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On strongly δ-continuous functions

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Abstract. In this paper attempt has been made to study a new class of function called strongly δ-continuous function and special emphasis is given on homotopy and retraction related properties.

1. Introduction

Here we have studied a new class of functions called strongly δ-continuous functions whose definition is equivalent to the definition of super continuous functions as introduced by Munhsi and Bassan [4]. This class is contained in the class of continuous functions; however we find a condition under which a continuous function is strongly δ-continuous. In the first two sections we give some basic properties of this class and characterise such function by means of graph topology. This class turns out to be an useful tool in studying different types of compactness like properties. Last sections are concerned with strong δ-retract and strong δ-homotopy; using N-R topology [6], we have proved some results on the function space of strongly δ-continuous functions. Throughout the paper spaces mean topological spaces on which no separation axioms are assumed unless explicitly stated and $SD(X, Y)$ would denote the set of all strongly δ-continuous functions from a topological space X to a topological space Y.

2. Prerequisites and Basic Properties

Definition 2.1. [5] *A subset S of a space X is said to be regular open (respectively regular closed) if Int.(cl.(S))=S (respectively Cl.(int.(S))=S), where cl.(S) (respectively int.(S)) denotes the closure (respectively interior) of S. A point $x \in X$ is said to be δ-cluster point of S if $S \cap U \neq \emptyset$, for every regular open set U containing x. The set of all δ-cluster points of S is called the δ-closure of S and is denoted by $[S]_\delta$. If $[S]_\delta = S$ then S is said to be δ-closed. The complement of a δ-closed set is called a δ-open set. Equivalently δ-open set can be defined as : a set G is said to be δ-open if for each $x \in G$, $\exists$ a regular open set H such that $x \in H \subseteq G$ i.e., G is expressible as an arbitrary union of regular open sets.*

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Definition 2.2. [3]*A set $A \subset (X, \tau)$ is said to be N-closed in X or simply N-closed, if for any cover of A by τ-open sets, there exists a finite sub-collection the interiors of the closures of which cover A; interiors and closures are of course w.r.t τ.*
A space (X, τ) is said to be nearly compact iff X is N-closed in X. A subset $A \subset (X, \tau)$, will be called nearly compact iff A is nearly compact w.r.t its subspace topology.

Definition 2.3. [2] *A space is said to be semiregular if every point of the space has a fundamental system of regular open neighbourhoods.*

Definition 2.4. [5] *A function $f : X \to Y$ is called a δ-continuous function iff for every regular open set V of Y, $f^{-1}(V)$ is δ-open in X.*

This can be alternatively defined as follows : a function $f : X \to Y$ is δ-continuous at a point $x \in X$ iff for every regular open nbd. V of $f(x)$ in Y, $\exists$ a δ-open nbd. U of x such that $f(U) \subseteq V$.

Definition 2.5. [3] *A space (X, τ) will be called locally nearly compact if every point has a neighbourhood whose closure is N-closed.*

Theorem 2.1. [3] *The following conditions are equivalent for a T_2 space :*
(i) the space (X, τ) is locally nearly compact;
(ii)for each x in X and each regular open nbd. U of x there is an open set V such that $x \in V \subset \bar{V} \subset U$ and $\bar{V}$ is N-closed.

Definition 2.6. [7] *A function $f : X \to Y$ is strongly δ-continuous at a point $x \in X$ iff for any open nbd. V of $f(x)$ in Y, $\exists$ a δ-open nbd. U of x in X such that $f(U) \subseteq V$; instead of taking an arbitrary nbd. of $f(x)$ we could take a sub-basic open set containing $f(x)$ as well.*

Definition 2.7. [6] *Let X and Y be topological spaces and let*

$$T(C, U) = \{f \in Y^X : f(C) \subset U\};$$

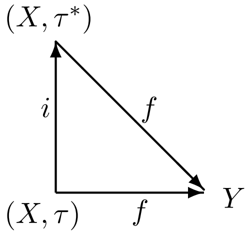
let N denote the N-closed set in X and $\mathcal{R}$ denote the class of all regular open sets in Y; then

$$\{T(C, U) : C \in N \text{ and } U \in \mathcal{R}\}$$

is a subbase for some topology on Y^X; we call this topology the N-R topology on Y^X.

Theorem 2.2. *For a function $f : X \to Y$ the following are equivalent:*
(a) f is strongly δ-continuous .
(b) The inverse image of a closed set is δ-closed.
(c) The inverse image of an open set is δ-open.
(d) For each $x \in X$ and each net $x_\lambda \xrightarrow{\delta} x$, the net $f(x_\lambda) \to f(x)$.

Remark 2.1. We know that for every topological space (X, τ), the collection of all δ-open sets forms a topology for X, which is weaker than τ. This topology τ^*, has a base consisting of all regular open sets in (X, τ). Thus a set is δ-closed in (X, τ) iff it is closed in (X, τ^*). Hence we can conclude that $f : (X, \tau) \to Y$ is strongly δ-continuous iff $f : (X, \tau^*) \to Y$ is continuous.



Observe that $i : (X, \tau) \to (X, \tau^*)$ in the above figure is continuous since $\tau^* \subset \tau$. Thus several results about strongly δ-continuous functions are obtained from the known facts about continuous functions.

Theorem 2.3. *Let $f, g : (X, \tau) \to Y$ be strongly δ-continuous functions and let Y be a T_2 space. Then the set $A = \{x : f(x) = g(x)\}$ is δ-closed in X.*

Theorem 2.4. *Let $f : X \to Y$ be a strongly δ-continuous injective function and Y be a T_2 space . Then X is T_2.*

Theorem 2.5. *If $f : X \to Y$ is strongly δ-continuous and $g : Y \to Z$ is continuous then the composition $g_o f : X \to Z$ is strongly δ-continious.*

Moreover the composition of two strongly δ-continuous function is strongly δ-continuous.

Corollary 2.1. *Let $f : X \to Y$ and $g : Y \to Z$ be any two mappings such that $g_o f$ is strongly δ-continuous. If one of f and g is an open, one-one and onto mapping then the other is strongly δ-continuous.*

Theorem 2.6. *Let $f : X \to \prod_{\alpha \in \Lambda} X_\alpha$ be a mapping. Then f is strongly δ-continuous iff its composition with each projection $\prod_\alpha$ is strongly δ-continuous.*

Proof : If f is strongly δ-continuous, then $\prod_\alpha f$ is strongly δ-continuous by the continuity of $\prod_\alpha$ and by Theorem 1.13.

