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Dedicated to Professor Ioan A. RUS on the occasion of his 70^{th} anniversary

Dynamical localization conditions for dc-trigonal electric fields proceeding beyond the nearest neighbor description

MARIA ANASTASIA JIVULESCU

ABSTRACT. Dynamic localization conditions proceeding beyond the nearest-neighbor description are derived by applying the quasi-energy description, in the case of the dc-trigonal electric field like $E_0 + E_1(t)$, for which $\omega_B = P\omega/Q$, where $\omega_B = eaE_0/\hbar$ and $\omega = 2\pi/T$ stand for, respectively, the Bloch and ac field frequencies, while P and Q are mutually prime integers. Concrete manifestations of dynamical localization have been presented for particular cases.

1. INTRODUCTION

In the last past decade the quantum-mechanical description of a charged particle, say electrons, moving on one-dimensional (1D) lattices under the influence of periodic time dependent electric fields has attracted attention $[1-2]$. It was proved there is a periodic return of the electron to the initially occupied site when the ratio of the field magnitude to its frequency is a root of the ordinary Bessel function of order zero [3]. These behaviors serve as a signature to the onset of the dynamic localization effects. Such results are able to be reproduced by resorting to the quasi-energy description, too. In this latter case, the dynamic localization conditions rely on the so called collapse points of the quasi-energy bands, as discussed before [4]. The dynamic localization properties of electrons on the 1D lattice under the influence of dc-trigonal electric field like [12]

$$E(t) = E_0 + E_1(t) = E_0 + \begin{cases} -E_1(1+\frac{4}{T}t), & -\frac{T}{2} \leq t < 0 \\ -E_1(1-\frac{4}{T}t), & 0 \leq t \leq \frac{T}{2} \end{cases} \tag{1.1}$$

are of a special interest for several applications in quantum electronics, with a special emphasis on semiconductor supper-lattices. We shall discuss further details concerning localization attributes characterizing such fields, now by proceeding beyond the nearest neighbor description. For this purpose a general energy dispersion law like

$$E_d(k) = \sum_{n=0}^{\infty} R_n \cos(nka) \tag{1.2}$$

will be used. Here k stands for the wave number, a denotes the lattice spacing characterizing the one-dimensional 1D lattice, while R_n are pertinent expansion

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coefficients. Concrete manifestations of dynamic localization conditions will then be established by resorting to the collapse points characterizing general quasi-energy formulae established before [5]. To this aim a commensurability condition such as given by:

$$\frac{\omega_0}{\omega} = \frac{P}{Q} \tag{1.3}$$

where P and Q are mutually prime integers will be accounted for. Note that $\omega_0 = eE_0a/\hbar$ stands for the Bloch-frequency [7], while $\omega = 2\pi/T$. There are reasons to say that dynamic localization conditions established before [12] have to be updated by accounting for (1.3). Similar results for dynamical localization conditions have been established by applying the quasi-energy description in the case of dc-ac electric field [13].

2. Preliminaries and notations

Considering the fact that the Hamiltonian of this system incorporates a sequence of successive next to nearest neighbors (NNN) hopping effects, the discrete time-dependent Schrödinger equation is:

$$\mathcal{H}_d(n \geq 0)\psi_m = \sum_{n=0}^{\infty} V_n(\psi_{m+n} + \psi_{m-n}) - meaF(t)\psi_m = i\frac{d}{dt}\psi_m(t) \tag{2.4}$$

where $-e < 0$ is the electric charge of the electron. This proceeds via $R_n = 2\hbar V_m$ as well as by virtue of the rule

$$k \to \frac{P_{op}}{\hbar} = -i\frac{\partial}{\partial x} \tag{2.5}$$

which also means that the momentum operator P_{op} is responsible for the related sequence of translations. Accordingly, the field free Hamiltonian implemented by (1.1) proceeds as

