y(\tau + h))$, $t \in T(\tau + h)$;
- 3) the quality of the open-loop control $u^0(t|\tau + h, y(\tau + h))$, $t \in T(\tau + h)$, may be less than the quality of the control $u^0(t|\tau, y(\tau))$, $t \in T(\tau)$ obtained at moment τ .

Discuss the first situation. Let $s(\tau)$ be the time the controller has spent on the construction of the open-loop control $u^0(t|\tau, y(\tau))$, $t \in T(\tau)$. To reduce the time for constructing $u^0(t|\tau + h, y(\tau + h))$ at moment $\tau + s(\tau)$ the optimal controller covers the η -vicinity of the measurement $y(\tau)$ with some finite ξ -grid $Y^\varepsilon(\tau)$. For every node $y^k \in Y^\varepsilon(\tau)$ the controller solves problems (2.7) with $\Delta\alpha^*(\tau + h) = \alpha^*(\tau) - \alpha^{*k}(\tau)$, $\Delta\alpha_s(\tau + h) = \alpha_s(\tau) - \alpha_s^k(\tau)$, where $\alpha_{si}^k(\tau) = h'_{(i)} F(t^*, \tau) y^k - \xi \|h'_{(i)} F(t^*, \tau)\|^* - \omega \int_0^{\tau} \|h'_{(i)} F(t^*, s)\|^* ds$, $\alpha_i^{*k}(\tau) = h'_{(i)} F(t^*, \tau) y^k + \xi \|h'_{(i)} F(t^*, \tau)\|^* + \omega \int_0^{\tau} \|h'_{(i)} F(t^*, s)\|^* ds$, $i \in I$. To obtain the solution $u^0(t|\tau, y^k(\tau))$ to problem (2.7) for the node y^k the controller corrects the solution $u^0(t|\tau, y(\tau))$ of problem (2.7) corresponding to the measurement $y(\tau)$ by the dual method [1]. The values η and ε are selected so that the time for solving problems (2.7) for every $y^k \in Y^\varepsilon(\tau)$ doesn't exceed $h - s(\tau)$. Then it seeks the closest to $y(\tau + h)$ node y^k and corrects the solution of problem (2.7) corresponding with the node y^k to the control $u(t|\tau + h, y(\tau + h))$, $t \in T(\tau + h)$ by the dual method [1].

In the second situation the controller sets $u^*(\tau + h) = u^*(\tau)$. Then, as in the case of the previous situation, it covers the η -vicinity of the measurement $y(\tau + h)$ with some finite ε -grid $Y^\varepsilon(\tau + h)$. For every $y^k \in Y^\varepsilon(\tau + h)$ the controller solves problem (2.7) with additional constraint $u^*(\tau) \leq u(\tau + h) \leq u^*(\tau)$ and $\Delta g(\tau + h) = [\int_0^{\tau+2h} H F(t^*, s) b(s) ds] u^*(\tau)$.

Let problems (2.7) with the nodes \bar{y}^k to possess an accessible control. At moment $\tau + 2h$ the optimal controller seeks the closest to $y(\tau + 2h)$ node \bar{y}^k and corrects the solution of problem (2.7) corresponding with the node \bar{y}^k to the control $u(t|\tau + 2h, y(\tau + 2h))$, $t \in T(\tau + h)$. In the third case to increase the quality of the optimal feedback the controller should act as in the previous situation. But to obtain the guaranteed result the optimal controller can work ignoring the third situation unlike the second one.

REFERENCES

- [1] N.V. Balashevich, R.Gabesov, F.M. Kirillova, Numerical computing for the program and positional optimization of linear systems, Comput. Math. Math. Phys. 40(2000), pp.838-859
- [2] R.Gabesov, F.M. Kirillova, E.A. Kostina, Closed state feedback for optimization of uncertain control systems. Part I, Automat. Remote Control, 57(1996), pp. 1008-1015
- [3] R.Gabesov, F.M. Kirillova and O.I. Kostyukova, Constructing of optimal controls of feedback type in a linear problem, Soviet Math. Dokl., 44 (1992), pp.608-613.

Received: 25.09.2002
Revised: 10.01.2003
Accepted: 10.01.2003

Institute of Mathematics,

National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: tanya_posetskaya@yahoo.com

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru

Editorial office: Institute of Mathematics,
National Academy of Sciences of Belarus,
2, Institute of Mathematics, 220072 Minsk,
Belarus, e-mail: viktor_ko@list.ru