Conversely, let V be a sub-basic open set in $\prod_{\alpha \in \Lambda} X_\alpha$. Then

$$V = \textstyle\prod_\alpha^{-1}(W)$$

for some open set W in X_α. Then

$$f^{-1}(V) = f^{-1}(\prod_\alpha^{-1}(W)) = (\prod_\alpha f)^{-1}(W)$$

is δ-open by the strong δ-continuity of $\prod_\alpha f$. Thus f is strongly δ-continuous.

Definition 2.8. [1] *Let $f : X \to Y$ be a function then the mapping $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ is called the graph function of f.*

Corollary 2.2. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$, given by $g(x) = (x, f(x))$. Then f is strongly δ-continuous if g is strongly δ-continuous. Further if $g : X \to X \times Y$ is strongly δ-continuous then X is semi-regular.*

Proof : The first part is rather obvious. If g is strongly δ-continuous and $x \in X$, then for any open set U containing x, $U \times Y$ is open in $X \times Y$ and contains $g(x) = (x, f(x))$. Then there exists an open set V containing x such that $g(Int.cl.V) \subset U \times Y$. Consequently $x \in V \subset Int.cl.V \subset U$. Thus X is semi-regular.

Lemma 2.1. *Let $U_{\alpha_i} \subset X_{\alpha_i}$ for each $i = 1, 2, \ldots, n$. Then*

$$U_{\alpha_1} \times U_{\alpha_2} \times \cdots \times U_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha \subset \prod_{\alpha \in \Lambda} X_\alpha$$

is δ-open iff U_{α_i} is δ-open in X_{α_i} for each $i = 1, 2, \ldots, n$.

Proof: Suppose $U_{\alpha_i} \subset X_{\alpha_i}$ is δ-open in X_{α_i} for each $i = 1, 2, \ldots, n$. Then for each $i = 1, 2, \ldots, n$ and each $x_{\alpha_I} \in U_{\alpha_i}$, there exists an open set V_{α_i} containing x_i such that $x_i \in V_{\alpha_i} \subset Int.cl.(V_{\alpha_i}) \subset U_{\alpha_i}$. Thus for each

$$\{x_\alpha\} \in U_{\alpha_1} \times U_{\alpha_2} \times \cdots \times U_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha,$$

$$\{x_\alpha\} \in V_{\alpha_1} \times V_{\alpha_2} \times \cdots \times V_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha \subset Int.cl.(V_{\alpha_1}) \times Int.cl.(V_{\alpha_2}) \times$$

$$\cdots \times Int.cl.(V_{\alpha_n}) \times \prod_{\alpha \neq \alpha_i} X_\alpha \subset U_{\alpha_1} \times U_{\alpha_2} \times \cdots \times U_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha.$$

This clearly shows that $U_{\alpha_1} \times U_{\alpha_2} \times \cdots \times U_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha$ is δ-open.

The converse is obvious.

Theorem 2.7. *Define $\prod_{\alpha \in \Lambda} f_\alpha : \prod_{\alpha \in \Lambda} X_\alpha \to \prod_{\alpha \in \Lambda} Y_\alpha$ by $\{x_\alpha\} \to \{f_\alpha(x_\alpha)\}$. Then $\prod_{\alpha \in \Lambda} f_\alpha$ is strongly δ-continuous iff each $f_\alpha : X_\alpha \to Y_\alpha$ is strongly δ-continuous.*

Proof: Let $V = V_{\alpha_1} \times V_{\alpha_2} \times \cdots V_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha$ be a basic open set in $\prod_{\alpha \in \Lambda} Y_\alpha$. Then if $f_{\alpha_i}^{-1}(V_{\alpha_i})$ is δ-open in X_{α_i} for each α_i, we have

$$(\prod_{\alpha \in \Lambda} f_\alpha)^{-1}(V) = f_{\alpha_1}^{-1} V_{\alpha_1} \times f_{\alpha_2}^{-1} V_{\alpha_2} \times \cdots f_{\alpha_n}^{-1} V_{\alpha_n} \times \prod_{\alpha \neq \alpha_i} X_\alpha$$

is δ-open in $\prod_{\alpha\in\Lambda} X_\alpha$ by Lemma 1.18. This implies that $\prod_{\alpha\in\Lambda} X_\alpha$ is strongly δ-continuous.

Conversely, suppose $\prod_{\alpha\in\Lambda} f_\alpha$ is strongly δ-continuous . Let $V_{\alpha_i} \subset Y_{\alpha_i}$ be open. Then $V = V_{\alpha_1} \times V_{\alpha_2} \times \cdots \times V_{\alpha_n} \times \prod_{\alpha\neq\alpha_i} Y_\alpha$ is a sub-basic open set in $\prod_{\alpha\in\Lambda} Y_\alpha$ and

$$(\prod_{\alpha\in\Lambda} f_\alpha)^{-1}(V) = f_{\alpha_1}^{-1}(V_{\alpha_1}) \times f_{\alpha_2}^{-1}(V_{\alpha_2}) \times \cdots \times f_{\alpha_1}^{-1}(V_{\alpha_n}) \times \prod_{\alpha\neq\alpha_i} X_\alpha$$

is δ-open . Thus $f_{\alpha_i}^{-1}(V_{\alpha_i})$ is δ-open in X_{α_i}, for each $i = 1, 2, \ldots, n$, which implies that f_{α_i} is strongly δ-continuous.

3. Sufficient condition for strong δ-continuity

Theorem 3.8. *Let $f : X \to Y$ be continuous . If Y is semiregular , then f is strongly δ-continuous.*

Proof : Let $x \in X$ and let V be an open set in Y containing $f(x)$. Since Y is semi regular $\exists$ open set W such that $f(x) \in W \subset Int.cl.(W) \subset V$. This fact along with the continuity of f implies

$$x \in f^{-1}(W) \subset Int.cl.f^{-1}(W) \subset f^{-1}(Int.cl.W) \subset f^{-1}(V).$$

Now let $U = f^{-1}(W)$. Then $f(Int.cl.U) \subset V$. Thus f is strongly δ-continuous.

