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CONTROLLABILITY OF TIME-VARYING DYNAMICAL SYSTEMS WITH DELAY

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Abstract. Controllability problems for sufficiently wide cass of linear timevarying systems of differential equations are studied. The unified method of researching controllability for time-invariant and time-varying systems of ordinary differential equations, singularly perturbed equations and linear differential equations with delay is suggested. Some effective conditions of complete as well as relative controllability are obtained for these type of systems in terms of the components of solutions to matrix algebraic defining equations.

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1. Introduction

Conrollability and observability problems for discrete and ordinary differential systems were formulated and solved originally by R.Kalman in 1960. These problems as before play a central role in modern control theory, in particular for time-varying ordinary differential systems, functional-differential systems, singularly perturbed dynamic systems (SPDS), SPDS with delay (SPDSD). The concept of controllability appears as necessary and sometimes as sufficient conditions for existence of a solution to most control problems. There exists several approaches of studying the controllability problems for linear time-varying dynamic systems. The efficiency of introducing *the defining equations* (i.e., matrix algebraic recurence equations) to investigate the controllability problem of time-varying differential systems with delay was shown in [1]. This method is very useful for studying the stabilizability problem of singularly perturbed dynamic systems with delay [2], for the row-by-row decoupling problem for linear delay systems [3], etc.

In this paper *the unified method* of investigating controllability problems for various types of systems (time-invariant and time-varying ordinary differential systems, SPDS, SPDSD, etc.) is suggested. This method combines the state space method and the method of the defining equations and takes into account the specific character of the objects under investigation (their time-dependent behaviour, singularity, the presence or lack of delay). In terms of the components of the solutions to the defining equations we formulate controllability conditions for these systems. The rules for constructing the defining equations are very simple and reflect the type of the objects, under consideration, by a natural way.

2. Statement of the problems

Let us consider the following linear time-varying SPDS (LTVSPDS)

$$\dot{x}(t) = \mathcal{A}_1(t,p)\,x(t) + \mathcal{C}_1(t,p)\,y(t) + \mathcal{B}_1(t)\,u(t),$$

$$\mu\,\dot{y}(t) = \mathcal{A}_2(t,p)\,x(t) + \mathcal{C}_2(t,p)\,y(t) + \mathcal{B}_2(t)\,u(t), \tag{2.1}$$

with the initial states

$$x_0(\cdot) = \{x(\vartheta) = \varphi(\vartheta),\ \vartheta \in [t_0 - h, t_0),\ x(t_0) = x_0\},\ x_0 \in \Re^{n_1},$$

$$y_0(\cdot) = \{y(\vartheta) = \psi(\vartheta),\ \vartheta \in [t_0 - h, t_0),\ y(t_0) = y_0\},\ y_0 \in \Re^{n_2}, \tag{2.2}$$

$$\dot{x}_0(\cdot) \ = \ \{\dot{x}(\vartheta) = \dot{\varphi}(\vartheta),\ \vartheta \in [t_0 - h, t_0]\},$$

$$\dot{y}_0(\cdot) \ = \ \{\dot{y}(\vartheta) = \dot{\psi}(\vartheta),\ \vartheta \in [t_0 - h, t_0]\}, \tag{2.3}$$

where $x(\cdot) \in C([t_0 - h, t_1]\,\Re^{n_1}$, x is a slow variable, $y(\cdot) \in C([t_0 - h, t_1]\,\Re^{n_2}$, y is a fast variable, h is some constant delay, $h = const > 0$, u is a control, $u(\cdot) \in U \subset C(T, \Re^r)$, $\Re^n$ is n-dimensional Euclidean space, $C([a,b], \Re^p)$ is a Banach space of continuous functions mapping $[a,b]$ in $\Re^p$ with the topology of uniform convergence, $t \in T = [t_0, t_1]$, μ is a small parameter, $\mu \in (0, \mu^0]$, $\mu^0 \ll 1$, p is a differentiation operator: $px(t) \equiv \dot{x}(t)$; $\exp(-ph)$ is a shift operator of function's argument: $\exp(-ph)\,x(t) \equiv x(t-h)$, so that $p \cdot \exp(-ph)x(t) \equiv \dot{x}(t-h)$;

$A_i(t,p)$, $C_i(t,p)$, $i=1,2$, are operators of the form

$$\mathcal{A}_i(t,p) = A_{i0}(t) + A_{i1}(t)\exp(-ph) + A_{i2}(t)p\cdot\exp(-ph),$$

$$C_i(t,p) = C_{i0}(t) + C_{i1}(t)\exp(-ph) + C_{i2}(t)p\cdot\exp(-ph). \tag{2.4}$$

