Bul. Ştiinţ. Univ. Baia Mare, Ser. B,
Matematică-Informatică, Vol. XVIII(2002), Nr. 2, 331 - 336

ON HERMITE MULTIVARIATE INTERPOLATION

Dana SIMIAN

Abstract. The Hermite interpolation problem given by the set of conditions $\Lambda = \{\lambda_{j,k} : j = 1,\ldots,m;\ k = 0,\ldots,l_j-1\}$, with $\lambda_{j,k} = \delta_{x_j} \circ q_{j,k}(D)$, $q_{j,k} = x^{\alpha_{j,k}}$, $\alpha_{j,k} \in N^2$, $x \in R^2$, $Q_j = span\{q_{j,k} : k = 0,\ldots,l_j-1\}$ is D- invariant, $\sum_{j=1}^{m} l_j = N$ is studied. In order to obtain a Newton form for the interpolant, three divided differences are introduced. Some results related to these divided differences are given.

MSC:41A05, 65D05

Keywords: Hermite interpolation, divided differences, interpolation block

1.Introduction

The aim of this article is to study a particular Hermite interpolation problem by polynomials in two variables. In view of this end we use the connection between the generalization of the Hermite univariate interpolation scheme introduced by T. Sauer and Y. Xu in [3], ideal interpolation schemes and Newton basis. Let Λ be a set of linear functionals, linear independent and let denote by Π_n^d the space of all polynomials of degree less or equal n in d variables. The polynomial interpolation problem with conditions Λ consists in finding a polynomial subspace $\mathcal{P} \subseteq \Pi_n^d$ such that for any $f \in \mathcal{F}$, with $\mathcal{F}$ a function subspace which includes polynomials, there is an unique $p \in \mathcal{P}$ which satisfies the conditions $\lambda(p) = \lambda(f)$, $\forall \lambda \in \Lambda$. If more, $ker\ \Lambda$ is a polynomial ideal then we call the pair $(\mathcal{P}, \Lambda)$ an ideal interpolation scheme. It is known (see [1]) that the "local" Hermite interpolation conditions connected to some point $\theta \in \Theta \subset R^d$ correspond to the primary decomposition of the ideal $ker\ \Theta$. The following theorem allows us to identify an ideal interpolation scheme with an Hermite one:

Theorem 1.(T.Sauer, [5]) *Let $\Lambda \in (\Pi^d)'$ a finite set of linear functionals linear independent. Then $ker\ \Lambda$ is an ideal in Π^d if and only if there are points $x_1,\ldots,x_m \in R^d$, and D- invariant subspaces $Q_1,\ldots,Q_m \subset \Pi^d$ such that*

$$span\ \Lambda = span\{\delta_{x_j} \circ q_j(D);\ q_j \in Q_j;\ j = 1,\ldots,m\} \tag{1}$$

A particular case of ideal interpolation spaces are the minimal degree interpolation spaces. We generalize the definition of minimal degree interpolation spaces with respect to a set of points X_n, given by T. Sauer in [4], for an arbitrary set of conditions Λ:

Definition 1. Let Λ be a set of N linear functionals, linear independent. A subspace $\mathcal{P}(\Lambda) \subset \Pi_n^d$ is called minimal interpolation space of order n,with respect to Λ if:

1. The pair $(\Lambda, \mathcal{P}(\Lambda))$ is correct; 2. Λ defines an ideal interpolation scheme;

3. The interpolation scheme $(\Lambda, \mathcal{P}(\Lambda))$, $\mathcal{P}(\Lambda) \subset \Pi_n^d$, is degree reducing (or equivalent, the interpolation problem with respect to Λ is not poised in any subspaces of Π_{n-1}^d).

Another notion we will need is the Newton basis for an interpolation space with respect to an arbitrary set of linear functionals.

