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Dedicated to Professor Ioan A. RUS on the occasion of his 70^{th} anniversary

A family of graphs whose independence polynomials are both palindromic and unimodal

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ABSTRACT. A *stable* (or *independent*) set in a graph is a set of pairwise non-adjacent vertices. The *stability number* $\alpha(G)$ is the size of a maximum stable set in the graph G. The *independence polynomial* of G is defined by

$$I(G;x) = s_0 + s_1x + s_2x^2 + ... + s_\alpha x^\alpha,\ \alpha = \alpha(G),$$

where s_k equals the number of stable sets of cardinality k in G (I. Gutman and F. Harary, 1983).

In this paper, we build a family of graphs whose independence polynomials are palindromic and unimodal. We conjecture that all these polynomials are also log-concave.

1. INTRODUCTION

Throughout this paper $G = (V, E)$ is a simple (i.e., a finite, undirected, loopless and without multiple edges) graph with vertex set $V = V(G)$ and edge set $E = E(G)$. If $A \subset V$, then $G[A]$ is the subgraph of G induced by A. By $G - W$ we mean the subgraph $G[V - W]$, if $W \subset V(G)$. We also denote by $G - F$ the partial subgraph of G obtained by deleting the edges of F, for $F \subset E(G)$. We write shortly $G - a$, whenever $\{a\} \subseteq V(G) \cup E(G)$. The *neighborhood* of a vertex $v \in V$ is the set $N_G(v) = \{w : w \in V \ \textit{and}\ vw \in E\}$, and $N_G[v] = N_G(v) \cup \{v\}$; if there is no ambiguity on G, we use $N(v)$ and $N[v]$, respectively. K_n, P_n, C_n denote respectively, the complete graph on $n \geq 1$ vertices, the chordless path on $n \geq 1$ vertices, and the chordless cycle on $n \geq 3$ vertices.

The *disjoint union* of the graphs G_1, G_2 is the graph $G = G_1 \cup G_2$ having as vertex set the disjoint union of $V(G_1), V(G_2)$, and as edge set the disjoint union of $E(G_1), E(G_2)$. In particular, nG denotes the disjoint union of $n > 1$ copies of the graph G.

If G_1, G_2 are disjoint graphs, $A_1 \subseteq V(G_1), A_2 \subseteq V(G_2)$, then the *Zykov sum* of G_1, G_2 with respect to A_1, A_2, is the graph $(G_1, A_1) + (G_2, A_2)$ with $V(G_1) \cup V(G_2)$ as vertex set and $E(G_1) \cup E(G_2) \cup \{v_1v_2 : v_1 \in A_1, v_2 \in A_2\}$ as edge set. If $A_1 \subset V(G_1)$ and $A_2 = V(G_2)$, we simply write $(G_1, A_1) + G_2$, while if both $A_1 = V(G_1)$ and $A_2 = V(G_2)$, we use $G_1 + G_2$.

The *corona* of the graphs G and H with respect to $A \subseteq V(G)$ is the graph $(G, A) \circ H$ obtained from G and $|A|$ copies of H, such that each vertex of A is joined to all vertices of a copy of H. If $A = V(G)$ we denote by $G \circ H$ instead of $(G, V(G)) \circ H$ (see Figure 1 for an example).

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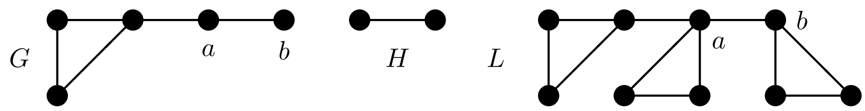

FIGURE 1. G, H and $L = (G, A) \circ H$, where $A = \{a, b\}$.

A *stable* set in G is a set of pairwise non-adjacent vertices. A stable set of maximum size will be referred to as a *maximum stable set* of G, and the *stability number* of G, denoted by $\alpha(G)$, is the cardinality of a maximum stable set in G.

Let s_k be the number of stable sets of cardinality k in a graph G. The polynomial

$$I(G; x) = s_0 + s_1 x + s_2 x^2 + ... + s_\alpha x^\alpha,\ \alpha = \alpha(G),$$

is called the *independence polynomial* of G, (Gutman and Harary, [5]). For a survey on independence polynomials the reader is referred to [11].