In (2.1), (2.4) $B_i(t)$, $A_{ij}(t)$, $C_{ij}(t)$, $i=1,2$, $j=\overline{0,2}$, are suitably differentiable matrix functions on T and assumed to be compatible size; in (2.2) $\varphi(t)$, $\psi(t)$ are continuously differentiable vector-functions; in (2.3) $\dot{\varphi}(t)$, $\dot{\psi}(t)$ are continuous n_1- and n_2- vector-functions correspondingly. System (2.1)-(2.3) with matrices $\mathcal{A}_i(t,p)$, $C_i(t,p)$ as in (2.4) is said to be the linear time-varying control SPDS with the deviating argument of neutral type (LTVSPDSNT).

It is readily to note that the system (2.1) contains 6 models of control time-varying differential systems for studying. If we suppose in the system (2.1): a) $\mathcal{A}_i(t,p), C_i(t,p), i=1,2$ as in (2.4) we deal with LTVSPDSNT; b) $A_{i2}(t)=0_{n_i\times n_1}$, $C_{i2}=0_{n_i\times n_2}, i=1,2$ in $\mathcal{A}_i(t,p), C_i(t,p), i=1,2$, we have LTVSPDS with constant delay h (LTVSPDSD); c) in (2.4) $A_{i1}(t)=A_{i2}(t)=0_{n_i\times n_1}, C_{i1}(t)=C_{i2}(t)=0_{n_i\times n_2}, i=1,2$, we have LTVSPDS; d) for $\mu=0, \mathcal{A}_2(t,p)=0_{n_2\times n_1}, C_2(t,p)=0_{n_2\times n_2}, B_2(t)=0_{n_2\times r}$, we have a control differential system of neutral type; e) for $A_{i2}(t)=0_{n_i\times n_1}, C_{i2}(t)=0_{n_i\times n_2}, i=1,2$ in the case d) we have a control differential system with constant delay; h) for $A_{i1}(t)=A_{i2}(t)=0_{n_i\times n_1}, C_{i1}(t)=C_{i2}(t)=0_{n_i\times n_2}, i=1,2$ in the case d) we get a linear time-varying control system of ordinary differential equations (LTVSOD).

Definition 2.1. LTVSPDSNT (2.1)-(2.4), $\mu\in(0,\mu^0]$, is called $\{x,y\}-$relatively controllable ($x-$relatively controllable; $y-$relatively controllable) on T, if for any vector $\{x_1,y_1\}\in\Re^{n_1+n_2}$ and any initial states (2.2), (2.3) there exists an admisible control $u(t)\in U$ such, that the corresponding solution $\{x(t,\mu),y(t,\mu)\}$ of the system (2.1) satisfies the condition $\{x(t_1,\mu),y(t_1,\mu)\}=\{x_1,y_1\}\in\Re^{n_1+n_2}$, $(x(t_1,\mu)=x_1\in\Re^{n_1}; y(t_1,\mu)=y_1\in\Re^{n_2})$.

Problem. ($\{x,y\}-$relative, $y-$relative controllability problem). Find conditions of $\{x,y\}-x-,y-$relative controllability for LTVSPDSNT (2.1)-(2.4), $\mu\in(0,\mu^0]$, expressed through its parameters $A_{ij}(t), C_{ij}(t), B_i(t), i=1,2, j=\overline{0,2}$.

Definition 2.2. LTVSPDSNT (2.1)-(2.4), $\mu\in(0,\mu^0]$ is $\{x,y\}-$relatively ($x-$relatively, $y-$relatively) controllable on T if the corresponding controllability problem is solved for any initial states (2.2), (2.3).

3. The defining equations for time-varying control system

To find algebraic controllability conditions for LTVSPDSNT (2.1) we write it as

$$px(t) = \mathcal{A}_1(t,p)x(t) + \mathcal{C}_1(t,p)y(t) + B_1(t)u(t),$$

$$\mu py(t) = \mathcal{A}_2(t,p)x(t) + \mathcal{C}_2(t,p)y(t) + B_2(t)u(t), \tag{3.1}$$

where $x \in \Re^{n_1}, y \in \Re^{n_2}, u \in \Re^r$. The defining equations for LTVSOD, differential systems with time delay in the state and/or control, LTVSPDSNT (2.1) can be readily obtain similarly [4] using the Laplace transform. By virtue of difficulties in applying Laplace transform to time-varying control systems we suggest another aproach. Let us introduce some correspondences [5]-[7] between vector-functions $x(t), y(t), u(t)$, the operator p, small parameter μ of the system (3.1) and new matrix functions $X_k^i(t,s) \in \Re^{n_1 \times r}, Y_k^i(t,s) \in \Re^{n_2 \times r}, U_k^i(t,s) \in \Re^{r \times r}$ of two arguments t, s (t reflects the time-dependence behaviour of system, s reflects the presence of delay h in it), new operators Δ_+, Δ^+, D according to the rules:

$$x(t) \ \rightarrow \ X_k^i(t,s), \ \ y(t) \rightarrow Y_k^i(t,s),$$

$$u(t) \ \rightarrow \ U_k^i(t,s), \ \ p \rightarrow \Delta_+ + D, \mu \rightarrow \Delta^+. \tag{3.2}$$

In (3.2) Δ_+ (Δ^+) is the shift operator to the right on 1 for lower (upper) index of some matrix function, D is the differentiation operator with respect to t of the some matrix function, so that for example $(\Delta_+ + D)Z_k^i(t,s) \equiv Z_{k+1}^i(t,s) + \dot{Z}_k^i(t,s)$. The index $k+j$ $(j = 0,1)$ at matrices X_{k+j}^i, Y_{k+j}^i corresponds to $j-$derivative of vectors x, y from (2.1); the index $i+m$ $(m = 0,1)$ at X_k^{i+m}, Y_k^{i+m} corresponds to power m of μ at derivatives x, y of the system (2.1). The time-dependent behaviour of the system (2.1) is reflected due to (3.2) by the term D in the correspondence $p \rightarrow \Delta_+ + D$ in contrast to time-invariant case. The correspondence (3.2) transform $\dot{x}(t), \mu\dot{y}(t)$ from (2.1) to $px(t) \rightarrow (\Delta_+ + D)X_k^i(t,s) \equiv X_{k+1}^i(t,s) + \dot{X}_k^i(t,s)$, $\mu py(t) \rightarrow \Delta^+(\Delta_+ + D)Y_k^i(t,s) \equiv Y_{k+1}^{i+1}(t,s) + \dot{Y}_k^{i+1}(t,s)$. Obviously for time-invariant system (3.1) we have $x(t) \rightarrow X_k^i(s)$, $y(t) \rightarrow Y_k^i(s)$ instead of (3.1) and we obtain correspondingly $px(t) \rightarrow (\Delta_+ + D)X_k^i(s) \equiv X_{k+1}^i(s)$, $\mu py(t) \rightarrow \Delta^+(\Delta_+ + D)Y_k^i(s) \equiv Y_{k+1}^{i+1}(s)$. Due to (3.1), (2.4) $\exp(-\Delta_+ h)$ is a shift operator of the argument t for some matrix function: $\exp(-\Delta_- h)X_k^i(t,s) \equiv X_k^i(t-h,s)$; $\exp(-Dh)$ is a shift operator of the argument s for some matrix function: $\exp(-D_+ h)X_k^i(t,s) \equiv X_k^i(t,s-h)$, so that

$\exp(-(\Delta_+ + D)h)X_k^i(t,s) \equiv X_k^i(t-h, s-h)$. In virtue of (3.2) and properties of operators Δ^+, Δ_+, D we have for the system (3.1) the following matrix algebraic recurrence on k, i equations:

$$X_{k+1}^i(t,s) - \overset{i}{X_k}(t,s) =$$

$$= \mathcal{A}_1(t, \Delta_+ + D)X_k^i(ts) + \mathcal{C}_1(t, \Delta_+ + D)Y_k^i(T,s) + B_1(t)U_k^i(t,s), \tag{3.3}$$

$$Y_{k+1}^{i+1}(t,s) | \overset{i+1}{Y_k}(t,s) =$$

$$= \mathcal{A}_2(t, \Delta_+ + D)X_k^i(t,s) + \mathcal{C}_2(t, \Delta_+ - D)Y_k^i(t,s) + B_2(t)U_k^i(t,s),$$

$i, k = 0, 1, 2, \ldots$

To define uniquely the solution $\{X_k^i(t,s)Y_k^i(t,s)\}$ for (3.3) we introduce the initial conditions:

$$U_0^0(t,0) = E_r, \; U_k^i(t,s) = O_r, \; k \neq 0 \vee i \neq 0,$$

$$X_k^i(t,s) = O_{n_1 \times r}, \; Y_k^i(t,s) = O_{n_2 \times r}, \; k \leq 0, \tag{3.4}$$

where $O_{(l_1 \times l_2)}, O_l$ are zero square $(l_1 \times l_2)-$ and $(l \times l)-$matrices correspondingly; E_l is the identity $(l \times l)-$matrix. Matrix algebraic recurrence on k, i equations (3.3) we shall call *the defining equations* of LTVCSPDSNT (2.1).