Definition 2. Let $\Lambda \subset (\Pi^d)'$ be a set of N linear functionals, linear independent, $\mathcal{P}(\Lambda) \subset \Pi^d_n$. I_k, with $k = 0, \ldots, n$ a set of multiindices satisfying the following conditions:

$$I_0 \subset I_1 \subset \ldots \subset I_n;\ I_{-1} = \Phi;\ I_k \setminus I_{k-1} \subseteq \{\alpha :\ |\alpha| = k\};\ k = 0, \ldots, n \tag{2}$$

$$I'_k = \{\alpha \in N^2 :\ |\alpha| \leq k\} \setminus I_k;\ k = 0, \ldots, n \tag{3}$$

$$I_n \setminus I_{n-1} \neq \Phi; \qquad card\ I_n = dim\ \mathcal{P}(\Lambda); \tag{4}$$

We say that the space $\mathcal{P}(\Lambda)$ admits a Newton basis of order n with respect to the set of functionals Λ, if there exist a set of multiindices $\mathcal{I} = (I_0, \ldots, I_n)$ satisfying (2)-(4) and having the following additional properties :

1.The functionals in Λ may be reindexed in the blocks: $\Lambda^{(k)} = \{\lambda_\alpha :\ \lambda_\alpha \in \Lambda;\ \alpha \in I_k \setminus I_{k-1}\},\ k = 0, \ldots, n;\quad \Lambda = \{\lambda_\beta :\ \beta \in I_n\}$;

2. There exists a basis $p_\alpha \in \Pi^d_{|\alpha|},\ \alpha \in I_n$ of $\mathcal{P}(\Lambda)$ such that $\lambda_\beta(p_\alpha) = \delta_{\alpha,\beta};\ \beta \in I_n;\ |\beta| \leq |\alpha|$;

3. There exists the complementary polynomials $p^\perp_\alpha \in \Pi^d_{|\alpha|},\ \alpha \in I'_n$ such that $\Lambda(p^\perp_\alpha) = 0$ and $\Pi^d_n = span\{p_\alpha :\ \alpha \in I_n\} \oplus span\{p^\perp_\alpha :\ \alpha \in I'_n\}$. The number of functionals in the block $\Lambda^{(k)}$ is $n_k = dim\ \mathcal{P}(\Lambda) \cap \Pi^0_k \leq dim\ \Pi^0_k$, with Π^0_k the space of homogeneous polynomials of degree k.

The following theorem illustrates the connection between the Newton basis and ideal interpolation schemes:

Theorem 2. *Let Λ be a set of linear independent functionals. The polynomial subspace $\mathcal{P}(\Lambda)$ is a minimal interpolation space of order n, with respect to Λ, if and only if there exist a Newton basis on order n for $\mathcal{P}(\Lambda)$ with respect to Λ.*

A constructive proof is given in [7]. For a given set of functionals, Λ, there exists an unique minimal interpolation space of n order, $\mathcal{P}(\Lambda)$ if and only if $\mathcal{P}(\Lambda) = \Pi^d_n$. We proofed in [7] that the space $H_\Lambda\downarrow$ given by

$$H_\Lambda\downarrow = span\{g\downarrow : g \in H_\Lambda\};\ H_\Lambda = span\{\lambda^\nu;\ \lambda \in \Lambda\}, \tag{5}$$

is a minimal degree interpolation space for the conditions Λ. We denote by $g\downarrow = T_j g$ the least term, with j the smallest integer for which $T_j g \neq 0$ and $T_j g$ the Taylor polynomial of degree $\leq j$. Let λ^ν be the generating function of the functional $\lambda \in \Lambda$. The expression of λ^ν is given by

$$\lambda^\nu(x) = \lambda(e_x)\ \text{with}\ e_z(x) = e^{z \cdot x} \tag{6}$$

We defined in [8] the λ - divided difference which allows us to give a Newton form for the interpolation operator from $H_\Lambda\downarrow$.

Definition 3. Let $(p_\alpha),\ \alpha \in I_n$ be the Newton basis for the minimal interpolation space of n order $\mathcal{P}(\Lambda)$ and $\Lambda^{(k)}$ the proper blocks of functionals. The λ-divided difference is defined recursively by: $d_0[\lambda; f] = \lambda(f)$, $d_{k+1}[\Lambda^{(0)}, \dots, \Lambda^{(k)}, \lambda; f] = d_k[\Lambda^{(0)}, \dots, \Lambda^{(k-1)}, \lambda; f] - \sum_{\alpha \in J_k} d_k[\Lambda^{(0)}, \dots, \Lambda^{(k-1)}, \lambda_\alpha; f]\lambda(p_\alpha)$, with $J_k = I_k \setminus I_{k-1}$.

