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Dedicated to Professor Ioan A. RUS on the occasion of his 70^{th} *anniversary*

Integral equations, large forcing, strong resolvents

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ABSTRACT. We consider an integral equation $x(t) = a(t) - \int_0^t C(t,s)x(s)ds$, a resolvent $R(t,s)$, and a variation of parameters formula $x(t) = a(t) - \int_0^t R(t,s)a(s)ds$. We present three general results concerning the behavior of solutions based on the signs of $C(t,s)$ and its derivatives, in conjunction with the magnitude of $a(t)$ or of $a'(t)$. The first result shows that $(x(t) - a(t))^2$ is bounded even when $a(t)$ is unbounded. The second result, under a different set of conditions on the derivatives of C, shows that $x(t)$ is bounded when a' is bounded.

Classical theory favors the idea that $x(t)$ follows $a(t)$ when the kernel is nice. In three recent papers we disputed this, providing many results in which $\int_0^t R(t,s)a(s)ds$ faithfully duplicates $a(t)$, even when $a(t)$ is unbounded, resulting in $x(t)$ being bounded and, most interestingly, bearing absolutely no relation to $a(t)$. Most of this set of results was based on certain smallness conditions on $C(t,s)$. By taking into account sign conditions on C and its derivatives we find that both views can be defended even when $a(t)$ is unbounded.

There is now ample evidence that there are two strikingly different theories about the relation of the solution to $a(t)$. These should be very important and rewarding areas for investigation.

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