$$\mathcal{H}_d^{(0)}\psi(x) = E_d\left(-i\frac{d}{dx}\right)\psi(x) \tag{2.6}$$

which produces the hopping terms characterizing (2.4) in terms of the discretization $\psi_m = \psi(ma)$. It is clear that usual nearest neighbor (NN) equation gets reproduced as soon as $V_n = 0$ for $n \geq 2$. In addition, the $n = 0$-term in (2.4) can be incorporated in a pure phase factor:

$$\psi_m(t) = e^{-i2V_0t}c_m(t) \tag{2.7}$$

so that

$$\mathcal{H}_d(n \geq 1)c_m(t) = i\frac{d}{dt}c_m(t) \tag{2.8}$$

where

$$\mathcal{H}_d(n \geq 1) = \mathcal{H}_d^{(0)}(n \geq 1) - meaE(t). \tag{2.9}$$

Resorting to a orthonormalized Wannier basis, say $< m|m' >= \delta_{m,m'}$, we have to realize that the Fourier-transform (1.2) relies on the matrix element of the underlying free-field Hamiltonian as follows [5, 9]

$$< 0|H_0|m >= \frac{1}{2\pi}\int_{-\pi}^{\pi} E_d(\widetilde{k}) \exp(-i\widetilde{k}m) d\widetilde{k} = 0 \tag{2.10}$$

where by now $\widetilde{k}$ stands for ka. We have restricted ourselves to the first Brillouin zone $\widetilde{k} \in [-\pi, \pi]$ as usual.

3. Deriving quasi-energy formulae

The Hamiltonian characterizing (2.4) is periodic in time with period T. This opens the way to apply the Floquet factorization:

$$C_m(t) = exp(-iEt)u_m(t) \tag{3.11}$$

where $u_m(t + T) = u_m(t)$ such as discussed in some more detail before [5, 6]. In order to handle the commensurability condition (1.3), one resorts to an extra wave-number discretization like

$$\widetilde{k} = s + 2\pi\frac{l}{Q} \tag{3.12}$$

where $s \in [-\pi/Q, \pi/Q)$ and $l = 0, 1, 2, ..., Q - 1$. This later equation also shows that the Q- denominator is responsible for the number of quasi-energy bands. The quasi-energy is then given by [5]

$$\varepsilon_{n_1}(s) = \frac{1}{T}\sum_j < 0|H_0|Qj > \exp(iQjs)\int_0^T dt \exp(iQj\theta(t)) + \frac{\omega n_1}{Q} \tag{3.13}$$

where j and n_1 are integers. The external electric field (1.1) can be represented in Fourier series [12]

$$E(t) = E_0 + \sum_{l=0}^{\infty}\frac{8E_1}{\pi^2(2l+1)^2}\cos\frac{2(2l+1)\pi}{T}t \tag{3.14}$$

where l are integers such that

$$\theta(t) = ea\int_0^t E(t')dt' = \omega_B t + \sum_{l=0}^{\infty}\frac{4eaE_1T}{\pi^3(2l+1)^3}\sin\frac{2(2l+1)\pi}{T}t. \tag{3.15}$$

Using intermediary relationships like

$$\exp(iz\sin\omega t) = \sum_{m=-\infty}^{\infty} J_m(z)\exp(im\omega t) \tag{3.16}$$

and

$$\int_{-\pi}^{\pi} d\widetilde{k}cos(j\widetilde{k})\cos(n\widetilde{k}) = \pi\delta_{j,n} \tag{3.17}$$

we can deduce a reasonable "center" of the quasi-energy band for $s = 0$:

(3.18)
$$\varepsilon_0(0) = \sum_j < 0|H_0|Qj > \sum_{n_1,n_2,.../Pj+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Qj\beta_0)J_{n_1}(Qj\beta_1)...J_{n_l}(Qj\beta_l)...$$

