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# Evaluation of the DWT filters for image compression

OVIDIU COSMA

ABSTRACT. The characteristics of the digital filter used for computing the Discrete Wavelet Transform (DWT) determine the performance of the compression scheme. This article presents an evaluation of the DWT filters, from an image compression point of view.

## 1. INTRODUCTION

The Discrete Wavelet Transform (DWT) [1], [4] is one of the most important components of a modern image compression scheme. The result of the DWT is a complete description of the analysed image. The DWT coefficients depend on the properties of the mother wavelet $\psi$ used in the transform process. If $\psi$ has more null moments, the coefficients in the high subbands will be smaller, and the image data can be efficiently coded, because it's energy is concentrated in the low frequency subbands.

Relation (1.1) gives the condition that the first $N$ moments of the mother wavelet $\psi(t)$ vanish [1], [4].

$$\int_R x^m \psi(t)dt = 0, \quad \text{for } m = 0, \ldots, N-1 \tag{1.1}$$

The vanishing moments condition is also expressed in relation (1.2), where $g(k)$ are the coefficients of the detail filter, $h(k)$ are the coefficients of the smoothing filter and $L$ is the length of the filters.

$$\begin{aligned} &\sum_{k=0}^{L-1} k^m g(k) = 0 \\ &\text{or} \qquad\qquad \text{for } m = 0, \ldots, N-1 \\ &\sum_{k=0}^{L-1} (-1)^k k^m h(k) = 0 \end{aligned} \tag{1.2}$$

If $N$ is large the wavelets are smooth and the filter has good approximating properties. Usually $N$ is large in the case of the long filters, which have the disadvantage that require more operations to compute the transform.

This article presents an evaluation of some of the best DWT filters for image compression, and the application that was built for this purpose.

## 2. Performance evaluating application

The block scheme of the application that was built for evaluating the filters is presented in figure 1. The first module computes the DWT of the test image, using the coefficients of the wavelet digital filter. Next the image subbands are coded at different bit rates with the SPIHT-HIA algorithm [2], [5]. The following blocks perform the inverse transforms, and the comparator computes the PSNR [6] for the reconstructed image.

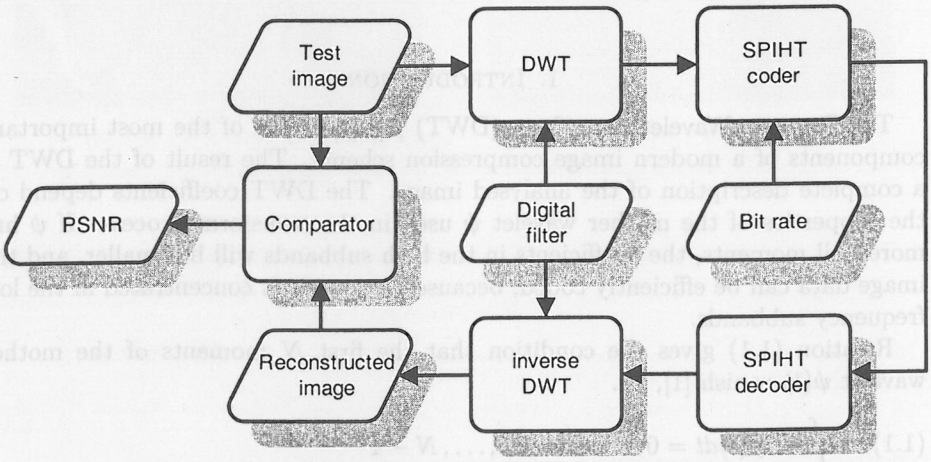



Figure 1: The performance evaluating application's block scheme.

A number of 25 filters were evaluated with this application, including the best ones for image compression. They can be found in [3], [7], [8], [9] and [10]. Both orthogonal and biorthogonal filters were tested, but the best results were obtained with the last category.

## 3. Results

The results for the best of the evaluated filters are presented in figures 2 – 5. The graphs present the increase of the PSNR against the case in which the Vilassenor 9/7 filter is used. This filter was chosen as reference, because it is one of the best for image compression.