A totality $\{X_k^i(t,s), Y_k^i(t,s)\} \in \Re^{(n_1+n_2)\times r}$ $(k = 0, 1, 2, \ldots, \; i = 0, 1, 2, \ldots)$ is said to be *a solution of the definig equation* (3.3), (3.4). Matrices $X_k^i(t,s) \in \Re^{n_1 \times r}$, $Y_k^i(t,s) \in \Re^{n_2 \times r}$ calculated according to (3.3), (3.4) we shall call *the components of the solutions* $\{X_k^i(t,s), Y_k^i(t,s)\} \in \Re^{(n_1+n_2)\times r}$ $(k = 0, 1, 2, \ldots, \; i = 0, 1, 2, \ldots)$ to these equations.

Remark 3.1. Note that for the case f), i.e. for LTVSOD the defining equations (3.3), (3.4) coincide with the well known ones before [8]-[10] up to factors $B(t)U_k(t)$, $B(t)U_k(t,s)$. For time-invariant systems with the deviating argument of neutral type equations coincide with [11], but for time-varying systems of such type they are new equations and differ from known previously. For LTVSPDSNT (2.1) and its partial cases of LTVSPDSD, LTVSPDS the defining equations (3.3) are introduced for the first time.

4. Main results

Now we shall find some conditions of relative controllability for NCSPDSNT (2.1)-(2.4) expressed through the components $X_k^i(t,s), Y_k^i(t,s)$ of the solutions to the appropriate defining equations (3.3), (3.4). First we shall demonstrate the main idea of the paper by investigating the controllability problem for the most simple case f), namely for linear time-varying SPDS of ordinary differential equations (LTVSPDSOD) (2.1)-(2.3). Let us represent this system in an extended space of dimension n_1+n_2 in the form

$$\dot{z}(t)=\mathcal{A}(t,\mu)\,z(t)+\mathcal{B}(t)u(t),\ z\in\Re^{n_1+n_2},\ t\in T=[t_0,t_1],\ z(t_0)=z_0, \qquad (4.1)$$

where $\mathcal{A}(t,\mu)\in\Re^{(n_1+n_2)\times(n_1+n_2)}$, $z(t)=col(x(t),y(t))$,

$$\mathcal{A}(t,\mu)=\begin{bmatrix}\mathcal{A}_1(t) & \mathcal{C}_1(t)\\ \mathcal{A}_2(t)/\mu & \mathcal{C}_2(t)/\mu\end{bmatrix}=\begin{bmatrix}\mathcal{A}_{10}(t) & \mathcal{C}_{10}(t)\\ \mathcal{A}_{20}(t)/\mu & \mathcal{C}_{20}(t)/\mu\end{bmatrix},\ \mathcal{B}(t)=\begin{bmatrix}B_1(t)\\ B_2(t)/\mu\end{bmatrix}.$$

Note that for $\mu\to 0$ system (4.1) singularly depends on μ.

Definition 4.1. LNSPDSOD (4.1), $\mu\in(0,\mu^0]$, is completely controllable on T if for any $z_1\in\Re^{n_1+n_2}$ and for any initial state $z_0\in\Re^{n_1+n_2}$ there exists an admissible control $u(t)\in U$ such that the corresponding solution $z(t,\mu), t\in T$, satisfies the condition $z(t_1,\mu)=z_1$.

For each fixed μ, $\mu\in(0,\mu^0]$, the system (4.1) is LTVSOD. Sufficient conditions of complete controllability for this system are well known [12], Theorem 20.1. In terms of solutions $Z_k(t)$ of the defining equations (3.3), (3.4) these conditions have the form:

$$rank\{Z_k(t),\ k=\overline{1,n}\}=n,\ \exists\, t\in T,$$

where

$$Z_{k+1}(t)+\dot{Z}_k(t)=\mathcal{A}(t)Z_k(t)+\mathcal{B}(t)U_k(t), \qquad (4.2)$$

$$U_0(t)=E_r,\ U_k(t)=O_r,\ k\neq 0,\ Z_k(t)=O_{n_1\times r},\ k\leq 0. \qquad (4.3)$$

Note that solutions $X_k(t), Y_k(t)$ of the defining equations (3.3), (3.4) for system (2.1), (2.4) and solutions $Z_k(t)$ of the defining equations (4.2) with the initial states (4.3) are connected by the relation:

$$rank\{Z_k(t),\ k=\overline{1,n_1+n_2}\}=rank\ Z(t,\mu), \qquad (4.4)$$

which is analogous to [7], p.42. Here $\mu\in(0,\mu^0]$,

$$Z(t,\mu)=\begin{bmatrix}\sum\limits_{m=0}^{i-1}\mu^m X_i^{i-m-1}(t)\\ \\ \sum\limits_{m=0}^{i-1}\mu^m Y_i^{i-m}(t)\end{bmatrix} \quad i=\overline{1,n_1+n_2}$$

The proof of this and consequent propositions are analogous to proofs of Theorem 1, Corollaries 1, 2 of [7] and we drop their in this paper.

