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The virtual shape of a spherical body moving along Ox – axis with a radial velocity V

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ABSTRACT. In this paper is determined the quadrivelocity $u(u^0, u^1, u^2, u^3)$ from the space-time metrics given for a frame with the origin in the center of gravity of radial velocity V. The body's surface equation is given by $u^0 = u^0_{surface} = ct$.

1. NOTATION

Let us consider two spherical bodies of masses m_1 and m_2 respectively, relatively far-off so that their shapes could be considered spherical.

In this case a reference frame can be chosen for both bodies, so that the mass m_1 should be in the origin O of the system of axes $Oxyzt$, and mass m_2 should be in the origin O' of the system of axes $O'x'y'z't'$ [1, 2].

We choose the axis Ox and $O'x'$ on the strainght line OO'.

Schwarzschild's metrics, associated with the body of mass m_1 situated in the origin of the reference frame $Oxyzt$ is:

$$\begin{aligned} ds_1^2 &= \left(1 - \frac{2m_1}{r}\right) dt^2 - \left(1 - \frac{2m_1}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \\ &- r^2 \sin^2\theta d\phi^2 \\ &= g_{ij} dx^i dx^j \end{aligned} \tag{1.1}$$

where

$$x^0 = t = ct_{ph}; \quad x^1 = r = r_{ph}; \quad x^2 = \theta = \theta_{ph}; \quad x^3 = \phi = \phi_{ph} \tag{1.2}$$

are the geometrized and generalized spherical coordinates. The geometrized time and mass have length dimensions (in meters).

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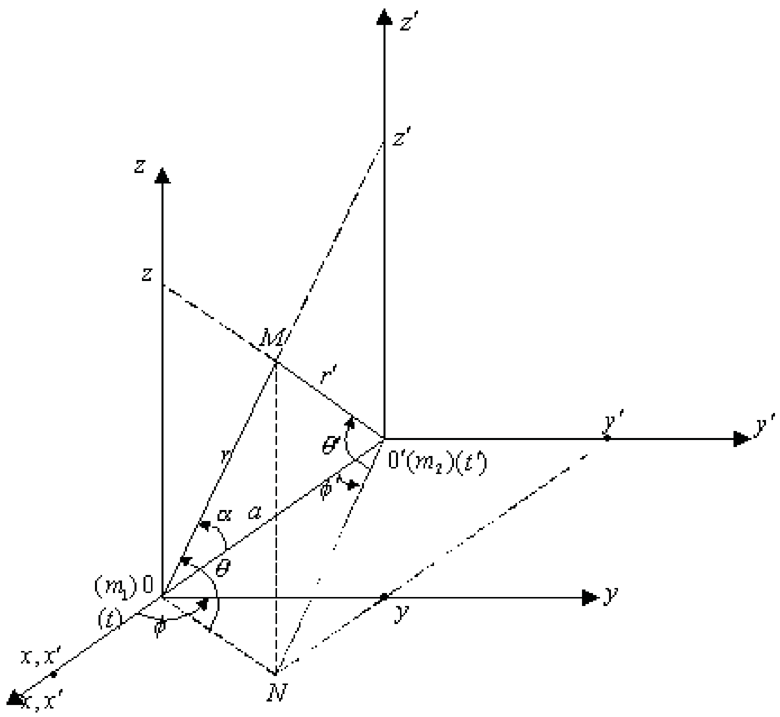

$$m = \frac{G\,M_{ph}}{c^2} \tag{1.3}$$

where index *ph* showing that the respective magnitude is measured in physical units and c and G represent the speed of the light and the universal constant of gravity, respectively.

To mass m_2 situated in the origin O' of the axes of coordinates $O'x'y'z't'$ corresponds the metrics

$$ds_2^2 = \left(1 - \frac{2m_2}{r'}\right) dt'^2 - \left(1 - \frac{2m_2}{r'}\right)^{-1} dr'^2 - r'^2 d\theta'^2 - r'^2 \sin^2\theta' d\phi'^2 \tag{1.4}$$

In both cases, we will consider the meter as a length unit. A point M from the space-time Universe has the Cartesian coordinates (x, y, z, t) measured from O and (x', y', z', y') measured from O'. We have noted the radial distances by

$$OM = r; \quad O'M = r'; \quad OO' = a \tag{1.5}$$

(see the next figure)

The relation between the Cartesian coordinates and spherical ones is given by the formulas:

$$(1.6) \qquad \begin{cases} x = r\cos\theta\cos\phi \\ y = r\cos\theta sin\phi \\ z = r\sin\theta \end{cases} \qquad \begin{cases} x' = r'\cos\theta'\cos\phi' \\ y' = r'\cos\theta' sin\phi' \\ z' = r'\sin\theta' \end{cases}$$

and

$$(1.7) \qquad t' = t + r - E$$

where

$$(1.8) \qquad E = \sqrt{a^2 + r^2 + 2ar\cos\theta\cos\phi}$$

and

$$(1.9) \qquad F = \sqrt{a^2 + r^2\cos^2\theta + 2ar\cos\theta\cos\phi}$$

2. The quadrivelocity components

Measured from O after the corresponding replacements in metrics (4), we get [2]

$$(2.1) \qquad \begin{aligned} ds_2^2 &= \frac{1}{E^2}\left(1 - \frac{2m_2}{E}\right)A^2 - \frac{1}{E^2}\left(1 - \frac{2m_2}{E}\right)^{-1}B^2 \\ &- \frac{1}{E^2F^2}C^2\frac{r^2\sin\theta}{F^4(a + r\cos\theta\cos\phi)^2}D^2 \end{aligned}$$