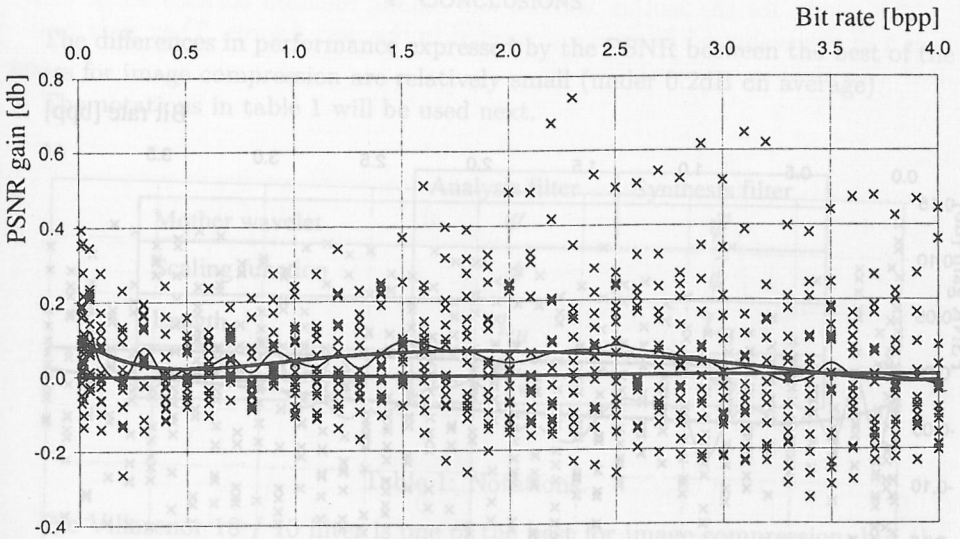
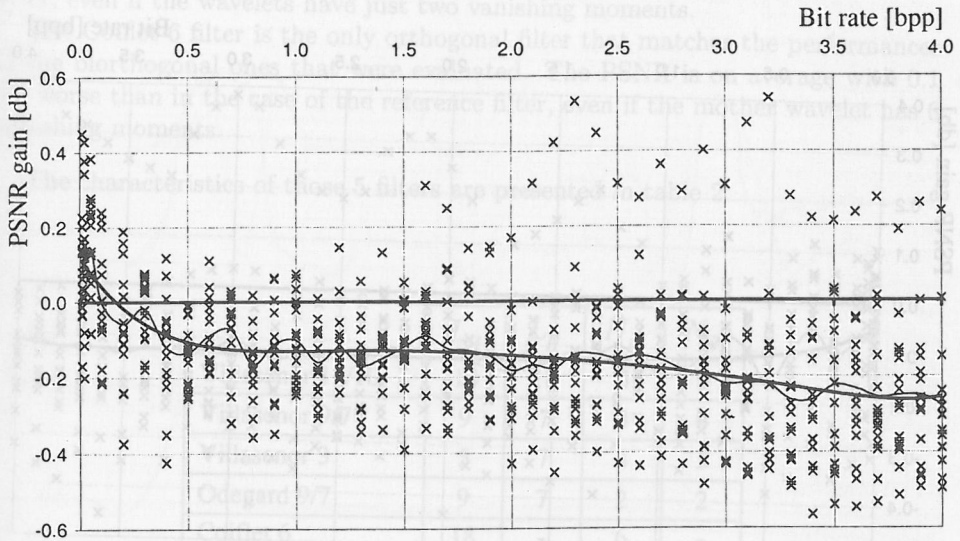



Figure 2: PSNR gain with the Villasenor 18/10 filter.