This "center" of the quasi-energy band can be written down just by inserting $s = 0$ instead of $s \in [-\pi/Q, \pi/Q)$.
This amounts to consider selected sequence $Q\widetilde{k}/2\pi = 0, 1, ..., Q-1$ instead of $\widetilde{k}/2\pi \in [0, 1)$. At this stage, we have to establish, for the moment, the collapse points of the quasi-energy band in terms of parameter values for which

$$\varepsilon_0(s = 0; \omega_B/\omega, eaE_1/\omega) = 0. \tag{3.19}$$

Equivalently, the dynamical localization condition is given by

(3.20)
$$\varepsilon_0(0) = \sum_j R_{Qj} \sum_{n_1,n_2,.../Pj+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Qj\beta_0)J_{n_1}(Qj\beta_1)...J_{n_l}(Qj\beta_l)... = 0$$

where $\beta_l = \frac{4eaE_1T}{\pi^3(2l+1)^3}$ and j is a positive integer.

4. Concrete realization of the dynamic localization condition

Considering fixed value of P, Q characterizing (1.3) and assuming that $R_Q \neq 0$, but $R_{2Q} = R_{3Q} = ... = 0$, then the dynamic localization condition is given by

$$F_1 \equiv R_Q \sum_{n_1,n_2,.../P+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Q\beta_0)J_{n_1}(Q\beta_1)...J_{n_l}(Q\beta_l)... = 0 \tag{4.21}$$

The relation (4.21) generalizes the case when ω_B/ω is integer($Q = 1$) [12]. Because the expression of F_1 contains the times of large numbers of the Bessel functions, it is hard to be get collapse points of quasi-energy band. In fact, when l increases, then $Q\beta_l$ decreases rapidly, so we only need to calculate the times of the first small numbers of Bessel functions. For particular case when $P = 1$, $Q = 2$ and $x = eaE_1/\omega$ we get the collapse points of quasi-energy band from relation

$$F_1 \equiv \sum_{n_1,n_2,.../1+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(\frac{16x}{\pi^2 1^3})J_{n_1}(\frac{16x}{\pi^2 3^3})...J_{n_l}(\frac{16x}{\pi^2(2l+1)^3})... = 0 \tag{4.22}$$

The graphical representation of F_1 presents the collapse points for this particular case, as follows from figure 1.

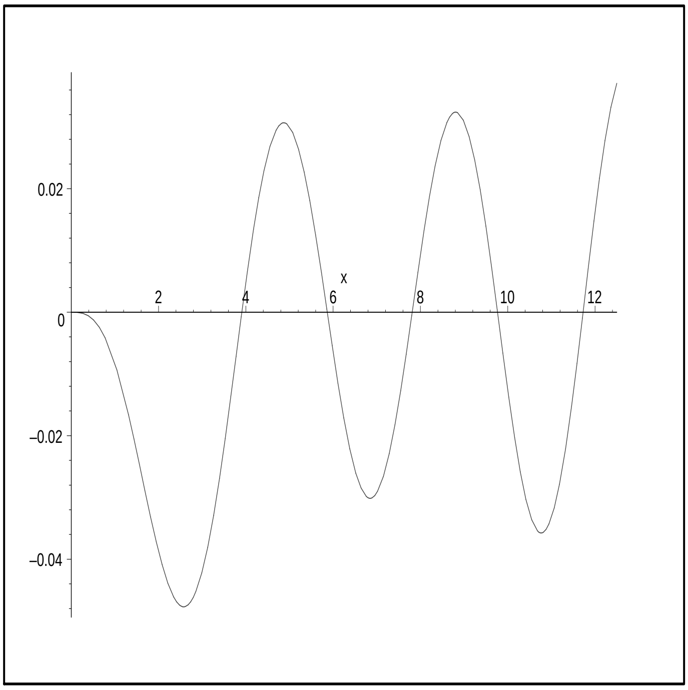

Fig.1 The eaE_1/ω-dependence of F_1

Proceeding one step further we consider that case when $R_Q \neq 0$, $R_{2Q} \neq 0$ but $R_{3Q} = R_{4Q} = .. = 0$. This time (3.20) yields the dynamic localization conditions like:

$$(4.23)\quad F_2 \equiv R_Q \sum_{n_1,n_2,.../P+\sum\limits_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Q\beta_0)J_{n_1}(Q\beta_1)...J_{n_l}(Q\beta_l)...$$

$$+ R_{2Q} \sum_{n_1,n_2,.../2P+\sum\limits_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(2Q\beta_0)J_{n_1}(2Q\beta_1)...J_{n_l}(2Q\beta_l)... = 0.$$

