Dynamical localization conditions for dc-trigonal electric fields proceeding beyond the nearest neighbor description

Maria Anastasia Jivulescu

Abstract. Dynamic localization conditions proceeding beyond the nearest-neighbor description are derived by applying the quasi-energy description, in the case of the dc-trigonal electric field like \( E_0 + E_1(t) \), for which \( \omega_B = P \omega/Q \), where \( \omega_B = eaE_0/\hbar \) and \( \omega = 2\pi/T \) stand for, respectively, the Bloch and ac field frequencies, while \( P \) and \( Q \) are mutually prime integers. Concrete manifestations of dynamical localization have been presented for particular cases.

1. Introduction

In the last past decade the quantum-mechanical description of a charged particle, say electrons, moving on one-dimensional (1D) lattices under the influence of periodic time dependent electric fields has attracted attention [1 - 2]. It was proved there is a periodic return of the electron to the initially occupied site when the ratio of the field magnitude to its frequency is a root of the ordinary Bessel function of order zero [3]. These behaviors serve as a signature to the onset of the dynamic localization effects. Such results are able to be reproduced by resorting to the quasi-energy description, too. In this latter case, the dynamic localization conditions rely on the so called collapse points of the quasi-energy bands, as discussed before [4]. The dynamic localization properties of electrons on the 1D lattice under the influence of dc-trigonal electric field like [12]

\[
E(t) = E_0 + E_1(t) = E_0 + \begin{cases} 
-E_1(1 + \frac{t}{T}), & -\frac{T}{2} \leq t < 0 \\
-E_1(1 - \frac{t}{T}), & 0 \leq t \leq \frac{T}{2}
\end{cases}
\]

are of a special interest for several applications in quantum electronics, with a special emphasis on semiconductor supper-lattices. We shall discuss further details concerning localization attributes characterizing such fields, now by proceeding beyond the nearest neighbor description. For this purpose a general energy dispersion law like

\[
E_d(k) = \sum_{n=0}^{\infty} R_n \cos(nka)
\]

will be used. Here \( k \) stands for the wave number, \( a \) denotes the lattice spacing characterizing the one-dimensional 1D lattice, while \( R_n \) are pertinent expansion

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coefficients. Concrete manifestations of dynamic localization conditions will then be established by resorting to the collapse points characterizing general quasi-energy formulae established before [5]. To this aim a commensurability condition such as given by:

\begin{equation}
\frac{\omega_0}{\omega} = \frac{P}{Q}
\end{equation}

where \( P \) and \( Q \) are mutually prime integers will be accounted for. Note that \( \omega_0 = eE_0a/\hbar \) stands for the Bloch-frequency [7], while \( \omega = 2\pi/T \). There are reasons to say that dynamic localization conditions established before [12] have to be updated by accounting for (1.3). Similar results for dynamical localization conditions have been established by applying the quasi-energy description in the case of dc-ac electric field [13].

2. Preliminaries and notations

Considering the fact that the Hamiltonian of this system incorporates a sequence of successive next to nearest neighbors (NNN) hopping effects, the discrete time-dependent Schrödinger equation is:

\begin{equation}
\mathcal{H}_d(n \geq 0)\psi_m = \sum_{n=0}^{\infty} V_n (\psi_{m+n} + \psi_{m-n}) - meaF(t)\psi_m = i\frac{d}{dt}\psi_m(t)
\end{equation}

where \(-e < 0\) is the electric charge of the electron. This proceeds via \( R_n = 2\hbar V_m \) as well as by virtue of the rule

\begin{equation}
k \rightarrow \frac{P_{op}}{\hbar} = -i \frac{\partial}{\partial x}
\end{equation}

which also means that the momentum operator \( P_{op} \) is responsible for the related sequence of translations. Accordingly, the field free Hamiltonian implemented by (1.1) proceeds as

\begin{equation}
\mathcal{H}_d^{(0)}(x) = E_d \left( -i \frac{d}{dx} \right) \psi(x)
\end{equation}

which produces the hopping terms characterizing (2.4) in terms of the discretization \( \psi_m = \psi(ma) \). It is clear that usual nearest neighbor (NN) equation gets reproduced as soon as \( V_n = 0 \) for \( n \geq 2 \). In addition, the \( n = 0 \)-term in (2.4) can be incorporated in a pure phase factor:

\begin{equation}
\psi_m(t) = e^{-i2V_0t} c_m(t)
\end{equation}

so that

\begin{equation}
\mathcal{H}_d(n \geq 1)c_m(t) = i\frac{d}{dt}c_m(t)
\end{equation}

where

\begin{equation}
\mathcal{H}_d(n \geq 1) = \mathcal{H}_d^{(0)}(n \geq 1) - meaE(t).
\end{equation}
Resorting to an orthonormalized Wannier basis, say \( < m | m' > = \delta_{m,m'} \), we have to realize that the Fourier-transform (1.2) relies on the matrix element of the underlying free-field Hamiltonian as follows [5, 9]

\[
< 0 | H_0 | m > = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_\omega(\tilde{k}) \exp(-i\tilde{k}m)d\tilde{k} = 0
\]

where by now \( \tilde{k} \) stands for \( ka \). We have restricted ourselves to the first Brillouin zone \( \tilde{k} \in [-\pi, \pi] \) as usual.

### 3. Deriving Quasi-Energy Formulae

The Hamiltonian characterizing (2.4) is periodic in time with period \( T \). This opens the way to apply the Floquet factorization:

\[
C_m(t) = \exp(-iEt)u_m(t)
\]

where \( u_m(t + T) = u_m(t) \) such as discussed in some more detail before [5, 6].

In order to handle the commensurability condition (1.3), one resorts to an extra wave-number discretization like

\[
\tilde{k} = s + 2\pi \frac{l}{Q}
\]

where \( s \in [-\pi/Q, \pi/Q) \) and \( l = 0, 1, 2, ..., Q - 1 \). This later equation also shows that the \( Q \)-denominator is responsible for the number of quasi-energy bands. The quasi-energy is then given by [5]

\[
\varepsilon_{n_1}(s) = \frac{1}{T} \sum_j < 0 | H_0 | Qj > \exp(iQj \theta(t)) + \frac{\omega n_1}{Q}
\]

where \( j \) and \( n_1 \) are integers. The external electric field (1.1) can be represented in Fourier series [12]

\[
E(t) = E_0 + \sum_{l=0}^{\infty} \frac{8E_1}{\pi^2(2l + 1)^2} \cos \frac{2(2l + 1)\pi}{T} t
\]

where \( l \) are integers such that

\[
\theta(t) = ea \int_0^t E(t')dt' = \omega_B t + \sum_{l=0}^{\infty} \frac{4eaE_1 T}{\pi^3(2l + 1)^3} \sin \frac{2(2l + 1)\pi}{T} t.
\]

Using intermediary relationships like

\[
\exp(i\pi \sin \omega t) = \sum_{m=-\infty}^{\infty} J_m(z) \exp(i\pi t)
\]

and

\[
\int_{-\pi}^{\pi} d\tilde{k} \cos(j \tilde{k}) \cos(n \tilde{k}) = \pi \delta_{j,n}
\]
we can deduce a reasonable “center” of the quasi-energy band for \( s = 0 \):

\[
\varepsilon_0(0) = \sum_{j} <0|H_0|Qj> \sum_{n_1,n_2,.../Pj+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Qj\beta_0)J_{n_1}(Qj\beta_1)...J_{n_l}(Qj\beta_l)...
\]

This ”center” of the quasi-energy band can be written down just by inserting \( s = 0 \) instead of \( s \in [-\pi/Q, \pi/Q) \).

This amounts to consider selected sequence \( \tilde{k}/2\pi = 0, 1, ..., Q - 1 \) instead of \( \tilde{k}/2\pi \in [0, 1) \). At this stage, we have to establish, for the moment, the collapse points of the quasi-energy band in terms of parameter values for which

\[
(3.19) \quad \varepsilon_0(s = 0; \omega_B/\omega, eaE_1/\omega) = 0.
\]

Equivalently, the dynamical localization condition is given by

\[
(3.20) \quad \varepsilon_0(0) = \sum_{j} R_{Qj} \sum_{n_1,n_2,.../Pj+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Qj\beta_0)J_{n_1}(Qj\beta_1)...J_{n_l}(Qj\beta_l)... = 0
\]

where \( \beta_l = \frac{4eaE_1T}{\pi^3(2l+1)^3} \) and \( j \) is a positive integer.