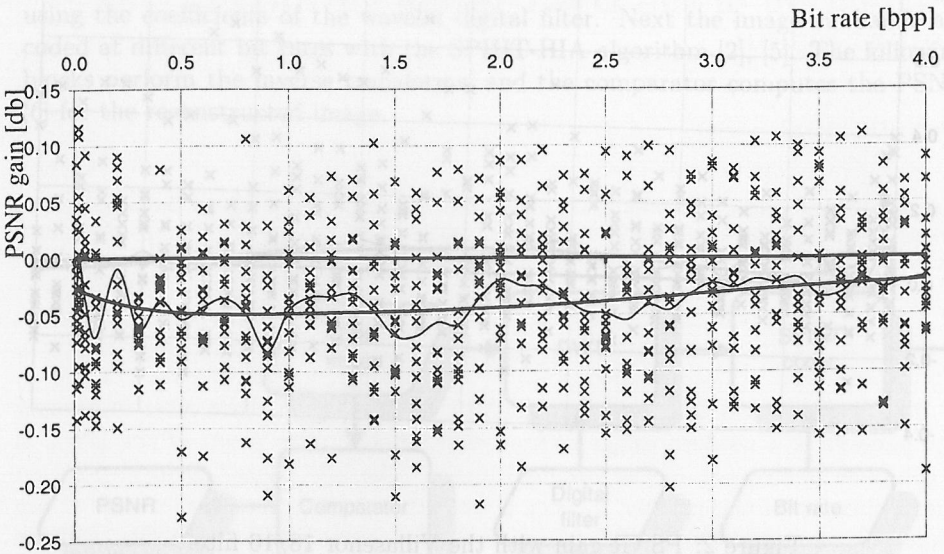
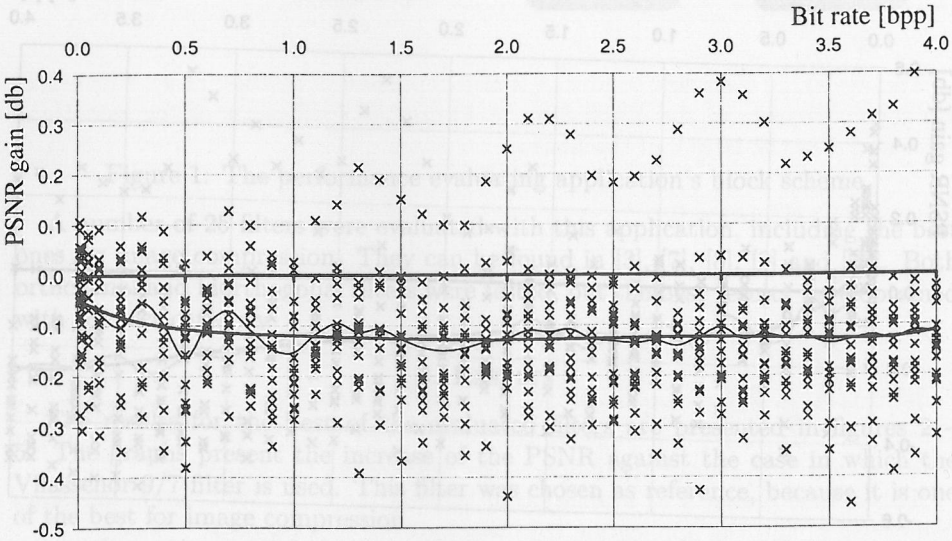



Figure 4: PSNR gain with the Odegard 9/7 filter.



## 4. Conclusions

The differences in performance expressed by the PSNR between the best of the filters for image compression are relatively small (under 0.2dB on average).

The notations in table 1 will be used next.

| | Analysis filter | Synthesis filter |
|---|---|---|
| Mother wavelet | $\psi$ | $\tilde{\psi}$ |
| Scaling function | $\phi$ | $\tilde{\phi}$ |
| Length | $L_H$ | $L_{\tilde{H}}$ |
| Vanishing moments | $N_\psi$ | $N_{\tilde{\psi}}$ |

Table 1: Notations.

The Villasenor 18 / 10 filter is one of the best for image compression, but the differences against the reference 9/7 filter are relatively small. They are situated under 0.1dB on average. The reference filter has the advantage that is shorter, and the transform can be computed more efficiently.

The Villasenor 9/7 filter was chosen as reference, because it usually gives the best results. Its performance expressed by the PSNR is surpassed only by the Villa 18/10 filter, and at high compression ratios by the Villasenor 3 filter, which gave the best results at low bitrates.

The Odegard 9/7 filter's performances are very close to the ones of the reference filter, even if the wavelets have just two vanishing moments.

The Coiflet 6 filter is the only orthogonal filter that matches the performances of the biorthogonal ones that were evaluated. The PSNR is on average with 0.1 dB worse than in the case of the reference filter, even if the mother wavelet has 6 vanishing moments.