Theorem 4.2. *Let us assume that* $A_{i0}(t) \in C^{n_1+n_2-2}(T, \Re^{n_i \times n_1})$, $C_{i0}(t) \in C^{n_1+n_2-2}(T, \Re^{n_i \times n_2})$, $B_i(t) \in C^{n_1+n_2-1}(T, \Re^{n_i \times r})$, $i = 1, 2$. *Then*

1. the condition

$$rank\ Z(t, \mu) = n_1 + n_2 \tag{4.5}$$

is a sufficient one of $\{x, y\}$*-complete controllability for LNSPDSOD,* $\mu \in (0, \mu^0]$, *(4.1), (4.2) for some* $t \in T$;

2. if elements of matrices $A_{i0}(t)$, $C_{i0}(t)$, $B_i(t)$, $i = 1, 2$ *are analytical functions on* T *then the condition (4.5) is necessary one as well.*

Corollary 4.1 gives sufficient conditions and corollary 4.2 gives necessary condition of $\{x, y\}-$complete controllability for LNSPDSOD (2.1)–(2.4).

Corollary 4.1. *Suppose that conditions of Theorem 1 are fulfilled. Then if for some set of integers* l_1, l_2, $l_i = \overline{1, n_1 + n_2}$, $i = 1, 2$ *there exists* m, $1 \leq m \leq n_1 + n_2 - \min(l_1 + l_2)$, *for which*

$$rank \begin{bmatrix} X_i^{i-m-l_i-1}(t) & \\ Y_i^{i-m-l_i}(t) & i = \overline{1, n_1 + n_2} \end{bmatrix} = n_1 + n_2,$$

then there exists $\mu^* > 0$ *such that LNSPDSOD (2.1)–(2.4) is completely controllable on* T *for all* $\mu \in (0, \mu^*]$.

Corollary 4.2. *Let us assume that elements of matrices* $A(t)$, $B(t)$ *are analytical functions on* T. *If LTVSPDSOD (4.1) is completely controllable on* T, *then for* $\mu \in (0, \mu^0]$

$$rank \left[\sum_{m=0}^{i-1} \mu^m X_i^{i-m-1}(t), \quad i = \overline{1, n_1 + n_2} \right] = n_1,$$

$$rank \left[\sum_{m=0}^{i-1} \mu^m Y_i^{i-m}(t), \quad i = \overline{1, n_1 + n_2} \right] = n_2.$$

For LTVSPDSNT (2.1) – (2.4) let us create the matrix

$$Z(t, \mu) = \begin{bmatrix} \sum\limits_{m=0}^{i-1} \mu^m X_i^{i-m-1} & \\ & i = \overline{1, n_1 + n_2}, s = \overline{0, lh} \\ \sum\limits_{m=0}^{i-1} \mu^m Y_i^{i-m} & \end{bmatrix}$$

where $\mu \in (0, \mu^0]$; $X_k^i(t,s), Y_k^i(t,s)$ are the components of solutions of the defining equations (3.3), (2.1)

Theorem 4.3.. *Let* $A_{ij}(t) \in C^{n_1+n_2+2}(T, \Re^{n_i \times n_1})$, $C_{ij}(t) \in C^{n_1+n_2-2}(T, \Re^{n_i \times n_2})$, $B_i(t) \in C^{n_1+n_2-1}(T, \Re^{n_i \times r})$, $i = 1, 2;\ j = 0, 1, 2$. *Then*

1. if for some $t \in T$, $\mu \in (0, \mu^0]$

$$rank\ Z(t, \mu) = n_1 + n_2 \tag{4.6}$$

then LTVSPDSNT (2.1) – (2.4) is $\{x, y\}$*-relatively controllable on* T.

2. if elements of matrices $A_{ij}(t)$, $C_{ij}(t)$, $B_i(t)$, $i = 1, 2;\ j = 0, 1, 2$ *are analytical functions on* T, *then (4.6) is necessary condition of* $\{x, y\}$*-relative controllability for (2.1) – (2.4) as well.*

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