Bul. Ştiinţ. Univ. Baia Mare, Ser. B,
Matematică-Informatică, Vol. XVIII(2002), Nr. 2, 373 - 382

ON CONCURRENCY-DEGREES FOR JUMPING PETRI NETS

Cristian VIDRAŞCU, Toader JUCAN

Abstract. This paper treats the notion of degrees of concurrency in jumping Petri nets. It will present a more general definition of concurrency-degrees for them, which takes into consideration the auto-concurrency (i.e the case of the transitions concurrently enabled with themselves), thus replacing the old definitions given in [4], which ignore the auto-concurrency. Also, this paper will introduce a finer notion, namely the concurrency-degrees w.r.t. a set of transitions and it will point out how these more general concurrency-degrees can be computed.

MSC: 68Q85

Keywords: parallel distributed systems, Petri nets, concurrency, computability

1 Introduction

A Petri net is a mathematical model used for the specification and the analysis of parallel and distributed systems. It is very important to introduce a measure of concurrency for such systems. What is the meaning of the fact that in the system S_1 the concurrency is greater than in the system S_2 ? We will study the problem of concurrency for Petri nets, but, since the Petri nets are used as suitable models for real-world parallel or distributed systems, the results will be applicable also to these systems. We will see that the number of transitions which can fire simultaneously in a Petri net which models a given real system, can be used as an intuitive measure of the concurrency of that system.

It is well-known that the behaviour of some distributed systems cannot be adequately modelled by classical Petri nets. Many extensions which increase the computational and expressive power of Petri nets have been thus introduced. One direction has led to various modifications of the firing rule of nets, like that of jumping Petri nets, introduced in [5].

The notion of concurrency-degree for jumping Petri nets was introduced in [4], but that definition has one great drawback: it takes into consideration only distinct transitions which can fire simultaneously; thus, it ignores the auto-concurrency, i.e. the case of a transition fireable simultaneously with itself at a marking. Therefore, we will give in this paper a more general definition of concurrency-degree for jumping Petri nets, which takes into consideration also the transitions concurrently enabled with themselves. Also, we will introduce a finer notion, namely the concurrency-degree w.r.t. a set of transitions, and we will show how we can compute these concurrency-degrees.

2 Preliminaries

We will assume to be known the basic terminology and notation about sets, relations and functions, vectors, multisets and formal languages. Let us just briefly remind that a *multiset* m, over a non-empty set S, is a function $m : S \to N$, usually represented as a formal sum: $\sum_{s \in S} m(s) s$. The operations and relations on multisets are componentwise defined. S_{MS} denotes the set of all multisets over S. The *size* of m is defined as $|m| = \sum_{s \in S} m(s)$.

This section will establish the basic terminology, notation, and results concerning Petri nets (for details the reader is referred to [1], [3]). Mainly, it will follow [3]

2.1 Petri nets

A *Place/Transition net*, shortly *P/T-net*, (finite, with infinite capacities), abbreviated *PTN*, is a 4-tuple $\Sigma = (S, T, F, W)$, where S and T are two finite non-empty sets (of *places* and *transitions*, resp.), $S \cap T = \emptyset$, $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation* and $W : (S \times T) \cup (T \times S) \to N$ is the *weight function* of Σ verifying $W(x, y) = 0$ iff $(x, y) \notin F$. A *marking* of a *PTN* Σ is a function $m : S \to N$, i.e. a multiset over S; it will be sometimes identified with a $|S|$-dimensional vector. N^S denotes the set of all markings of Σ.

A *marked PTN*, abbreviated *mPTN*, is a pair $\gamma = (\Sigma, m_0)$, where Σ is a *PTN* and m_0, called the *initial marking* of γ, is a marking of Σ.

Let Σ be a P/T-net, $t \in T$ and $w \in T^*$. The functions $t^-, t^+ : S \to N$ and $\Delta t, \Delta w : S \to Z$ are defined by: $t^-(s) = W(s,t)$, $t^+(s) = W(t,s)$, $\Delta t(s) = t^+(s) - t^-(s)$ and $\Delta w(s) = \sum_{i=1}^{n} \Delta t_i(s)$, if $w = t_1 t_2 \ldots t_n$, or $\Delta w(s) = 0$, if $w = \lambda$.