The characteristics of those 5 filters are presented in table 2.

| | $L_H$ | $L_{\tilde{H}}$ | $N_\psi$ | $N_{\tilde{\psi}}$ |
|---|---|---|---|---|
| Villasenor 18/10 | 10 | 18 | 5 | 9 |
| Villasenor 9/7 | 9 | 7 | 4 | 4 |
| Villasenor 3 | 5 | 7 | 3 | 5 |
| Odegard 9/7 | 9 | 7 | 2 | 2 |
| Coiflet 6 | 18 | - | 6 | - |

The graphs for the mother wavelet and scaling function corresponding to the best 5 of the evaluated filters are presented in the next figures. They were generated with the cascade algorithm [4].

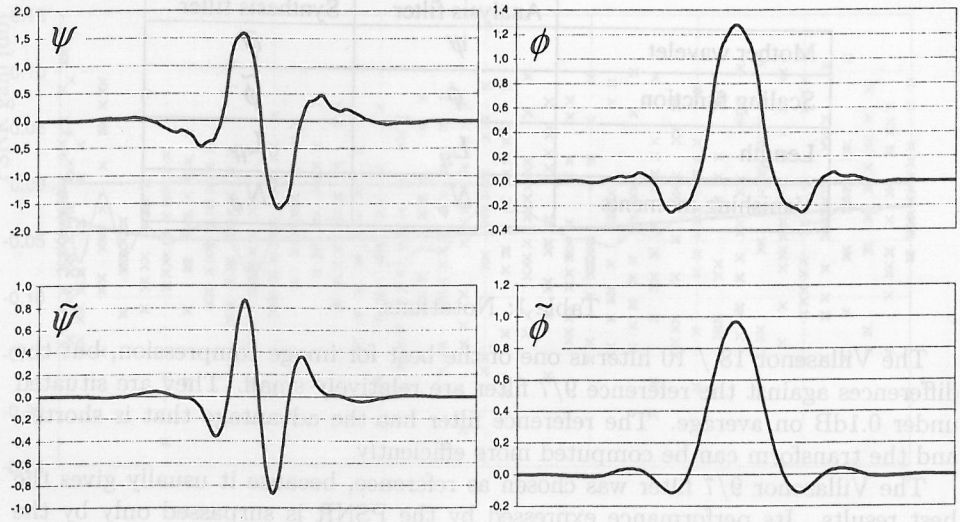
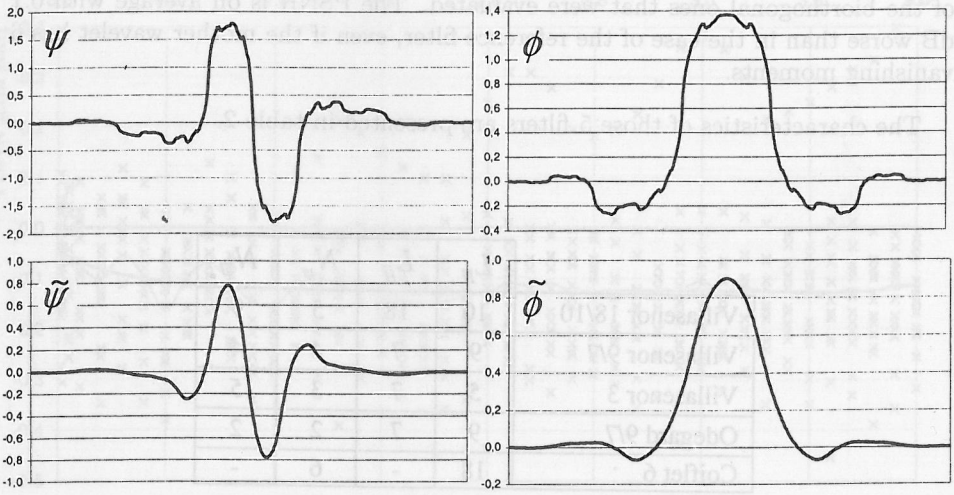



Figure 6. Graphs for the *Villasenor 18/10* filter.



