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SCHWARZSCHILD'S METRIC GENERATED BY A BODY OF MASS m_1 PERTURBED BY A MOVING BODY OF MASS m_2

István Huba Atilla SASS

Abstract. This paper approaches the translation of the coordinate axes of coordinates having the origin O' in the centre of a spherical body of mass m_2 in the reference point with the origin O of a spherical body of mass m_1, the straight line OO' being the support of the axes $O'x'$ and Ox. This translation being necessary for studying the influence of the body with mass m_2 on that with mass m_1. We assume that the body of mass m_2 moves compared to the body of mass m_1 at a radial speed V.

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1. Notations.

Let us consider two spherical bodies of masses m_1 and m_2, respectively, relatively for-off so that their shapes could be considered spherical.

In this case a references point can be chosen for both bodies, so that mass m_1 should be in the origin O of the system of axes $Oxyzt$ and mass m_2 should be in the origin O' of the system of axes $O'x'y'z'$.

We choose the axes Ox and $O'x'$ on the straight line OO'.

Schwarzschild's metrics, associated with the body of mass m_1 situated in the origin of the references point $Oxyzt$.

$$ds_1^2 = \left(1 - \frac{2m_1}{r}\right)dt^2 - \left(1 - \frac{2m_1}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 = g_{ij}dx^i dx^j \qquad (1)$$

where

$$x^0 = t = ct_{ph}; \quad x^1 = r = r_{ph}; \quad x^2 = \theta = \theta_{ph}; \quad x^3 = \phi = \phi_{ph} \qquad (2)$$

are the generalized and geometrized spherical coordinates. The geometrized time and mass have length dimensions (in meters).

$$m = \frac{GM_{ph}}{c^2} \tag{3}$$

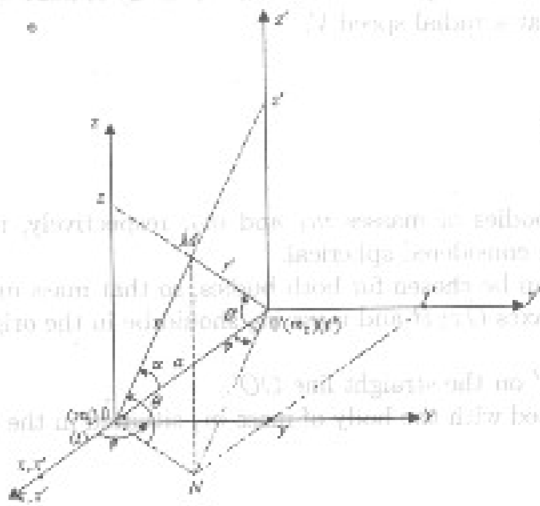
where index ph showing that the respective magnitude is measured in physical units and c and Grepresent the speed of the light and the universal constant of gravity, respectively.

To mass m_2 situated in the origin O' of the axes of coordinates $O'x'y'z't'$ corresponds the metrics

$$ds_2^2 = \left(1 - \frac{2m_2}{r^1}\right) dt'^2 - \left(1 - \frac{2m_2}{r^1}\right)^{-1} dr'^2 - r'^2 d\theta'^2 - r'^2 \sin^2\theta' d\phi'^2 \tag{4}$$

In both cases, we will consider the meter as a leght unit. A point M from the space-time Universe has the Cartesian coordinates (x, y, z, t) measured from O and (x', y', z', t') measure from O'. We have noted the radial distances by

$$OM = r; \ O'M = r'; \ OO' = a \tag{5}$$



(see figure 1)

The relation between the Cartesian coordinates and the spherical ones is given by the formulas:

$$\begin{cases} x = r\cos\theta\cos\phi \\ y = r\cos\theta sin\phi \\ z = r\sin\theta \end{cases} \qquad \begin{cases} x' = r'\cos\theta'\cos\phi' \\ y' = r'\cos\theta' sin\phi' \\ z' = r'\sin\theta' \end{cases} \tag{6}$$

On the other hand

$$\begin{cases} x' = a + x \\ y' = y \\ z' = z \end{cases} \tag{7}$$

In the triangle MMO' we have

$$r'^2 = a^2 + r^2 - 2ar\cos\alpha \tag{8}$$

But

$$r\cos(180^0 - \alpha) = -r\cos\alpha = x, \tag{9}$$

from (8) and (9) it results that $r'^2 = a^2 + r^2 + 2ax$, and replacing x from first formula (6) we get

$$r' = E = \sqrt{a^2 + r^2 + 2ar\cos\theta\cos\phi} \tag{10}$$

An event occurring in the point M at the moment $t_{M_{ph}}$ measured after the clock in M will be seen in O after the arrival of the luminous signal at the speed of the light c at the moment $t_{ph} + \frac{r}{c}$, and in O'at the moment $t'_{ph} + \frac{r'}{c}$. Passing to generalized units:

$$t_M = t + r = t' + r' \tag{11}$$

and consequently

$$t' = t + r - E \tag{12}$$

As $z = z'$ from (6) and (10), it results

$$\sin\theta' = \frac{r\sin\theta}{E} \tag{13}$$

and

$$\cos\theta' = \frac{F}{E} \tag{14}$$

where

$$F = \sqrt{a^2 + r^2\cos^2\theta + 2ar\cos\theta\cos\phi} \tag{15}$$

As $y' = y$, from (6) and (15) also results that

$$\sin\phi' = \frac{r\cos\theta\sin\phi}{F} \tag{16}$$

and

$$\cos\phi' = \frac{a + r\cos\theta\cos\phi}{F} \tag{17}$$

2. The transcription of Schwarzschild's metric associated with mass m_2 in the

coordinates from m_1

Supposing that mass m_2 moves as referred to m_1, at a radial speed V_{ph} that is

$$da = V dt \tag{18}$$

where

$$V = \frac{V_{ph}}{c} \tag{19}$$

From the formula

$$r' \sin\theta = r\sin\phi \tag{20}$$

and differentiating the formulae (10),(12),(16),(13) and (15), we get

$$\begin{aligned} dr' = dE = \tfrac{1}{E}\,[(a + r\cos\theta\cos\phi)\,V dt + (r + a\cos\theta\cos\phi)dr - \\ -ar\sin\theta\cos\phi\, d\theta - ar\cos\theta\sin\phi\, d\,\phi] \end{aligned} \tag{21}$$

$$dt' = dt + dr - dE \tag{22}$$

$$d\theta' = \frac{1}{F}\left(\sin\theta dr + r\cos\theta d\theta - r\sin\theta\frac{dE}{E}\right) \tag{23}$$

$$\begin{aligned} d\phi' = \frac{1}{a + r\cos\theta\cos\phi} \\ \left(\cos\theta\sin\phi dr - r\sin\theta\sin\phi d\theta + r\cos\theta\cos\phi d\phi - r\cos\theta\sin\phi\frac{dF}{F}\right) \end{aligned} \tag{24}$$

$$\begin{aligned} dF = \frac{1}{F}\,[(a + r\cos\theta\cos\phi)V dt + (r\cos\theta + a\cos\phi)\,\cos\theta dr - \\ -(r + a)r\sin\theta\cos\phi d\theta - ar\cos\theta\sin\phi d\,\phi] \end{aligned} \tag{25}$$

After the corresponding replacements in metrics (4), it is demonstred the following theorem

Theorem. *The Schwarzschild's metric generated by a body with mass m_1 is perturbed by moving body with mass m_2 with radial speed V situated at a distance a, with the supplementary metric:*

$$ds_2^2 = \\ = \frac{1}{E^2}\left(1-\frac{2m_2}{E}\right)A^2 - \frac{1}{E^2}\left(1-\frac{2m_2}{E}\right)^{-1}B^2 - \frac{1}{E^2F^2}C^2 - \frac{r^2\sin\theta}{F^4(a+r\cos\theta\cos\phi)^2}D^2 \tag{26}$$

where

$$A = [E-(a+r\cos\theta\cos\phi)V\,dt + [E-(r+a\cos\theta\cos\phi)]\,dr + \\ +ar\sin\theta\cos\phi d\theta + ar\cos\theta\sin\phi\,d\phi \tag{27}$$

$$B = (a+r\cos\theta\cos\phi)V\,dt + (r+a\cos\theta\cos\phi)dr - \\ -ar\sin\theta\cos\phi d\theta - ar\cos\theta\sin\phi d\phi \tag{28}$$

$$C = (a+r\cos\theta\cos\phi)Vr\sin\theta dt - a\sin\theta(a+r\cos\theta\cos\phi)dr - \\ -r\left[(a^2+r^2)\cos\theta + ar\cos\phi(1+\cos^2\theta)\right]d\theta - ar^2\sin\theta\cos\theta\sin\phi d\phi \tag{29}$$

$$D = (a+r\cos\theta\cos\phi)Vr\cos\theta\sin\phi dt - a\cos\theta\sin\phi(a+r\cos\theta\cos\phi)dr + \\ +r\sin\theta\sin\phi\left[a^2+r^2\cos^2\theta + r(a-r)\cos\theta + \cos\theta\cos\phi\right]d\theta - \\ -r\cos\theta\left[ar\cos\theta(1+\cos^2\phi) + (a^2+r^2\cos^2\theta)\cos\phi\right]d\phi \tag{30}$$

$\square$

Metrics (26) shows us the modifications that have been broght about by the body of mass m_2 of the space-time seen from the body of mass m_1.

The body of mass m_2 moves of a radial speed v, compared to the body of mass m_1. In our case, metrics (2.6) diferens from the corresponding metrics (16) from the previous paper for the static case, only thought the temporary components h_{0j} which also include O indices.

REFERENCES

[1] Landau L., Lifchitz E., 1966, "Physique théorique. Théorie du champ", Ed. Mir, Moscou

[2] Sass I. H.A., 2000, "Late Stages in Stellar Evolution", Monographical Booklets in Applied&Computer Mathematics, PAMM, Budapest

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Department of Mathematics and Computer Science
North University of Baia Mare, Str. Victoriei nr. 76
4800 Baia Mare, ROMANIA