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Very well-covered graphs with log-concave independence polynomials

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ABSTRACT. If s_k equals the number of stable sets of cardinality k in the graph G, then $I(G;x) = \sum_{k=0}^{\alpha(G)} s_k x^k$ is the *independence polynomial* of G (Gutman and Harary, 1983). Alavi, Malde, Schwenk and Erdös (1987) conjectured that $I(G;x)$ is unimodal whenever G is a forest, while Brown, Dilcher and Nowakowski (2000) conjectured that $I(G;x)$ is unimodal for any well-covered graph G. Michael and Traves (2002) showed that the assertion is false for well-covered graphs with $\alpha(G) \geq 4$, while for very well-covered graphs the conjecture is still open.

In this paper we give support to both conjectures by demonstrating that if $\alpha(G) \leq 3$, or $G \in \{K_{1,n}, P_n : n \geq 1\}$, then $I(G^*;x)$ is log-concave, and, hence, unimodal (where G^* is the very well-covered graph obtained from G by appending a single pendant edge to each vertex).

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