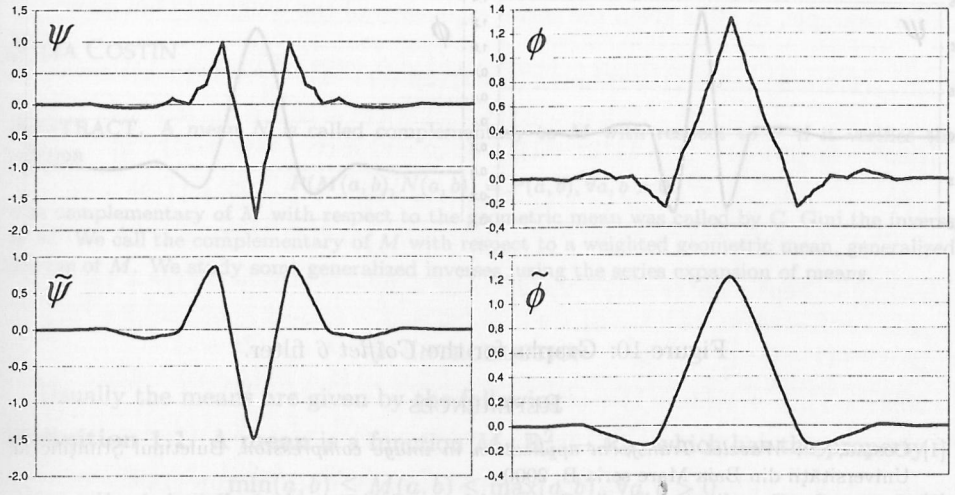
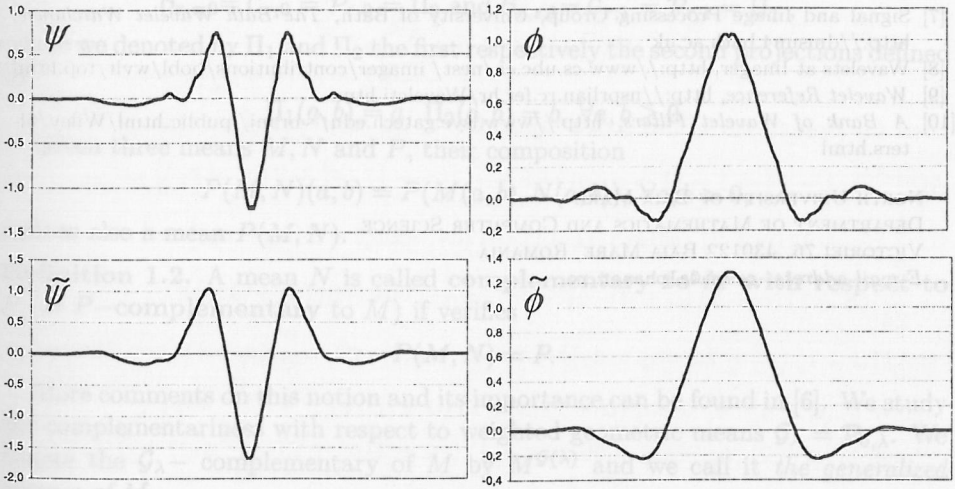



Figure 8: Graphs for the *Villasenor 9/7* filter.



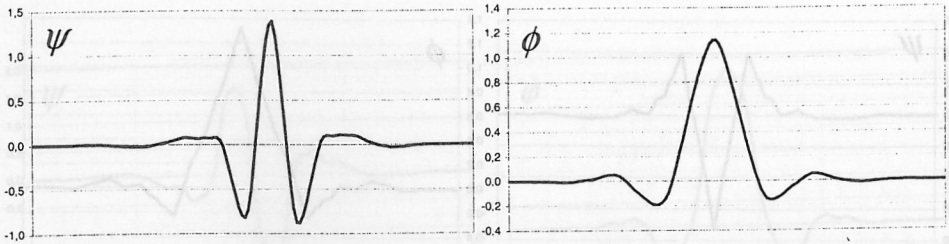



Figure 10: Graphs for the *Coiflet 6* filter.

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North University of Baia Mare
Department of Mathematics and Computer Science
Victoriei 76, 430122 Baia Mare, Romania
*E-mail address*: cosma@alphanet.ro