The sequential behaviour of a P/T-net Σ is given by the *firing rule*, which consists of a) the *enabling rule*: a transition t is *enabled* at a marking m in Σ (or t is *fireable* from m), abbreviated $m[t\rangle_\Sigma$, iff $t^- \le m$; b) the *computing rule*: if $m[t\rangle_\Sigma$, then t may *occur* yielding a new marking m', abbreviated $m[t\rangle_\Sigma m'$, defined by $m' = m + \Delta t$.

In fact, for any transition t of Σ we have a binary relation on N^S, denoted by $[t\rangle_\Sigma$ and given by: $m[t\rangle_\Sigma m'$ iff $t^- \le m$ and $m' = m + \Delta t$. If $t_1, t_2, \ldots, t_n$, $n \ge 1$, are transitions of Σ, $[t_1 t_2 \ldots t_n\rangle_\Sigma$ will denote the classical product of the relations $[t_1\rangle_\Sigma, \ldots, [t_n\rangle_\Sigma$. Moreover, we also consider the relation $[\lambda\rangle_\Sigma$ given by $[\lambda\rangle_\Sigma = \{(m, m) | m \in N^S\}$.

Let $\gamma = (\Sigma, m_0)$ be a *mPTN*, and $m \in N^S$. The word $w \in T^*$ is called a *transition sequence* from m in Σ if there exists a marking m' of Σ such that $m[w\rangle_\Sigma m'$. Moreover, the marking m' is called *reachable* from m in Σ. $RS(\Sigma, m) = [m\rangle_\Sigma = \{m' \in N^S | \exists w \in T^* : m[w\rangle_\Sigma m'\}$ denotes the set of all reachable markings from m in Σ. In the case $m = m_0$, the set $RS(\Sigma, m_0)$ is abbreviated by $RS(\gamma)$ (or $[m_0\rangle_\gamma$) and it is called the *reachability set* of γ.

The marking m is *coverable* in γ if there exists a marking $m' \in [m_0\rangle_\gamma$ such that $m \le m'$. The first, and well-known, reduced reachability tree/graph was that introduced by Karp and Miller, and A. Finkel introduced the minimal coverability set, tree, and graph (in [2]).

Concurrency-degrees for P/T-nets.

In [8] was given a more general definition of concurrency-degree for P/T-nets, which takes into consideration also the auto-concurrency. Let us recall this definition.

Let $\Sigma = (S, T, F, W)$ be a P/T-net and $m \in N^S$. A *step* Y is a non-empty and finite multiset over T. A step Y is *enabled* at the marking m in Σ (or Y is *fireable* from m), and we say also that Y is a multiset of transitions *concurrently enabled* at m, abbreviated $m[Y\rangle_\Sigma$, iff $\sum_{t\in T} Y(t) \cdot t^- \le m$. Moreover, if $m[Y\rangle_\Sigma$, then by occuring Y at m is produced $m' = m + \sum_{t\in T} Y(t) \cdot \Delta t$, abbreviated $m[Y\rangle_\Sigma m'$. Y is a *maximal* step enabled at m in Σ iff there exists no step Y' enabled at m in Σ with $Y' > Y$.

The *concurrency-degree of* Σ *at the marking* m is defined as the supremum number of transitions concurrently enabled at m: $d(\Sigma, m) = \sup\{|Y| \mid Y$ is a step enabled at m in $\Sigma\}$.

Let $\gamma = (\Sigma, m_0)$ be a *mPTN*. The *inferior and superior concurrency-degree of* γ are defined as the minimum, and resp., the supremum, number, at any moment, of transitions maximal concurrently enabled: $d^-(\gamma) = \min\{d(\Sigma, m) | m \in [m_0\rangle_\gamma\}$ and $d^+(\gamma) = \sup\{d(\Sigma, m) | m \in [m_0\rangle_\gamma\}$. If $d^-(\gamma) = d^+(\gamma)$, then this number is called the *concurrency-degree of* γ and it is denoted by $d(\gamma)$.

For more comments about the intuitive meaning of concurrency-degrees for P/T-nets, the reader is reffered to [8]. All these concurrency-degrees are computable ([8]).

2.2 Jumping Petri nets

Jumping Petri nets ([5]) are an extension of P/T-nets, which allows them to perform "spontaneous jumps" from one marking to another (this is similar to λ-moves in automata theory).