Definition 3.9. *If $f : X \to Y$ is a function and $G(f) = \{(x, f(x)) : x \in X\}$ denotes the graph of f, we define $G(f)$ to be δ-closed w.r.t $X \times Y$ if for each $(x, y) \notin G(f)$ $\exists$ δ-open sets U and V containing x and y respectively such that $(U \times V) \cap G(f) = \emptyset$.*

With this definition we prove another sufficient condition for strongly δ-continuity.

Theorem 3.9. *Let $f : X \to Y$ have a δ-closed graph w.r.t $X \times Y$. If Y is compact, then f is strongly δ-continuous.*

Proof : Let $x \in X$ and let V be an open set containing $f(x)$. Then $Y \setminus V$ is closed and for each $y \in Y \setminus V$, $(x, y) \notin G(f)$. Then by the above definition, $\exists$ two δ-open sets $U_y(x)$ and $W(y)$ containing x and y respectively such that

$$(U_y(x) \times W(y)) \cap G(f) = \emptyset \quad \Rightarrow \quad f(U_y(x)) \cap W(y) = \emptyset.$$

The collection $\{W(y) : y \in Y \setminus V\}$ forms an open cover of $Y \setminus V$. Since Y is compact so $Y \setminus V$, being closed subset of a compact set is closed. Consequently there is a finite collection $\{W(y_i) : i = 1, 2, \ldots, n\}$ such that $Y \setminus V \subset \cup_{i=1}^n W(y_i)$. Now let $U = \cap_{i=1}^n U_{y_i}(x)$. Then U is an δ-open set and $f(U) \subset V$. Hence f is strongly δ-continuous.

Definition 3.10. *The graph of $f : X \to Y$ is called δ-closed w.r.t X if for each $(x, y) \notin G(f)$, $\exists$ δ-open set U and open set V containing x and y respectively such that $(U \times V) \cap G(f) = \emptyset$.*

Obviously every strongly δ-continuous function is δ-continuous. The next theorem gives a criterion for a δ-continuous function to be strongly δ-continuous function.

Definition 3.11. *A space X is said to be rim-N-closed if the boundary of every basic open set is N-closed.*

Theorem 3.10. *If Y is rim-N-closed and almost regular and $f : X \to Y$ is a δ-continuous function whose graph is δ-closed w.r.t X, then f is a strongly δ-continuous function.*

Proof : Let $x \in X$ and let W be an open set containing $f(x)$. Since Y is rim-N-closed, $\exists$ an open set V such that $f(x) \in V \subset W$ with BdV (BdV denotes the boundary of the basic open set V) N-closed$\cdots$(1).
Let $y \in$ BdV, then $y \neq f(x)$ (as $f(x) \in V$) and then $(x, y) \notin G(f)$. Since $G(f)$ is δ-closed, $\exists$ open ndbs. U_y^x & U_y of x and y respectively such that

$$Int.cl.(U_y^x \times U_y) \cap G(f) = \emptyset \text{ i.e., } (Int.cl.U_y^x \times Int.cl.U_y) \cap G(f) = \emptyset$$

and thus

$$f(Int.cl.U_y^x) \cap Int.cl.U_y = \emptyset \cdots (2).$$

Now $\{Int.cl.U_y : y \in \text{ Bd}V\}$ is a covering of BdV by regular open sets and thus has a finite subcovering say $Int.cl.U_{y_1}, Int.cl.U_{y_2}, \ldots, Int.cl.U_{y_n}$. Now f is δ-continuous at x and so there exists an open nbd.

$$U_0 \text{ of } x \text{ such that } f(Int.cl.U_0) \subset Int.cl.V.$$

Let $U = Int.cl.U_0 \cap (\cap_{i=1}^n Int.cl.U_{y_i}^x)$; then U is a regular open nbd. of x. Also

$$\begin{gathered} f(U) \cap (Y \setminus V) = f(U) \cap \text{ Bd}V \text{ [as } f(Int.cl.U_0) \subset Int.cl.V] \\ \subset f(U) \cap [\cup_{i=1}^n Int.cl.U_{y_i}] \subset \cup_{i=1}^n (f(U) \cap Int.cl.U_{y_i}) \subset \\ \cup_{i=1}^n (f(Int.cl.U_{y_i}^x) \cap Int.cl.U_{y_i}) = \emptyset \text{ (by (2))}. \end{gathered}$$

Thus $f(U) \subset V$. U being a regular open nbd. of x f is strongly δ-continuous.

Theorem 3.11. *Let Y be a compact space. If $f : X \to Y$ has a graph which is δ-closed w.r.t X, then f is strongly δ-continuous.*

Proof : Let $x \in X$ and let V be an open set containing $f(x)$. Then for each $y \in Y \setminus V$, we have $(x, y) \notin G(f)$. Then by the condition of δ-closed graph of f, $\exists$ δ-open set $U_y(x)$ and open set $W(y)$ containing x and y respectively such that

$$(U_y(x) \times W(y)) \cap G(f) = \emptyset \;\Rightarrow\; f(U_y(x)) \cap W(y) = \emptyset.$$

Now $\{W(y) : y \in Y \setminus V\}$ is an open cover of the closed subset of the compact set space Y i.e. the compact set $Y \setminus V$. So it has a finite sub-cover such that $Y \setminus V \subset \cup_{i=1}^{n} W(y_i)$. Let $U = \cap_{i=1}^{n} U_{y_i}(x)$. Then U is an δ-open set containing x and $f(U) \cap [\cup_{i=1}^{n} W(y_i)] = \emptyset$ so $f(U) \subset V$ proving that f is strongly δ-continuous.

Theorem 3.12. *If $f : X \to Y$ is a strongly δ-continuous mapping and Y is T_2 then $G(f)$ is δ-closed w.r.t X.*

Proof : Let $x \in X$ and let $y \neq f(x)$. Then by the T_2-ness of Y there are open sets U and V containing $f(x)$ and y respectively such that $U \cap V = \emptyset$. Since f is strongly δ-continuous, there exists a δ-open set W containing x such that $f(W) \subset U$. Therefore

$$f(W) \cap V = \emptyset \ \Rightarrow \ (W \times V) \cap G(f) = \emptyset.$$

Thus $G(f)$ is δ-closed w.r.t X.