4. CONCRETE REALIZATION OF THE DYNAMIC LOCALIZATION CONDITION

Considering fixed value of \( P, Q \) characterizing (1.3) and assuming that \( R_{Q} \neq 0 \), but \( R_{2Q} = R_{3Q} = ... = 0 \), then the dynamic localization condition is given by

\[
(4.21) \quad F_1 \equiv R_{Q} \sum_{n_1,n_2,.../P+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Q\beta_0)J_{n_1}(Q\beta_1)...J_{n_l}(Q\beta_l)... = 0
\]

The relation (4.21) generalizes the case when \( \omega_B/\omega \) is integer \( Q = 1 \) [12]. Because the expression of \( F_1 \) contains the times of large numbers of the Bessel functions, it is hard to be get collapse points of quasi-energy band. In fact, when \( l \) increases, then \( Q\beta_l \) decreases rapidly, so we only need to calculate the times of the first small numbers of Bessel functions. For particular case when \( P = 1, Q = 2 \) and \( x = eaE_1/\omega \) we get the collapse points of quasi-energy band from relation

\[
(4.22) \quad F_1 \equiv \sum_{n_1,n_2,.../1+\sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(\frac{16x}{\pi^22^3})J_{n_1}(\frac{16x}{\pi^22^3})...J_{n_l}(\frac{16x}{\pi^2(2l+1)^3})... = 0
\]

The graphical representation of \( F_1 \) presents the collapse points for this particular case, as follows from figure 1.
Proceeding one step further we consider that case when $R_Q \neq 0$, $R_{2Q} \neq 0$ but $R_{3Q} = R_{4Q} = \ldots = 0$. This time (3.20) yields the dynamic localization conditions like:

\[
F_2 \equiv R_Q \sum_{n_1, n_2, \ldots / P + \sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(Q/\beta_0)J_{n_1}(Q/\beta_1)\ldots J_{n_l}(Q/\beta_l)\ldots + R_{2Q} \sum_{n_1, n_2, \ldots / 2P + \sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}(2Q/\beta_0)J_{n_1}(2Q/\beta_1)\ldots J_{n_l}(2Q/\beta_l)\ldots = 0.\]

The case when $P = 1, Q = 2, R_Q = 1$ and $R_{2Q} = 2$ yields the dynamic localization conditions like:

\[
F_2 \equiv \sum_{n_1, n_2, \ldots / P + \sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}\left(\frac{16x}{\pi^2 1^3}\right) J_{n_1}\left(\frac{16x}{\pi^2 2^3}\right)\ldots J_{n_l}\left(\frac{16x}{\pi^2 (2l+1)^3}\right)\ldots + 2 \sum_{n_1, n_2, \ldots / 2P + \sum_{l=0}^{\infty} n_l(2l+1)=0} J_{n_0}\left(\frac{32x}{\pi^2 1^5}\right) J_{n_1}\left(\frac{32x}{\pi^2 2^5}\right)\ldots J_{n_l}\left(\frac{32x}{\pi^2 (2l+1)^5}\right) + \ldots = 0
\]

and the collapse points are presented in figure 2.
In this article we succeeded to find the dynamic localization conditions for the motion of an electron in the 1D lattice considering long range intersite interactions in the presence of dc-trigonal electric fields like (1.1) for which the commensurability condition (1.3) is fulfilled. This proceeds in terms of the collapse points characterizing the center of the quasi-energy band (3.19), which amounts to consider that $s = 0$. Relation (4.21) can be viewed as a reasonable generalization of result presented in [12]. The dynamic localization conditions obtained in this manner are useful in the description of higher harmonics generation [10], but related resonance phenomena characterizing several areas of physics can also be invoked [11]. Moreover, the present results are also able to provide a better understanding of transport and optical properties. The generalization of dynamic localization conditions characterizing dc-ac electric fields have been given recently [13]. Also, of further interest is a generalization of dc-bichromatic electric fields discussed latter [9].

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5. CONCLUSIONS
REFERENCES


"POLITEHNICA" UNIVERSITY OF TIMISOARA
DEPARTMENT OF MATHEMATICS
P-ŢA VICTORIEI 2,300004, TIMISOARA, ROMANIA
E-mail address: mariajivulescu@yahoo.com