A *jumping P/T-net*, abbreviated *JPTN*, is a pair $\gamma = (\Sigma, R)$, where Σ is a *PTN* and R is a recursive binary relation on the set of markings of Σ, i.e., $R \subseteq N^S \times N^S$, called the *set of (spontaneous) jumps* of γ. The pairs $(m, m') \in R$ are referred to as *jumps* of γ. Σ is called the *underlying P/T-net* of γ. A *marking* of γ is any marking of Σ. If R is finite, then γ is called a *finite jumping P/T-net*, abbreviated *FJPTN*.

A *marked jumping P/T-net* is defined similarly as a marked P/T-net, by changing "Σ" into "Σ, R". The abbreviations used will be *mJPTN*, *mFJPTN*.

The behaviour of a jumping net γ is given by the *j-firing rule*, which consists of a) the *j-enabling rule*: a transition t is *j-enabled* at a marking m (in γ), abbreviated $m[t\rangle_{\gamma,j}$, iff there exists a marking m_1 such that $mR^*m_1[t\rangle_\Sigma$ (Σ being the underlying P/T-net of γ and R^* the reflexive and transitive closure of R); b) the *j-computing rule*: if $m[t\rangle_{\gamma,j}$, then the marking m' is *j-produced* by occurring t at m, abbreviated $m[t\rangle_{\gamma,j}m'$, iff there exists two markings m_1, m_2 such that $mR^*m_1[t\rangle_\Sigma m_2R^*m'$.

The notions of *transition j-sequence* and *j-reachable marking* are defined similarly as for P/T-nets (the relation $[\lambda\rangle_{\gamma,j}$ is defined by $[\lambda\rangle_{\gamma,j} = R^*$). The *j-reachability set* of a *mJPTN* $\gamma = (\Sigma, R, m_0)$ is denoted by $RS(\gamma)$ or by $[m_0\rangle_{\gamma,j}$.

All other notions from P/T-nets are defined for jumping Petri nets similarly as for P/T-nets, by considering the notion of *j-reachability* instead of *reachability* from P/T-nets.

The notions of reachability tree/graph and (minimal) coverability structures (set, and graph) for jumping Petri nets were introduced in [6]. The main result from [6] says the minimal coverability set and graph, denoted by $MCS(\gamma)$ and $MCG(\gamma)$, resp., are unique, finite, and computable for finite jumping Petri nets.

3 Defining concurrency-degrees for jumping Petri nets

The notion of concurrency-degree for jumping Petri nets was introduced in [4]. But that definition has the same drawback as in the case of P/T-nets: it ignores the auto-concurrency, i.e. the case of a transition fireable simultaneously with itself at a marking. We will give in this section a more general definition, which takes into consideration also the transitions concurrently enabled with themselves. Also, we will introduce a new, finer definition, namely the concurrency-degree w.r.t. a set of transitions.

First, we will define the notion of a step fireable at a marking as a multiset of transitions, and then the concurrency-degrees based on this notion of a step. The line we follow is [4].

Definition 3.1. Let $\gamma = (\Sigma, R)$ be a jumping P/T-net. A *step* Y is a non-empty and finite multiset over the set of transitions T of Σ. The set of all steps of γ will be denoted by $Y(\gamma)$.

Definition 3.2. The step-type concurrent behaviour of a jumping P/T-net $\gamma = (\Sigma, R)$ is given by the *step firing rule*, which consist of a) the *step enabling rule*: a step Y is *j-enabled* at a marking m in γ (or Y is *fireable* from m), and we say also that Y is a multiset of transitions *concurrently j-enabled* at m, abbreviated $m[Y\rangle_{\gamma,j}$, iff there exists a marking m_1 such that $mR^*m_1[Y\rangle_\Sigma$; b) the *step computing rule*: if $m[Y\rangle_{\gamma,j}$, then the marking m' is *j-produced* by occurring Y at the marking m, abbreviated $m[Y\rangle_{\gamma,j}m'$, iff there exists two markings m_1, m_2 such that $mR^*m_1[Y\rangle_\Sigma m_2R^*m'$. Intuitively, the notion of concurrency-degree at a marking m of a jumping P/T-net γ represents the supremum number of transitions concurrently j-enabled at the marking m.

Definition 3.3. Let γ be a jumping P/T-net, and m an arbitrary marking of γ. The *concurrency-degree at the marking* m of the net γ is defined by:

$$d(\gamma, m) = \sup\{\ |Y|\ |\ Y \in Y(\gamma) \wedge m[Y\rangle_{\gamma,j}\} \quad (1)$$

Proposition 3.1. For any jumping P/T-net $\gamma = (\Sigma, R)$ and any arbitrary marking m of γ,

$$d(\gamma, m) = \sup\{ d(\Sigma, m') \mid mR^* m' \}. \quad (2)$$

It follows from definitions.

