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Viscosity approximation methods for nonexpansive mapping in Banach spaces

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ABSTRACT. Let C be a closed convex subset of a uniformly smooth Banach space E and let $T : C \to C$ be a nonexpansive mapping such that $F(T) \neq \emptyset$. The initial guess $x_0 \in C$ is chosen arbitrarily and given sequences $\{\alpha_n\}_{n=0}^{\infty}$ in (0,1) and $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ in [0,1], the following conditions are satisfied:

(i) $\sum_{n=0}^{\infty} \alpha_n = \infty,\ \alpha_n \to 0;$

(ii) $(1+\beta_n)\gamma_n - 2\beta_n > a,$ for some $a \in [0,1);$

(iii) $\sum_{n=0}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty,\ \sum_{n=0}^{\infty} |\beta_{n+1} - \beta_n| < \infty$ and $\sum_{n=0}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty.$

Let $\{x_n\}_{n=1}^{\infty}$ be the composite process defined by

$$\begin{cases} z_n = \gamma_n x_n + (1-\gamma_n)Tx_n \\ y_n = \beta_n x_n + (1-\beta_n)Tz_n \\ x_{n+1} = \alpha_n f(x_n) + (1-\alpha_n)y_n. \end{cases}$$

Then $\{x_n\}_{n=1}^{\infty}$ converges strongly to a fixed point of T which solves some variational inequality.

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