It is easy to deduce that

$$I(G_1 \cup G_2; x) = I(G_1; x) \cdot I(G_2; x), I(G_1 + G_2; x) = I(G_1; x) + I(G_2; x) - 1.$$

The following equality, proved firstly in [5], is very useful in calculating of the independence polynomial for various families of graphs.

Proposition 1.1. *If $v \in V(G)$, then $I(G; x) = I(G - v; x) + x \cdot I(G - N[v]; x)$.*

A finite sequence of real numbers $(a_0, a_1, a_2, ..., a_n)$ is said to be:

- *unimodal* if there is some $k \in \{0, 1, ..., n\}$, called the *mode* of the sequence, such that $a_0 \leq ... \leq a_{k-1} \leq a_k \geq a_{k+1} \geq ... \geq a_n$;
- *logarithmically concave* (shortly, *log-concave*) if $a_i^2 \geq a_{i-1} \cdot a_{i+1}$ is valid for every $i \in \{1, 2, ..., n-1\}$.

Unimodal and log-concave sequences occur in many areas of mathematics, such as algebra, combinatorics, and geometry (see, for example, the survey [3]).

It is known that any log-concave sequence of positive numbers is also unimodal. As a well-known example, we recall that the sequence of binomial coefficients is log-concave.

A polynomial is called *unimodal (log-concave)* if the sequence of its coefficients is unimodal (log-concave, respectively).

For instance, the independence polynomial

- $I(K_{42} + 3K_7; x) = 1 + 63x + 147x^2 + 343x^3$ is log-concave;
- $I(K_{43} + 3K_7; x) = 1 + 64x + 147x^2 + 343x^3$ is unimodal, but non-log-concave, because $147^2 - 64 \cdot 343 = -343 < 0$;
- $I(K_{127} + 3K_7; x) = 1 + 148x + 147x^2 + 343x^3$ is non-unimodal.

For other examples, see [1], [11] and [12]. Moreover, Alavi, Malde, Schwenk and Erdös proved the following theorem.

Theorem 1.1. [1] *For every permutation π of $\{1, 2, ..., \alpha\}$ there exists a graph G with $\alpha(G) = \alpha$ such that $s_{\pi(1)} < s_{\pi(2)} < ... < s_{\pi(\alpha)}$.*

Nevertheless, for trees, it is conjectured in [1] that the independence polynomial of a tree is unimodal.

A graph is called *claw-free* if it has no induced subgraph isomorphic to $K_{1,3}$. Hamidoune [8] proved that the independence polynomial of a claw-free graph is log-concave. As a simple application of this statement, one can easily see that $I(P_n;x)$ and $I(C_n,x)$, i.e., the independence polynomials of chordless paths and chordless cycles, are log-concave.

A polynomial $P(x)=\sum\limits_{i=o}^{n} c_i x^i$ is called *palindromic* if $c_i=c_{n-i}, i=0,1,...,\lfloor n/2 \rfloor$. The palindromicity of matching polynomial and characteristic polynomial of a graph were examined in [10], while for independence polynomial we quote [6], [7] and [13].

It is worth noticing that if $\alpha(G)\leq 3$ and $I(G;x)$ is palindromic, then it is also log-concave. However, there exist graphs with stability number ≥ 4, whose independence polynomials are palindromic and non-unimodal; for example, $I(K_{52}+3K_4+4K_1;x)=1+68x+54x^2+68x^3+x^4$.

In this paper we define a family of graphs whose independence polynomials are both palindromic and unimodal. Characterizing graphs whose independence polynomials are palindromic is an open problem [13].

2. Graphs with palindromic and unimodal independence polynomials

Taking into account that $s_0=1$ and $s_1=|V(G)|=n$, it follows that the palindromicity of $I(G;x)$ implies that $s_0=s_\alpha=1$ and $s_1=s_{\alpha-1}=n$, i.e., G has only one maximum stable set, say S, and $n-\alpha(G)$ stable sets, of size $\alpha(G)-1$, that are not subsets of S.

In [13] three ways to build graphs having palindromic independence polynomials are presented. For our purpose, we recall the rule using the so-called "*clique cover of a graph*". A *clique cover* of a graph G is a spanning graph of G, each component of which is a clique. Now, if Ω is a clique cover of G, construct a new graph H from G, which we denote by $H=\Omega\{G\}$, as follows: for each clique $Q\in\Omega$, add two new non-adjacent vertices and join them to all the vertices of Q. Figure 2 contains an example: $\Omega=\{\{a,b,c\},\{d,e\},\{f\}\}$ is a clique cover of G that has a clique consisting of one vertex.