Definition 3.4. Let $\gamma = (\Sigma, R, m_0)$ be a marked jumping P/T-net.
i) The *inferior / superior concurrency-degree* of the net γ is defined by:

$$d^-(\gamma) = \min\{ d(\gamma, m) \mid m \in [m_0\rangle_{\gamma,j} \} \quad (3)$$

$$d^+(\gamma) = \sup\{ d(\gamma, m) \mid m \in [m_0\rangle_{\gamma,j} \} \quad (4)$$

ii) If $d^-(\gamma) = d^-(\gamma)$, then this number is called the *concurrency-degree* of γ and it is denoted by $d(\gamma)$.

Remark 3.1. i) Directly from definitions we have $0 \le d^-(\gamma) \le d^+(\gamma) \le \infty$.
ii) $d^-(\gamma)$ represents the minimum number of transitions maximal concurrently j-enabled at any j-reachable marking of γ. In other words, at any j-reachable marking m of γ there exist $d^-(\gamma)$ transitions concurrently j-enabled at m.
iii) $d^+(\gamma)$ represents the supremum number of transitions maximal concurrently j-enabled at any j-reachable marking of γ. In other words, at any j-reachable marking m of γ there exist at most $d^+(\gamma)$ transitions concurrently j-enabled at m.
iv) The concurrency-degree of the net γ means that at any j-reachable marking m of the net γ there exist $d(\gamma)$ transitions concurrently j-enabled at m, and there is no j-reachable marking m' of γ with more than $d(\gamma)$ transitions concurrently j-enabled at m'.

Remark 3.2. Sometimes it is useful to ignore some transitions of a net and to study the behaviour of the net w.r.t. the remaining transitions. Thus, we can define concurrency-degrees for jumping Petri nets w.r.t. to a subset of transitions, similarly as for P/T-nets.

Definition 3.5. Let $\gamma = (\Sigma, R)$ be a jumping net and $T' \subseteq T$ a subset of transitions of Σ.
a) A *T'-step* Y is a step with $Y(t) = 0$, for all $t \in T - T'$ (practically, Y is a non-empty and finite multiset over the subset T'). Thus, by $Y(\gamma, T') = Y(\gamma) \cap T'_{MS}$ we will denote the set of all T'-steps of γ. b) Let m be an arbitrary marking of γ. The *concurrency-degree w.r.t. T' at m* of γ, denoted by $d(\gamma, T', m)$, is defined by replacing $Y(\gamma)$ with $Y(\gamma, T')$ in (2).

Definition 3.6. Let $\gamma = (\Sigma, R, m_0)$ be a marked jumping P/T-net, and $T' \subseteq T$ a subset of transitions. The *inferior and superior concurrency-degree w.r.t. T'* of the net γ, denoted by $d^-(\gamma, T')$ and $d^+(\gamma, T')$, resp., are defined by replacing $d(\gamma, m)$ with $d(\gamma, T', m)$ in (3), (4).

4 Computing the concurrency-degrees for jumping nets

In this section we will show how we can compute the concurrency-degrees of a jumping net.

First of all, let us notice that the concurrency-degree (w.r.t. a subset of transitions) at any marking of a *JPTN* γ can be computed if γ has the property that $\{m' \mid mR^*m'\}$ is finite, for each marking m (this follows easily from Prop. 3.1, which holds similarly for the notion of concurrency-degrees w.r.t. a set of transitions, because we have to compute a supremum on a finite set).

Let us remark that any finite jumping net has this property. Thus, we have the result:

Theorem 4.1. *The concurrency-degree w.r.t. a set of transitions at a marking, $d(\gamma, T', m)$, is computable for any FJPTN γ, for any subset of transitions T', and for any marking m.*

Corollary 4.1. The concurrency-degree at a marking, $d(\gamma, m)$, is computable for any *FJPTN* γ and for any marking m.

The superior concurrency-degree of a marked finite jumping net can be computed as follows.

Theorem 4.2. *The superior concurrency-degree $d^+(\gamma)$ is computable for any mFJPTN γ. More exactly,*

$$d^+(\gamma) = \max\{\, d(\gamma, m) \mid m \in MCS(\gamma)\}\,.$$

This result has been proved in [7] as a consequence of a more general result, which stated that, given a marked jumping net γ and a monotone increasing function defined on the set of all markings of γ, the supremum of that function on the j-reachability set of γ can be computed using any finite coverability set of γ, like the minimal coverability set.