Theorem 3. *With the notations in the definition 3 the following equalities hold:*

$$\lambda(L_n(f)) = \sum_{\alpha \in I_n} d_{|\alpha|}[\Lambda^{(0)}, \dots, \Lambda^{(|\alpha|-1)}, \lambda_\alpha; f] \cdot \lambda(p_\alpha); \ \lambda \in \Pi' \tag{7}$$

$$\lambda(f - L_n(f)) = d_{n+1}[\Lambda^{(0)}, \dots, \Lambda^{(n)}, \lambda; f] \tag{8}$$

2. Hermite interpolation in blocks

Let consider the following choice of the set of interpolation conditions:

$$\Lambda = \{\lambda_{j,k} : j = 1, \dots, m; \ k = 0, \dots, l_j - 1\}, \tag{9}$$

with $\lambda_{j,k} = \delta_{x_j} \circ q_{j,k}(D)$; $q_{j,k} = x^{\alpha_{j,k}}$; $\alpha_{j,k} \in N^2$, $x \in R^2$, $Q_j = span\{q_{j,k} : k = 0, \dots, l_j - 1\}$ is $D-$ invariant, $\sum_{j=1}^{m} l_j = N$.

Proposition 1. If Q is a D- invariant polynomial space, generated by l monomyals, x^α, $\alpha \in I \subset N^2$, then, I is a lower set, that is, if $\alpha \in I$ and $\beta < \alpha$, then $\beta \in I$.

Taking into account theorem 1 the following proposition holds:

Proposition 2. The conditions Λ from (9) define an ideal interpolation scheme or, equivalently, an Hermite interpolation scheme.

Proposition 3. A minimal interpolation space with respect to Λ is $(H_\Lambda)\downarrow = \left(\sum_{j=1}^{m} e_{x_j} Q_j\right)\downarrow$

Proof: We use relation (5) and the fact that the generating function $\lambda^{\nu}_{j,k} = q_{j,k}(x)e_{x_j}(x)$. To characterize the set of conditions, we define, using the model from [3], for any point x_j, the following elements:

1. An index set $E_j = \{1 | \varepsilon^1_1, \varepsilon^1_2 | \dots | \varepsilon^{l_j-1}_1, \dots, \varepsilon^{l_j-1}_{l_j}\}$, with $\varepsilon^k_i \in \{0,1\}$, $\forall\ k = 1, \dots, l_j - 1$; $i = 1, \dots, k+1$.
2. A diagram T_j according to E_j, $T_j = \{x_j | y^1_1, y^1_2 | \dots | y^{l_j-1}_1, \dots, y^{l_j-1}_{l_j}\}$, $y^k_i \in R^2$ and $y^k_i = 0$, if $\varepsilon^k_i = 0$.

We said that E_j is of tree structure if for any $\varepsilon^k_i = 1$, $k > 1$, there exists an $j = j(i)$ uniquely determinated, such that $\varepsilon^{k-1}_j = 1$. In this case, ε^{k-1}_j is the predecessor of ε^k_i. If E_j is of tree structure, then we named T_j tree with root x_j and y^k_i for $\varepsilon^k_i = 1$ are called vertices of the tree. We denote by $|T_j|$ and call length of the tree, the number of nonzero vertices, including the root, in T_j. A sequence $\eta = (i_1, \dots, i_k)$ is called a chain in the tree structure E if $\varepsilon^1_{i_1} = \dots = \varepsilon^k_{i_k} = 1$, where $\varepsilon^l_{i_l}$ is the predecessor of $\varepsilon^{l+1}_{i_{l+1}}$, $\forall l = 1, \dots, k-1$. If $\varepsilon^k_{i_k}$ is not the predecessor of another element in E, the chain is called a maximal chain. A chain $\eta' = (i_1, \dots, i_j)$ is subordinate to $\eta = (i_1, \dots, i_k)$, if $j \le k$.

Let $\eta = (i_1, \dots, i_k)$ be a chain in E_j. We define the sequence of directions $\mathbf{y}^\eta = (y^1_{i_1}, \dots, y^k_{i_k})$, with $y_{i_l} \in R^2$ and the sequence of directional derivatives $D^{\sigma(\eta)}_{\mathbf{y}^\eta} = D_{y^1_{i_1}} \dots D_{y^k_{i_k}}$.