where

$$\begin{aligned} A = [E - (a + r\cos\theta\cos\phi)V]\,dt + [E - (r + a\cos\theta\cos\phi)]\,dr + \\ (2.2) \qquad +ar\sin\theta\cos\phi\; d\theta + ar\cos\theta\sin\phi\; d\phi \end{aligned}$$

$$\begin{aligned} B = (a + r\cos\theta\cos\phi)V dt + (r + a\cos\theta\cos\phi)dr - \\ (2.3) \qquad -ar\sin\theta\cos\phi\; d\theta - ar\cos\theta\sin\phi\; d\phi \end{aligned}$$

$$\begin{aligned} C = (a + r\cos\theta\cos\phi)Vr\sin\theta dt - a\sin\theta(a + r\cos\theta\cos\phi)dr - \\ (2.4) \quad -r\left[(a^2 + r^2)\cos\theta + ar\cos\phi(1 + \cos^2\theta)\right]d\theta - ar^2\sin\theta\cos\theta\sin\phi d\phi \end{aligned}$$

$$\begin{aligned} D = (a &+ r\cos\theta\cos\phi)Vr\cos\theta\sin\phi dt - a\cos\theta\sin\phi(a + r\cos\theta\cos\phi)dr + \\ &+ (a + r\cos\theta\cos\phi)ar\sin\theta\sin\phi d\theta \\ (2.5) \qquad &- r\cos\theta\left[ar\cos\theta(1 + \cos^2\phi) + (a^2 + r^2\cos^2\theta)\cos\phi\right]d\phi \end{aligned}$$

Theorem 1. *For a body with the mass m_2 moving compared to the body of mass m_1 with a radial speed V along of the axis Ox, measured from a body with the mass negligible m_1 situated in origin O, the quadrivelocity u have the components*

$$u^0 = \left\{ \frac{1}{E^2} \{ \left(1 + \frac{m_2}{E}\right) [E + (1+V) - V(r+a)(1+\cos\theta\cos\phi)]^2 - \right.$$

$$- \left(1 + \frac{m_2}{E}\right)^{-1} V^2 (r+a)^2 (1+\cos\theta\cos\phi)^2 \} -$$

$$\left. - \frac{V^2(r-a)^2 \sin^2\theta}{E^2 F^4} \left[(a + r\cos\theta\cos\phi)^2 F^2 + E^2 r^2 \cos^2\theta \sin^2\phi \right] \right\}^{-\frac{1}{2}}$$

$$u^1 = V u^0$$
$$u^2 = u^3 = 0$$

Proof: The quadivelocity u have the components

$$u^0 = \frac{dx^0}{ds} = \frac{dt}{ds}; \quad u^1 = \frac{dx^1}{ds} = \frac{dr}{ds}; \; u^2 = \frac{dx^2}{ds} = \frac{d\theta}{ds}; \; u^3 = \frac{dx^3}{ds} = \frac{d\phi}{ds}$$

We have

$$u^1 = \frac{dr}{ds} = \frac{dr}{dt} \cdot \frac{dt}{ds} = V \cdot u^0$$

$$u^2 = \frac{d\theta}{ds} = \frac{d\theta}{dt} \cdot \frac{dt}{ds} = 0 \cdot u^0 = 0$$

$$u^3 = \frac{d\phi}{ds} = \frac{d\phi}{dt} \cdot \frac{dt}{ds} = 0 \cdot u^0 = 0$$

Multiplying the quadivelocity (10) with $\frac{1}{ds_2^2}$ and replacing u^1 with Vu^0, and u^2,u^3 with 0, we get formless from the theorem.

When $V = 0$, we have

$$u^0 = \left(1 - \frac{2m_2}{E}\right)^{-\frac{1}{2}} \tag{2.6}$$

For a body m_1 when the mass m_2 is negligible, we have from metrics (1):

$$u(u^0, 0, 0, 0)$$

who

$$u^0 = \left(1 - \frac{2m_1}{r}\right)^{-\frac{1}{2}} \tag{2.7}$$

The components of the quadrivelocity are useful in determination of the three-dimensional acceleration components.

3. The body's surface equation

We consider the mass m_1 litle compared with m_2. The radius of body with mass m_2 is R.

In [3] it was demonstrated that the surface equation of a body with a mass m is

$$u^0 = u^0_{surface} = const. \tag{3.1}$$

At is surface of body m_2 we have

$$u^0_{surface} = \left(1 - \frac{2m_2}{R}\right)^{-\frac{1}{2}} \tag{3.2}$$

From (15) and (19) it results for the virtual surface shape of body m_2 looked from m_1, the equations:

$$\frac{1}{E^2}\left\{\left(1+\frac{m_2}{E}\right)\left[E+(1+V)-V(r+a)(1+\cos\theta\cos\phi)\right]^2 - \right.$$

$$\left. - \left(1+\frac{m_2}{E}\right)^{-1} V^2(r+a)^2)(1+\cos\theta\cos\phi)^2\right\} -$$

$$-\frac{V^2(r-a)^2\sin^2\theta}{E^2F^4}\left[(a+r\cos\theta\cos\phi)^2F^2 + E^2r^2\cos^2\theta\sin^2\phi\right]\Big\} =$$

$$1 - \frac{2m_2}{R} \tag{3.3}$$

where E and F is given by (8) and (9).

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