The inferior concurrency-degree of a marked finite jumping net can be computed as follows.

Let $\gamma = (\Sigma, R, m_0)$ be a *mFJPTN*. It is easy to remark that, although the j-reachability set $[m_0\rangle_{\gamma,j}$ can be finite or infinite, there exists a finite subset $\mathcal{M} \subseteq [m_0\rangle_{\gamma,j}$ such that

$$\forall m \in [m_0\rangle_{\gamma,j}, \exists m' \in \mathcal{M} \text{ such that } m' \leq m. \tag{5}$$

Indeed, we can consider $\mathcal{M}$ as being the set of minimal j-reachable markings of γ, i.e.

$$\mathcal{M} = minimal([m_0\rangle_{\gamma,j}) = \{\, m \in [m_0\rangle_{\gamma,j} \mid \forall m' \in [m_0\rangle_{\gamma,j} - \{m\} : m' \not\leq m\}.$$

By Dickson's lemma, any subset of N^k contains only finitely many incomparable vectors. Since the elements of $\mathcal{M}$ are incomparable, it follows that $\mathcal{M}$ is finite. Then, we have

$$\min\{d(\gamma, m) \mid m \in [m_0\rangle_{\gamma,j}\} = \min\{d(\gamma, m) \mid m \in \mathcal{M}\}, \tag{6}$$

which follows easily from (5) and from the fact that the concurrency-degree at a marking is a monotone increasing function. From (6) and Corollary 4.1, we obtain the result:

Theorem 4.3. *The inferior concurrency-degree $d^-(\gamma)$ is computable for any mFJPTN γ.*

Similar results to Theorem 4.2 and 4.3 hold for the superior and inferior concurrency-degree w.r.t. a subset of transitions.

5 Conclusion

In this paper we have presented a more general definition of concurrency-degrees for jumping Petri nets, which takes into consideration the auto-concurrency, i.e the case of the transitions concurrently enabled with themselves, and, also, a finer notion, of concurrency-degrees w.r.t. a set of transitions. Moreover, we have shown how we can compute the concurrency-degrees for finite jumping Petri nets.

Some problems remain to be studied, for example: a) finding better algorithms for computing the concurrency-degrees of finite jumping nets; b) extending the computability results regarding concurrency-degrees of finite jumping Petri nets for the larger class of jumping Petri nets; c) making some case studies on models of real-world systems.

REFERENCES

[1] Best, E., Fernandez, C.: Notations and Terminology on Petri Net Theory. Arbeitspapiere der GMD 195 (1986)

[2] Finkel, A.: The Minimal Coverability Graph for Petri Nets. In: Advanced in Petri Nets 1993. Lecture Notes in Computer Science, Vol. 674. Springer-Verlag (1993) 210–243

[3] Jucan, T., Ţiplea, F.L.: Reţele Petri. Teorie şi practică. Ed. Academiei Române (1999)

[4] Jucan, T., Vidraşcu, C.: Concurrency-Degrees for Petri Nets. In: Proc. 1st Conf. on Theoretical Computer Science and Informatics Technologies (2000) 108–114

[5] Ţiplea, F.L., Jucan, T.: Jumping Petri Nets. Found. of Comp. and Decision Sciences 19-4 (1994) 319–332

[6] Vidraşcu, C., Jucan, T.: On Coverability Structures for Jumping Petri Nets. Scientific Annals of the Univ. "Al. I. Cuza" of Iaşi, Comp. Sci. Section, Tome IX (2000) 1–26

[7] Vidraşcu, C.: Some Applications of the Minimal Coverability Structures. Scientific Annals of the Univ. "Al. I. Cuza" of Iaşi, Comp. Sci. Section, Tome X (2001) 55–77

[8] Vidraşcu, C., Jucan, T.: On Concurrency-Degrees for Petri Nets. Technical report TR 01-01, Faculty of Computer Science, Univ. "Al. I. Cuza" of Iaşi (2001)

[9] Vidraşcu, C.: Modelling a Sender-Receiver System. Acta Cybernetica, 16-1 (2003)

Received: 22.09.2003

Faculty of Computer Science, Univ. "Al.I.Cuza" of Iasi
16 Gen.Berthelot St., 6600 Iasi, România
E-mail: vidrascu, jucan@infoiasi.