If $\sigma(\eta) = 0$, then $D^0_{y^\eta} = I$, where I is the identity operator. We associate to each pair (x_j, Q_j) or equivalent, to each block of functionals $\Lambda_j = \{\lambda_{j,k} : k = 0, \ldots, l_j - 1\}$ a structure of tree, E_j and a tree T_j. Clearly, the directions in the trees T_j will be represented by $\varepsilon^1 = (1, 0)$ and $\varepsilon^2 = (0, 1)$. We obtain a new expression for the set of functionals Λ,

$$\Lambda = \{\mathcal{D}^{\sigma(\eta)}_{y^\eta},\ \forall \eta \in T_k,\ \eta \text{ maximal chain},\ k = 1, \ldots, m\} \tag{10}$$

We denote by $\mathcal{P}(\Lambda)$ a minimal space for the conditions Λ and call it "Hermite minimal interpolation space". We intended to put the set Λ into blocks and to derive the Newton basis in a way which is able to characterize the special structure of this functionals set. Any point in (9) has attached l_k interpolation conditions of Hermite type. Therefore we may consider x_k as a point of multiplicity l_k, or as a point having l_k identical copies. We denote the copies of x_k by $x^j_{k,i}$. Thus $x^j_{k,i} = x_k$, for $\varepsilon^j_{k,i} = 1$ and $\varepsilon^j_{k,i}$ are in the structure of tree E_k, with $\varepsilon^0_{k,1} = 1$. Let X_n be the set of all points and theirs copies:

$$X_n = \{x^j_{k,i} : \varepsilon^j_{k,i} = 1,\ k = 1, \ldots, m\}, \tag{11}$$

Obvious, $card(X) = dim\ \mathcal{P}(\Lambda) \leq dim\ \Pi^2_n$. The points in X_n are indexed using multi-indices $\beta \in I_n$. If $x_\beta = x^j_{k,i}$ and $\eta = (i_1, \ldots, i_j) \in T_k$ is a chain, we will denote $\sigma_\beta = \sigma(\eta)$ and $D^{\sigma_\beta}_{y_\beta} = D^{\sigma(\eta)}_{y^\eta}$, that is $\mathbf{y}_\beta = \mathbf{y}^\eta$. Using the previous notations, the interpolation conditions Λ are given by the pairs $(x_\beta, \mathcal{D}^{\sigma_\beta}_{y_\beta})$, $\beta \in I_n$.

Definition 4. The points set X_n given in (11) is block minimal derived from Λ, if we can indexed the points and the conditions into blocks

$$X^{(j)} = \{x_\beta : \beta \in I_j \setminus I_{j-1}\},\ j = 0, \ldots, n;\ card\ X^{(j)} = dim(\mathcal{P}(\Lambda) \cap \Pi^0_j) \leq j + 1, \tag{12}$$

such that for any $k = 1, \ldots, m$ the same level copies of x_k be in the same block, the level order of a point x_k is preserved in X_n blocks order and the interpolation problems with respect to the conditions $\tilde{X}_k = \{(x_\beta, D^{\sigma_\beta}_{y_\beta}) : \beta \in I_k\}$ are poised in $\Pi_k \cap \mathcal{P}(\Lambda)$, $\forall\ k = 0, \ldots, n - 1$. A minimal interpolation space with respect to a set of points block minimal derived from Λ is called block minimal Hermite interpolation space.

Proposition 4. If X_n is block minimal derived from Λ, then there exists a Newton basis, (p_α), $\alpha \in I_n$, such that

$$D^{\sigma_\beta}_{y_\beta} p_\alpha(x_\beta) = \delta_{\beta,\alpha},\ \forall \alpha, \beta \in I_n \text{ and } |\beta| \leq |\alpha| \tag{13}$$

Proof: Let (g_α), $\alpha \in I_n$ the base of space $\mathcal{P}(\Lambda)$, we used in proof of the theorem 2 and $V(\Lambda) = (\lambda_\beta(g_\alpha))_{\alpha,\beta \in I_n}$. $\mathcal{P}(\Lambda)$ is an interpolation space. Consequently, $det\ V(\Lambda) \neq 0$. If X_n is block minimal set derived from Λ, then the matrix $V(X_n) = (D^{\sigma_\beta}_{y_\beta} g_\alpha(x_\beta))$, $\alpha, \beta \in I_n$ differs from $V(\Lambda)$ only by some interchange of rows or columns. Hence

$$det\ V(X_n) \neq 0 \tag{14}$$

Taking into account definition 4, $card\ I_k = dim(\Pi_k \cap \mathcal{P}(\Lambda))$ and

$$det\ V(X_k) \neq 0,\ \forall\ k = 0, \ldots, n-1. \tag{15}$$

Let be $V_\gamma(X_n|x) = (D^{\sigma_\beta}_{y_\beta} g_\alpha(x^*_\beta)), \alpha, \beta \in I_n$, with $x^*_\beta = \begin{cases} x_\beta & \text{if } \beta \neq \gamma \\ x & \text{if } \beta = \gamma \end{cases}$

Then, polynomials $p_\gamma(x) = \frac{det\ V_\gamma(X_k|x)}{det\ V(X_k)}$; $|\gamma| = k$, $k = 0, \ldots, n$; $\gamma \in I_n$ satisfy the conditions (13), therefore represents the Newton basis. The next theorem results from the above proposition and from theorem 3.