Combining the above two Theorems we can state:

Theorem 3.13. *Let Y be a compact T_2 space. Then $f : X \to Y$ is strongly δ-continuous iff $G(f)$ is δ-closed w.r.t X.*

Theorem 3.14. *The graph mapping of a strongly δ-continuous function is strongly δ-continuous.*

Proof : Let $f : X \to Y$ be a strongly δ-continuous mapping and let $g : X \to X \times Y$ be the graph mapping of f. Let $x \in X$ and let W be an open nbd. of $g(x)$ in $X \times Y$. Then there exist open sets U_1 and U_2 of X and Y respectively such that $g(x) \in U_1 \times U_2 \subseteq M$. Since $g(x) = (x, f(x))$, it implies that $f(x) \in U_2$. Since f is strongly δ-continuous $\exists$ an open set V containing x such that $f(Int.cl.V) \subset U_2$. Let $M = V \cap U_1$. Then M is an open nbd. of x such that $g(Int.cl.M) \subseteq U_1 \times U_2 \subset W$. Hence g is strongly δ-continuous.

Theorem 3.15. *Let $f : X \to Y$ be a mapping and let $x \in X$. If $\exists$ a δ-open nbd. N of x such that the restriction of f to N is strongly δ-continuous at x, then f is strongly δ-continuous at x.*

Proof : Let U be an open set containing $f(x)$ in Y. Since $f_{|N}$ is strongly δ-continuous at x, therefore there is a δ-open set V_1 such that $x \in N \cap V_1$ and $f(N \cap V_1) \subset U$. Hence the result result follows from the fact that $N \cap V_1$ ia a δ-open nbd. of x.

Corollary 3.3. *Let $f : X \to Y$ be a mapping and let $\{G_\lambda : \lambda \in \Lambda\}$ be an δ-open cover of X. If for each $\lambda \in \Lambda$, $f_{|G_\lambda}$ is strongly δ-continuous at each point of G_λ, then f is strongly δ-continuous.*

Theorem 3.16. *Let $f : X \to Y$ be a mapping and let $X = X_1 \cup X_2$, where X_1 and X_2 are δ-closed and $f_{|X_1}$ & $f_{|X_2}$ are strongly δ-continuous, then f is strongly δ-continuous.*

Proof : Let A be a closed subset of Y. Since $f_{|X_1}$ & $f_{|X_2}$ are both strongly δ-continuous then $(f_{|X_1})^{-1}(A)$ and $(f_{|X_2})^{-1}(A)$ are both δ-closed in X_1 and X_2 respectively. Since X_1 and X_2 are δ-closed subsets of X, therefore $(f_{|X_1})^{-1}(A)$ and $(f_{|X_2})^{-1}(A)$ are δ-closed subsets of X. Also $f^{-1}(A) = (f_{|X_1})^{-1}(A) \cup (f_{|X_2})^{-1}(A)$. Thus $f^{-1}(A)$ is the union of two δ-closed set and hence δ-closed . Thus f is strongly δ-continuous.

Definition 2.14 : A point x of a subset A of a space X is called an δ-interior point of A if $\exists$ a δ-open nbd. U of x which lies wholly within A. A point of A is called a δ-boundary point of A if it is not a δ-interior point of A, the δ-boundary of A is composed of precisely of all δ-boundary points of A.

Theorem 3.17. *The set of all points of a space X at which $f : X \to Y$ is not strongly δ-continuous, is identical with the union of the δ-boundaries of the inverse images of open subsets of Y.*

Proof : Suppose f is not strongly δ-continuous at a point $x \in X$. Then $\exists$ an open set V containing $f(x)$ and for every δ-open set U containing x, we have $f(U) \cap (Y \setminus V) \neq \emptyset$. Thus for every δ-open set U containing x, we must have $U \cap (X \setminus f^{-1}(V)) \neq \emptyset$. Thus x cannot be an δ-interior point of $f^{-1}(V)$. But $x \in f^{-1}(V)$. So x is a point of the δ-boundary of $f^{-1}(V)$.

Now, let x belong to the δ-boundary of $f^{-1}(G)$, for some open subset G of Y. Then $f(x) \in G$. If f is strongly δ-continuous at x, then there is an δ-open set V such that $x \in V$ and $f(V) \subset G$. Thus $x \in V \subset f^{-1}(f(V)) \subset f^{-1}(G)$. Therefore x is a δ-interior point of $f^{-1}(G)$, which is a contradiction. Hence f is not strongly δ-continuous at x. This completes the proof.

4. Properties preserved by strongly δ-continuous functions

Theorem 4.18. *Let $f : X \to Y$ be a strongly δ-continuous function. If $A \subset X$ is an N-closed set then $f(A)$ is compact.*

Proof : Let A be an N-closed set in X and let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be an open cover of $f(A)$. Then for each $a \in A$, there is an open set $U_a \in \mathcal{U}$ such that $f(a) \in U_a$. Since f is strongly δ-continuous, there exists a δ-open set V_a containing a such that $f(V_a) \subset U_a$. Now the collection $\{V_a : a \in A\}$ forms a δ-open cover of A and so there exists a finite sub-collection $\{V_{a_1}, \ldots, V_{a_n}\}$ such that $A \subset \cup_{i=1}^{n} V_{a_i}$. Then $f(A) \subset f(\cup_{i=1}^{n} V_{a_i}) = \cup_{i=1}^{n}(f(V_{a_i})) \subset \cup_{i=1}^{n} U_{a_i}$ so that $\mathcal{U}$ has a finite sub-collection $\{U_{a_i} : i = 1, \ldots, n\}$ which covers $f(A)$. Thus $f(A)$ is compact.