The case when $P = 1$, $Q = 2$, $R_Q = 1$ and $R_{2Q} = 2$ yields the dynamic localization conditions like:

(4.24)

$$F_2 \equiv \sum_{n_1,n_2,.../1+\sum\limits_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}\left(\frac{16x}{\pi^2 1^3}\right)J_{n_1}\left(\frac{16x}{\pi^2 3^3}\right)...J_{n_l}\left(\frac{16x}{\pi^2(2l+1)^3}\right)...$$

$$+2 \sum_{n_1,n_2,.../2+\sum\limits_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}\left(\frac{32x}{\pi^2 1^3}\right)J_{n_1}\left(\frac{32x}{\pi^2 3^3}\right)...J_{n_l}\left(\frac{32x}{\pi^2(2l+1)^3}\right) + ... = 0$$

and the collapse points are presented in figure 2.

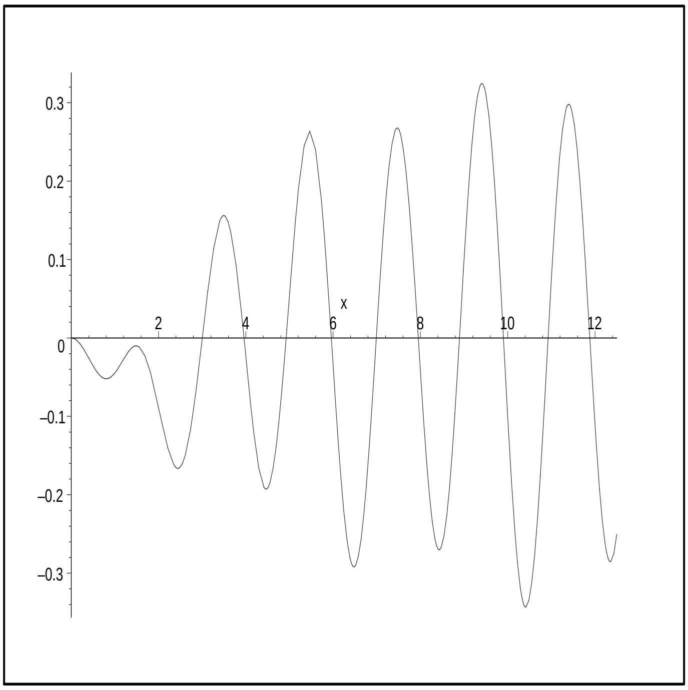

Fig.2 The eaE_1/ω-dependence of F_2

5. CONCLUSIONS

In this article we succeeded to find the dynamic localization conditions for the motion of an electron in the 1 D lattice considering long rang intersite interactions in the presence of dc-trigonal electric fields like (1.1) for which the commensurability condition (1.3) is fulfilled. This proceeds in terms of the collapse points characterizing the center of the quasi-energy band (3.19), which amounts to consider that $s = 0$. Relation (4.21) can be viewed as a reasonable generalization of result presented in [12]. The dynamic localization conditions obtained in this manner are useful in the description of higher harmonics generation [10], but related resonance phenomena characterizing several areas of physics can also be invoked [11]. Moreover, the present results are also able to provide a better understanding of transport and optical properties. The generalization of dynamic localization conditions characterizing dc-ac electric fields have been given recently [13]. Also, of further interest is a generalization of dc-bichromatic electric fields discussed latter [9].

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"POLITEHNICA" UNIVERSITY OF TIMIŞOARA
DEPARTMENT OF MATHEMATICS
P-ŢA VICTORIEI 2 ,300004, TIMIŞOARA, ROMANIA
E-mail address: mariajivulescu@yahoo.com