Theorem 4. *Let $H_n(f)$ be the projection of f on a minimal interpolation space of n order with respect to the set of functionals given in (10),*

$$\bar{X}^{(k)} = \{\delta_{x_\alpha} \circ D^{\sigma_\alpha}_{y_\alpha} :\ \alpha \in J_k\},\ \{\Lambda^{(0)}, \ldots, \Lambda^{(n)}\} = \{\bar{X}^{(0)}, \ldots \bar{X}^{(n)}\} \tag{16}$$

and d_n is the λ-divided difference given in definition 3. Then

$$\lambda(H_n(f)) = \sum_{\alpha \in I_n} d_{|\alpha|}[\bar{X}^{(0)}, \ldots, \bar{X}^{(|\alpha|-1)}, \delta_{x_\alpha} \circ D^{\sigma_\alpha}_{y_\alpha}; f] \cdot \lambda(p_\alpha),\ \lambda \in \Pi' \tag{17}$$

$$\lambda(f - H_n(f)) = d_{n+1}[\bar{X}^{(0)}, \ldots, \bar{X}^{(n)}, \lambda; f] \tag{18}$$

We want to give another form to theorem 4. For this end we introduce another two divided differences.

Definition 5. Let X_n be block minimal derived from Λ, $X^{(k)}$ the k level block and I_k the proper sets of multiindices. We define, recursively, the k order block divided difference b_k, $k \in \{0, \ldots, n+1\}$: $b_0[x; f] = f(x)$ $b_{n+1}[X^{(0)}, \ldots, X^{(n)}, x; f] = b_n[X^{(0)}, \ldots, X^{(n-1)}, x; f] - \sum_{\alpha \in J_n} D^{\sigma_\alpha}_{y_\alpha} b_n[X^{(0)}, \ldots, X^{(n-1)}, x_\alpha; f] \cdot p_\alpha(x)$,
with $D^{\sigma_\alpha}_{y_\alpha} b_n[X^{(0)}, \ldots, X^{(n-1)}, x_\alpha; f] = (D^{\sigma_\alpha}_{y_\alpha} b_n[X^{(0)}, \ldots, X^{(n-1)}, x; f])(x_\alpha)$.

$$b_{k+1}[X^{(0)}, \ldots, X^{(k)}, x; f] = f(x) - \sum_{\alpha \in I_k} D^{\sigma_\alpha}_{y_\alpha} b_{|\alpha|}[X^{(0)}, \ldots X^{(|\alpha|-1)}, x_\alpha; f] \cdot p_\alpha(x) \tag{19}$$

Theorem 5. *Let H_n be the projection of f on a minimal interpolation space of n order, $\mathcal{P}(\Lambda)$, with respect to the set X_n block minimal derived from Λ.* Then

$$(H_n(f))(x) = \sum_{\alpha \in I_n} D^{\sigma_\alpha}_{y_\alpha} b_{|\alpha|}[X^{(0)}, \ldots, X^{(|\alpha|-1)}, x_\alpha; f] \cdot p_\alpha(x) \tag{20}$$

$$(R_n(f))(x) = (f - H_n(f))(x) = b_{n+1}[X^{(0)}, \ldots, X^{(n)}, x; f] \tag{21}$$

Proof: To prove (20) is sufficient to verify the next conditions

$$\sum_{\alpha \in I_n} D^{\sigma_\alpha}_{y_\alpha} b_{|\alpha|}[X^{(0)}, \ldots, X^{(|\alpha|-1)}, x_\alpha; f] \cdot D^{\sigma_\beta}_{y_\beta} p_\alpha(x_\beta) = D^{\sigma_\beta}_{y_\beta} f(x_\beta),\ \forall \beta \in I_n. \tag{22}$$