Corollary 4.4. *If $f : X \to Y$ be a strongly δ-continuous surjective mapping with X is N-closed then Y is compact.*

Corollary 4.5. *A strongly δ-continuous real valued function defined on an N-closed space is bounded.*

Theorem 4.19. *Let $f : X \to Y$ be a strongly δ-continuous open surjective function. If X is locally nearly compact T_2 and Y is T_2 then Y is locally compact.*

Proof : Let $y \in Y$ and $x \in X$ be such that $f(x) = y$. Let $y \in W$ where W is open in Y; we show that there exist an open set U_y containing y such that $y \in U_y \subset \overline{U}_y \subset W$ where $\overline{U_y}$ is compact. Since f is strongly δ-continuous, there exist an open set U_x containing x such that $f(Int.cl.U_x) \subset W$. Since $Int.cl.U_x$ is a regular open nbd of x, there exists an open set V_x containing x such that $x \in V_x \subset \overline{V}_x \subset Int.cl.U_x$ where $\overline{V_x}$ is N-closed ([3] Theorem 3.2). Now f being open $f(V_x)$ is open nbd of y in Y. Again, by Theorem 3.1 $f(\overline{V}_x)$ is compact in $Y \cdots (1)$ and hence closed. Thus

$$y \in \overline{f(V_x)} \subset f(\overline{V_x}) \subset f(Int.cl.U_x) \subset W.$$

But $f(\overline{V_x})$ being closed in Y is compact. Hence $y \in \overline{f(V_x)} \subset f(\overline{V_x}) \subset W$ and thus Y is locally compact.

Definition 4.12. *A T_2 space X is called p-compact if each closed set in X is an N-closed set.*

Theorem 4.20. *Let $f : X \to Y$ be a strongly δ-continuous bijective function from a p-compact space X to a Hausdorff space Y, then X is homeomorphic to Y and both X, Y are compact.*

Proof : Since f is strongly δ-continuous, f is continuous. Further as X is p-compact so if $A \subset X$ is closed then A is an N-closed set so that $f(A)$ is compact by Theorem 3.1 and hence closed in the Hausdorff space Y. This shows that f is a homeomorphism from X onto Y. Now since X is itself an N-closed set, $f(X) = Y$ is compact again by Theorem 3.1 . It follows that both $X \,\&\, Y$ are compact since they are homeomorphic.

Theorem 4.21. *Let F be a mapping from $SD(X, Y) \times SD(Y, Z)$ to $SD(X, Z)$ by the rule $F(f, g) = g_\circ f$. If the topology of each of the function spaces is N-R topology and Y is locally nearly compact T_2, then F is continuous.*

Proof : Let $T(C, U)$ be a sub-basic open nbd. of $g_\circ f$ in $SD(X, Z)$ in the N-R topology, where C is an N-closed subset of X and U is a regular open set in Z. So

$$g_\circ f \in T(C, U) \;\Rightarrow\; (g_\circ f)(C) \subset U \;\Rightarrow\; f(C) \subset g^{-1}(U).$$

Since $f \in SD(X, Y)$ and C is N-closed then $f(C)$ is a compact subset of Y and $g \in SD(Y, Z)$ and U regular open in Z and hence open in Z so $g^{-1}(U)$ is δ-open in Y.

Since Y is locally nearly compact T_2 and $f(C)$ has a δ-open nbd. $g^{-1}(U)$, $\exists$ a regular open nbd. V of $f(C)$ such that $\overline{V} \subset g^{-1}(U)$ with $\overline{V}$ N-closed (by Theorem 1.16). Then clearly $T(C, V)$ is an open nbd. of f and $T(\overline{V}, U)$ is an open nbd. of g in the N-R topology.

It remains to show that $F(T(C,V) \times T(\overline{V},U)) \subset T(C,U)$.
Let $(f_1, g_1) \in T(C,V) \times T(\overline{V},U)$ then $f_1(C) \subset V$ & $g_1(\overline{V}) \subset U$. So

$$F(f_1, g_1)(C) = (g_{1\circ} f_1)(C) = g_1(f_1(C)) \subset g_1(V) \subset g_1(\overline{V}) \subset U.$$

Hence $(g_{1\circ} f_1) \in T(C,U)$, which proves that F is continuous.

Theorem 4.22. *Let $f : X \to Y$ be a strongly δ-continuous function and Y is locally nearly compact T_2. Then the map $f^+ : SD(Y,Z) \to SD(X,Z)$ induced by f defined by $f^+(g) = g_\circ f$ is continuous. The function spaces are endowed with N-R topology.*

Proof : Let $T(C,U)$ be a subbasic open nbd.of $g_\circ f$ in $SD(X,Z)$ in the N-R topology, where C is a N-closed subset of X and U a is regular open set in Z. So

$$g_\circ f \in T(C,U) \;\Rightarrow\; (g_\circ f)(C) \subset U \;\Rightarrow\; f(C) \subset g^{-1}(U).$$

Now $f \in SD(X,Y)$,$f(C)$ is a compact subset of Y and $g \in SD(Y,Z)$ then $g^{-1}(U)$ is δ-open in Y.

Since Y is locally nearly compact T_2 and $f(C)$ has a δ-open nbd. $g^{-1}(U)$, there exists a regular open nbd. V of $f(C)$ such that $\overline{V} \subset g^{-1}(U)$ with $\overline{V}$ N-closed. Then $T(\overline{V},U)$ is an open nbd. of g in the N-R topology. We claim that $f^+(T(\overline{V},U) \subset T(C,U)$.
Let $g_1 \in T(\overline{V},U) \;\Rightarrow g_1(\overline{V}) \subset U$ then

$$f^+(g_1)(C) = (g_{1\circ} f)(C) = g_1(f(C)) \subset g_1(V) \subset g_1(\overline{V}) \subset U.$$

So $(g_{1\circ} f) \in T(C,U)$ and hence $f^+(T(\overline{V},U)) \subset T(C,U)$. Thus f^+ is continuous.

Theorem 4.23. *Let $f : X \to Y$ be a strongly δ-continuous map. Then f induces a map $f_+ : SD(Z,X) \to SD(Z,Y)$ given by $f_+(g) = f_\circ g$ which is continuous.*

Proof : Let $T(C,U)$ be a subbasic open nbd. of $f_\circ g$ in $SD(Z,Y)$ in the N-R topology, where C is a N-closed subset of Z and U is regular open set in Y. So $f_\circ g \in T(C,U) \;\Rightarrow\; (f_\circ g)(C) \subset U \;\Rightarrow\; g(C) \subset f^{-1}(U)$. Since U is regular open and $f \in SD(X,Y)$ so $f^{-1}(U)$ is δ-open in X.Then clearly $T(C, f^{-1}(U))$ is an open nbd. of g in the N-R topology of $SD(Z,X)$. We claim that $f_+(T(C, f^{-1}(U))) \subset T(C,U)$.
Let $g_1 \in T(C, f^{-1}(U)) \;\Rightarrow g_1(C) \subset f^{-1}(U)$. So

$$f_+(g_1)(C) = (f_\circ g_1)(C) = f(g_1(C)) \subset f(f^{-1}(U)) \subseteq U.$$

Thus $f_+(T(C, f^{-1}(U))) \subset T(C,U)$, which shows f_+ is continuous.