The relations (22) and (21) can be proofed simultaneously by induction on n

Corollary 1. $b_{n+1}[X^{(0)},\ldots,X^{(k)},x;p_{\alpha}^{\perp}]=p_{\alpha}^{\perp},\ \forall\alpha\in I_n'$

For computational purposes, the block divided difference is difficult to use. Therefore we introduce another divided difference given in the following definition

Definition 6. Let $\mathbf{y}=(y_1,\ldots,y_\sigma)$, $y_i\in R^2$ be a sequence of directions in R^2. With the notation in definition 5 we define, recursively, the $\mathbf{y}$- directional divided difference of k order, $k\in\{0,\ldots n+1\}$, γ_k: $\gamma_0[x,\mathbf{y};f]=D_{\mathbf{y}}^{\sigma}f(x)$; $\gamma_{n+1}[X^{(0)},\ldots,X^{(n)},x,\mathbf{y};f]=\gamma_n[X^{(0)},\ldots,X^{(n-1)},x,\mathbf{y};f]-\sum_{\alpha\in J_n}\gamma_n[X^{(0)},\ldots,X^{(n)},x_\alpha,\mathbf{y}_\alpha;f]D_{\mathbf{y}}^{\sigma}p_\alpha(x)$

Theorem 6. *In conditions of theorem 5*

$$(H_n(f))(x)=\sum_{\alpha\in I_n}\gamma_{|\alpha|}[X^{(0)},\ldots,X^{|\alpha|-1},x_\alpha,\mathbf{y}_\alpha;f]\cdot p_\alpha(x) \tag{23}$$

Theorem 7. *The connection between the three divided differences, defined in this chapter, is given by the following equalities:*

$$\gamma_n[X^{(0)},\ldots,X^{(n-1)},x,\mathbf{y};f]=d_n[\bar{X}^{(0)},\ldots,\bar{X}^{(n-1)},\delta_x\circ D_{\mathbf{y}}^{\sigma};f] \tag{24}$$

$$\gamma_n[X^{(0)},\ldots,X^{(n-1)},x,\mathbf{y};f]=D_{\mathbf{y}}^{\sigma}b_n[X^{(0)},\ldots,X^{(n-1)},x;f] \tag{25}$$

Proof: To prove relations (24) and (25) we use induction on n.

Corollary 2. For any $f:R^2\to R$ and $x_\alpha\in X$ the following relations hold:

$$d_{n+1}[\bar{X}^{(0)},\ldots,\bar{X}^{(n)},\delta_{x_\alpha}\circ D_{\mathbf{y}_\alpha}^{\sigma_\alpha};f]\ =\ 0;\ \gamma_{n+1}[X^{(0)},\ldots,X^{(n)},x_\alpha,\mathbf{y}_\alpha;f]=0;$$
$$D_{\mathbf{y}_\alpha}^{\sigma_\alpha}b_n[X^{(0)},\ldots,X^{(n-1)},x_\alpha;f]\ =\ 0$$

REFERENCES

[1] de Boor C., Ron A. (1991) : On polynomial ideals of finite co dimension with application to box spline theory. J. Math. Anal. and Appl., 158, 168-193. [2] de Boor C., Ron A. (1992) :The least solution for the polynomial interpolation problem Math.Z., 220, 347-378.

[3] Sauer T., Xu Y. (1995) : On multivariate Hermite interpolation. Advances in Computational Mathematics, 4, 207-259.

[4] Sauer T. (1997) : Polynomial interpolation of minimal degree. Numer. Math., 78, 59-85.

[5] Sauer T. (1998) : Polynomial interpolation of minimal degree and Gröbner bases. Gröbner Bases and Applications (Proc. of the Conf. 33 Year of Gröbner Bases), vol. 251 of London Math. Soc, Lecture Notes, 483-494, Cambridge University Press.

[6] Gasca M., Sauer T., (1999): Polynomial interpolation in several variables Advances in Computational Mathematics

[7] Simian D. (2001) : On the bivariate Polynomial Interpolation, Acta Technicum Napocensis, Series Applied Mathematics and Mechanics, 44, vol. 1, 97-105.

[8] Simian D. (2001) : Ideal interpolation schemes. Proceedings of "The 9-th Symposium of Mathematics and its Applications", Timişoara, 145-153.

Received: 09.09.2002
"Lucian Blaga" University of Sibiu, Romania
E-mail: d_simian@yahoo.com