5. Retraction and fixed point property

Definition 5.13. *A subset A of a space X is said to be a strong δ-retract of X if there exists a strongly δ-continuous mapping $f : X \to A$ such that f is identity on A (i.e., $f(x) = x$, $\forall x \in A$).*

Theorem 5.24. *Strong δ-retract B of a strong δ-retract A of X is a strong δ-retract of X.*

Proof : Since A is a strong δ-retract of X, there exists a strongly δ-continuous mapping $f : X \to A$ such that f is identity on A. Since B is a strong δ-retract of A, there exists a strongly δ-continuous mapping $g : A \to B$ such that g is identity on B. Then the composition mapping $g_o f : X \to B$ is strongly δ-continuous by Theorem 1.13. If x be any point of B, then $x \in A$ and thus $(g_o f)(x) = g(f(x)) = g(x) = x$. Thus $g_o f$ is identity on B. Hence B is a strong δ-retract of X.

Theorem 5.25. *If X is a T_2 space and $A \subset X$ a strong δ-retract of X, then A is δ-closed in X.*

Proof : Since X is a T_2-space for any two points x and y in X there exist open sets U_x and U_y containing x and y such that $U_x \cap U_y = \emptyset \cdots (1)$. From (1), $U_x \cap \overline{U}_y = \emptyset$ i.e., $U_x \cap Int.cl.\overline{U}_y = \emptyset$ and $Int.cl.\overline{U}_y$ is an open set. Again,

$$Int.cl.\overline{U}_y \cap \overline{U}_x = \emptyset \Rightarrow Int.cl.\overline{U}_y \cap Int.cl.\overline{U}_x = \emptyset$$

and then x and y are strongly separated by regular open sets. Now, since A is a strong δ-retract of X, there exist a δ-continuous map $f : X \to A$ such that $f(x) = x$ for each $x \in A$.

Let $x \notin A$; then $f(x) \neq x$ in X, i.e., there exist two regular open sets U_1 and U_2 containing $f(x)$ and x respectively such that $U_1 \cap U_2 = \emptyset$. Again f being δ-continuous and U_1 being regularly open there exists a regular open nbd U_3 of x such that $f(U_3) \subset U_1$; but $U_2 \cap U_3 = U_4$(say) is again a regular open nbd of x and $f(U_4) \subset U_1$; if possible let $x \in [A]_\delta$; then $U_4 \cap A \neq \emptyset$ i.e., there exist $a \in A$ such that $a \in U_4$; but since $a \in A, f(a) = a$ and thus $a = f(a) \in f(U_4) \subset U_1$; but $U_4 \subset U_2$ and $U_2 \cap U_1 = \emptyset$ — and hence a contradiction. Thus $x \notin [A]_\delta$ hence $[A]_\delta = A$ i.e., A is δ closed.

Theorem 5.26. *A subset A of a space X is a strong δ-retract of X iff for every space Y, every strongly δ-continuous mapping $f : A \to Y$ can be extended to a strongly δ-continuous mapping from X into Y.*

Proof : First let A be a strong δ-retract of X. Then there exists a strongly δ-continuous mapping $g : X \to A$ such that g is identity on A. Let $f : A \to Y$ be any strongly δ-continuous mapping. Then the composition mapping $f_o g : X \to Y$ is a strongly δ-continuous mapping by Theorem 1.13. If $x \in A$ then

$$(f_o g)(x) = f(g(x)) = f(x).$$

Thus $f_o g$ is an extension of f.

Conversely suppose that for every space Y, every strongly δ-continuous mapping $f : A \to Y$ can be extended to a strongly δ-continuous mapping

$g : X \to Y$. Let $f : A \to A$ be the mapping defined by $f(a) = a$, $\forall a \in A$. Then f is strongly δ-continuous. By hypothesis $\exists$ a strongly δ-continuous mapping $g : X \to A$ such that $g_{|A} = f$. Let $x \in A$, then $g(x) = f(x) = x$. Hence A is a strong δ-retract of X.

Definition 5.14. *A space X is said to have the strong δ-fixed point property if for every strongly δ-continuous mapping $f : X \to X$, $\exists$ an $x \in X$ such that $f(x) = x$.*

Theorem 5.27. *If a space has the strong δ-fixed point property and A is a strong δ-retract of X, then A has the strong δ-fixed point property.*

Proof : Let $f : A \to A$ be any strongly δ-continuous mapping. Since A is a strong δ-retract of X, f can be extended to a strongly δ-continuous mapping $g : X \to A$ by Theorem 4.4. Since X has the strong δ-fixed point property, there exists an $x \in X$ such that $g(x) = x$. Since g is from X to A so $x \in A$. It follows that $f(x) = g(x) = x$. Thus x is a fixed point of A. Hence A has the strong δ-fixed point property.

6. Strong δ-homotopy

Definition 6.15. *Two mappings $f, g : X \to Y$ are said to be strong δ-homotopic if $\exists$ a strongly δ-continuous mapping $\Phi : I \times X \to Y$, where X, Y are any spaces and I is the closed interval $[0, 1]$ with the countable complement extension topology [i.e., a set O in I is open iff $O = U \setminus A$, where U is any open set in the usual topology and A is a countable subset of I] such that*

$$\Phi(0, x) = f(x) \quad \text{and} \quad \Phi(1, x) = g(x),\ \forall x \in X.$$

Definition 6.16. *Two mapping $f, g : X \to Y$ are said to lie in a common strong δ-path component if there exists a strongly δ-continuous mapping $\widehat{\Phi} : I \to SD(X, Y)$, where X, Y are any two spaces and $I = [0, 1]$ with the countable complement extension topology such that*

$$\widehat{\Phi}(0) = f \ \text{and}\ \widehat{\Phi}(1) = g.$$

Theorem 6.28. [7] *The N-R topology on $SD(Y, Z)$ is strongly δ-splitting i.e., for every space X, the strong δ-continuity of a map $\alpha : X \times Y \to Z$ implies the strong δ-continuity of the map $\widehat{\alpha} : X \to SD(Y, Z)$.*

Theorem 6.29. [7] *On the set $SD(Y, Z)$, the N-R topology is strongly δ-conjoining i.e., for every space X, the strong δ-continuity of a map $\widehat{\alpha} : X \to SD(Y, Z)$ implies the strong δ-continuity of the map $\alpha : X \times Y \to Z$, provided Y is locally nearly compact T_2 and Z is semiregular.*

Theorem 6.30. *Let the function space $SD(X, Y)$ be endowed with the N-R topology. If two mappings $f, g : X \to Y$ are strong δ-homotopic then they lie in a common strong δ-path component and conversely, if f and g*

lie in a common strong δ-path component then they are strong δ-homotopic provided X is locally nearly compact T_2 and Y is a semiregular space.

Proof : Let Φ be a strong δ-homotopy between $f \,\&\, g$ i.e., the mapping $\Phi : I \times X \to Y$ is strongly δ-continuous then by Theorem 5.3 the associated map $\widehat{\Phi} : I \to SD(X, Y)$ is strongly δ-continuous where $SD(X, Y)$ is endowed with N-R topology. Also

$$\Phi(0, x) = f(x) \text{ and } \Phi(1, x) = g(x)$$

for every x gives

$$[\widehat{\Phi}(0)](x) = \Phi(0, x) = f(x) \quad \text{and} \quad [\widehat{\Phi}(1)](x) = \Phi(1, x) = g(x) \text{ for all } x.$$

Thus

$$\widehat{\Phi}(0) = f \quad \text{and} \quad \widehat{\Phi}(1) = g.$$

Hence $\widehat{\Phi}$ is a strong δ-path joining f to g, i.e., f and g lie in a common strong δ-path component.

Conversely, suppose $\widehat{\Phi} : I \to SD(X, Y)$ be a strong δ-path joining f to g. Since X is locally nearly compact T_2 and Y is semi regular so by Theorem 5.4 the associated map $\Phi : I \times X \to Y$ is strongly δ-continuous. Also

$$\phi(0, x) = [\widehat{\Phi}(0)](x) = f(x) \quad \text{and} \quad \Phi(1, x) = [\widehat{\Phi}(1)](x) = g(x)\ \forall x \in X.$$

Thus $f \& g$ are strong δ-homotopic.

Theorem 6.31. *If the function space are endowed with the N-R topology and the two maps $f, g : X \to Y$ are strong δ-homotopic with Y a locally nearly compact T_2 space, then the maps $f^+, g^+ : SD(Y, Z) \to SD(X, Z)$ induce by f and g are strong δ-homotopic.*

Proof : Let Φ be the strong δ-homotopy between f and g, i.e., the mapping $\Phi : I \times X \to Y$ is strongly δ-continuous. Then by Theorem 5.3 the associated map $\widehat{\Phi} : I \to SD(X, Y)$ is strongly δ-continuous. Hence $\widehat{\Phi} \times 1 : I \times SD(Y, Z) \to SD(X, Y) \times SD(Y, Z)$ is strongly δ-continuous. We consider the mapping $F : SD(X, Y) \times SD(Y, Z) \to SD(X, Z)$, since Y is locally nearly compact T_2, F is continuous by Theorem 3.7. Hence the composition function $F_\circ(\widehat{\Phi} \times 1) : I \times SD(Y, Z) \to SD(X, Z)$ is strongly δ-continuous by Theorem 1.10.

Put $H = F_\circ(\widehat{\Phi} \times 1)$; for any

$$f_1 \in SD(Y, Z),\ H(0, f_1) = (F_\circ(\widehat{\Phi} \times 1))(0, f_1) = F(\widehat{\Phi}(0), f_1) = f_{1\circ}\widehat{\Phi}(0).$$

For any $x \in X$,

$$[H(0, f_1)](x) = [f_1(\widehat{\Phi}(0))(x)] = f_{1\circ} f(x)$$

giving $H(0, f_1) = f_{1\circ} f = f^+(f_1)$. Similarly $H(1, f_1) = g^+(f_1)$.

Thus $f^+ \,\&\, g^+$ are strong δ-homotopic.

Theorem 6.32. *If $f, g : X \to Y$ are strong δ-homotopic and if X is a locally nearly compact T_2 space, then the induced maps $f_+, g_+ : SD(Z, X) \to SD(Z, Y)$ are strong δ-homotopic.*

Proof : Let Φ be the strong δ-homotopy between $f\&g$, i.e., the mapping $\Phi : I \times X \to Y$ is strongly δ-continuous then by Theorem 5.3 the associated map $\widehat{\Phi} : I \to SD(X,Y)$ is strongly δ-continuous. Hence $1 \times \widehat{\Phi} : SD(Z,X) \times I \to SD(Z,X) \times SD(X,Y)$ is strongly δ-continuous. Since X is locally nearly compact T_2 so the map $F : SD(Z,X) \times SD(X,Y) \to SD(Z,Y)$, is continuous. Thus the composition map $F_\circ(1 \times \widehat{\Phi})$ is strongly δ-continuous by Theorem 1.10.
Set $H = F_\circ(1 \times \widehat{\Phi}) : SD(Z,X) \times I \to SD(Z,Y)$.
Then

$H(f_1, 0) = (F_\circ(1 \times \widehat{\Phi}))(f_1, 0) = F(f_1, \widehat{\Phi}(0))$ for every $f_1 \in SD(Z,X)$.
Thus for any $z \in Z$,

$$\begin{aligned}[H(f_1,0)](z) &= [F(f_1, \widehat{\Phi}(0))](z) = [\widehat{\Phi}(0)](f_1(z)) \\ &= \Phi(0, f_1(z)) = f(f_1(z)) = [f_+(f_1)](z).\end{aligned}$$

Thus for any $f_1 \in SD(Z,X)$, $H(f_1,0) = f_+(f_1)$.
Similarly $H(f_1,1) = g_+(f_1)$ for any $f_1 SD(Z,X)$.

Thus $f_+ \,\&\, g_+$ are strong δ-homotopic.

Definition 6.17. *A subspace A of X is said to have the strong δ-homotopy extension property in X w.r.t a space Y, if for every continuous mapping $f : X \to Y$ and every homotopy $h : A \times I \to Y$ such that $h(x,0) = f(x)$, $\forall x \in A$, there exists a strong δ-homotopy $g : X \times I \to Y$ such that $g(x,t) = h(x,t)$, $\forall (x,t) \in A \times I \,\&\, g(x,0) = f(x)$, $\forall x \in X$.*

Definition 6.18. *A subspace A of a space X is said to have the absolute strong δ-homotopy extension property in X if A has the strong δ-homotopy extension property w.r.t every space X.*

Theorem 6.33. *If A is a δ-closed subset of a space X, then A has the absolute strong δ-homotopy extension property in X iff $(X \times \{0\}) \cup (A \times I)$ is a strong δ-retract of $X \times I$.*

Proof : First, suppose that $(X \times \{0\}) \cup (A \times I)$ is a strong δ-retract of $X \times I$. Let Y be any space, $f : X \to Y$ be any cotinuous mapping and let $h : A \times I \to Y$ be a homotopy such that $h(x,0) = f(x)$ $\forall x \in A$. Let $g : (X \times \{0\}) \cup (A \times I) \to Y$ be the mapping defined by $g(x,t) = f(x)$ if $t = 0$ & $g(x,t) = h(x,t)$ if $t \neq 0$. Obviously g is a cotinuous mapping . Since $(X \times \{0\}) \cup (A \times I)$ is a strong δ-retract of $X \times I$ then by Theorem 4.4 g can be extended to a strongly δ-continuous mapping $g^* X \times I \to Y$. Since g^* is an extension of g so $g^*(x,t) = g(x,t)$ $\forall (x,t) \in A \times I$ & $g^*(x,0) = f(x)$ $\forall x \in X$. Hence A has the absolute strong δ-homotopy extension property in X.

Conversely, suppose A has the absolute strong δ-homotopy extension property in X. Let $Y = (X \times \{0\}) \cup (A \times I)$ and let $f : X \to X \times Y$ be a mapping defined by $f(x) = (x,0)$ $\forall x \in X$. Then f is continuous. Let $g : A \times I \to Y$ be the homotopy defined by $g(a,t) = (a,t)$ $\forall (a,t) \in A \times I$.

By hypothesis $\exists$ a strong δ-homotopy $h : X \times I \to Y$ such that $h(x,t) = g(x,t)\ \forall (x,t) \in A \times I \,\&\, h(x,0) = f(x)\ \forall x \in X$. Obviously h is a strongly δ-continuous mapping of $X \times I$ into $(X \times \{0\}) \cup (A \times I)$. Let $(x,y) \in Y$. If $(x,y) \in (X \times \{0\})$ then $y = 0$, so $h(x,y) = h(x,0) = f(x) = (x,0) = (x,y)$. Also if $(x,y) \in A \times I$ then $x \in A$ so $h(x,y) = g(x,y) = (x,y)$. Thus h is identity on Y. Hence Y is a strong δ-retract of $X \times I$.

Theorem 6.34. *If A is a δ-closed subset of X, then $(X \times \{0\}) \cup (A \times I) \cup (X \times \{1\})$ is a strong δ-retract of $X \times I$ iff for every space Y and each pair of continuous mapping $f, g : X \to Y$, any given homotopy between $f_{|A} \,\&\, g_{|A}$ can be extended to a strong δ-homotopy between $f \& g$.*

Proof : First suppose that $(X \times \{0\}) \cup (A \times I) \cup (X \times \{1\})$ is a strong δ-retract of $X \times I$. Let Y be any space and let $f : X \to Y$ & $g : X \to Y$ be continuous mapping. Let $h : A \times I \to Y$ be a homotopy between $f_{|A} \,\&\, g_{|A}$. Define a mapping $h^* : (X \times \{0\}) \cup (A \times I) \cup (X \times \{1\}) \to Y$ by

$$\begin{aligned} h^*(x,t) &= f(x) \quad ; \quad t = 0 \\ &= h(x,t) \quad ; \quad 0 < t < 1 \\ &= g(x) \quad ; \quad t = 1 \end{aligned}$$

Then h^* is continuous. Also since, $(X \times \{0\}) \cup (A \times I) \cup (X \times \{1\})$ is a strong δ-retract of $X \times I$, so h^* can be extended to a strongly δ-continuous mapping $h^{**} : X \times I \to Y$. Now h^{**} being an extension of h^*, $h^{**}(x,0) = h^*(x,0) = f(x)$ & $h^{**}(x,1) = h^*(x,1) = g(x)\ \forall x \in X$. Thus h^{**} is a strong δ-homotopy between $f_{|A} \,\&\, g_{|A}$.

Conversely, suppose that if Y is any space and $f : X \to Y$ & $g : X \to Y$ are continuous mappings, then given any homotopoy between $f_{|A} \,\&\, g_{|A}$ can be extended to a strong δ-homotopoy between $f \,\&\, g$. Let $Y = (X \times \{0\}) \cup (A \times I) \cup (X \times \{1\})$. Let $f : X \to Y$ be the mapping defined by $f(x) = (x,0)\, \forall x \in X$. Let $g : X \to Y$ be the mapping defined by $g(x) = (x,1)\, \forall x \in X$. Then $f \,\&\, g$ are continuous mappings. Let $h : A \times I \to Y$ be the homotopy between $f_{|A} \,\&\, g_{|A}$ defined by $h(a,t) = (a,t)\ \forall (a,t) \in A \times I$. Then h can be extended to a strong δ-homotopy $k : X \times I \to Y$ between $f \,\&\, g$. Let $(x,y) \in Y$. If $(x,y) \in X \times \{0\}$, then

$k(x,y) = k(x,0) = f(x) = (x,0) = (x,y)$. If $(x,y) \in X \times \{1\}$,

then $k(x,y) = k(x,1) = g(x) = (x,1) = (x,y)$. If $(x,y) \in A \times I$ then $k(x,y) = h(x,y) = (x,y)$. Thus k is identity on Y. Hence Y is a strong δ-retract of $X \times I$.

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