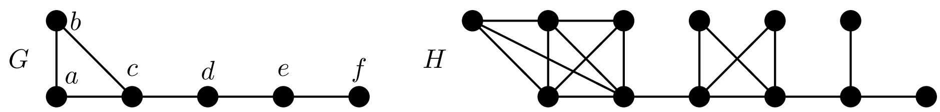

FIGURE 2. G and $H=\Omega\{G\}$.

The independence polynomials of G and $H=\Omega\{G\}$, from Figure 2, are

$$\begin{aligned} I(G;x) &= 1+6x+9x^2+2x^3, \\ I(H;x) &= 1+12x+48x^2+76x^3+48x^4+12x^5+x^6, \end{aligned}$$

but only $I(H;x)$ is palindromic.

Theorem 2.2. [13] *If Ω is a clique cover of G and $H=\Omega\{G\}$, then H has a palindromic independence polynomial.*

Let us remark that the non-isomorphic graphs G_1 and G_2, depicted in Figure 3, are obtained by the same construction rule presented above, only using different clique covers of P_5, namely $\Omega_1 = \{\{a\}, \{b, c\}, \{d, e\}\}$ and $\Omega_2 = \{\{a, b\}, \{c\}, \{d, e\}\}$.

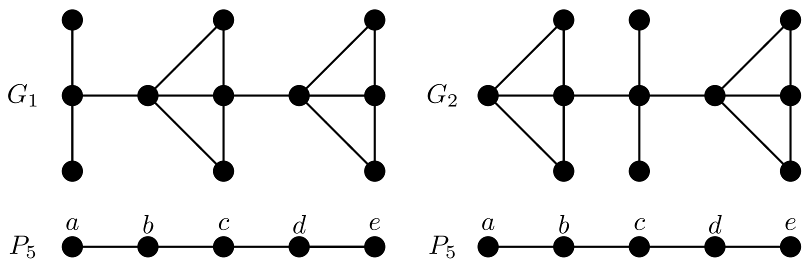

FIGURE 3. P_5 and $G_1 = \Omega_1\{P_5\}$, $G_2 = \Omega_2\{P_5\}$.

Let us notice that

$$I(P_5; x) = 1 + 5x + 6x^2 + x^3,$$

is unimodal, but not palindromic, while the independence polynomials of the two graphs coincide, namely,

$$I(G_1; x) = I(G_2; x) = 1 + 11x + 41x^2 + 63x^3 + 41x^4 + 11x^5 + x^6,$$

and, evidently, they are both unimodal and palindromic.

Lemma 2.1. *If ab is an edge of G, then for every graph H*

$$I((G, \{a, b\}) \circ H; x) = I\,(H; x) \cdot I((G, \{a, b\}) + H; x).$$

Proof. Let $G_1 = (G, \{a, b\}) \circ H$ and $G_2 = ((G, \{a, b\}) + H) \cup H$. According to Proposition 1.1, we obtain:

$$\begin{aligned} I((G_1; x) &= I(G_1 - a; x) + x \cdot I((G_1 - N[a]; x) \\ &= I((G - a), \{b\} \circ H; x) \cdot I(H; x) + x \cdot I(G - N[a], x) \cdot I(H; x) \end{aligned}$$

and also

$$\begin{aligned} I((G_2; x) &= I(G_2 - a; x) + x \cdot I((G_2 - N[a]; x) \\ &= I((G - a), \{b\} \circ H; x) \cdot I(H; x) + x \cdot I(G - N[a], x) \cdot I(H; x). \end{aligned}$$

Consequently, one may infer that $I(G_1; x) = I(G_2; x)$. □

Proposition 2.2. *If the clique cover Ω_m of P_n contains m vertices as cliques, then*

$$I(P_n \circ 2K_1; x) = (1 + x)^{n-m} \cdot I(\Omega_m\{P_n\}; x).$$

Proof. For $G = P_n$ and $\{a, b\} \in \Omega_m$, Lemma 2.1 assures that

$$\begin{aligned} I((P_n, \{a, b\}) \circ 2K_1; x) &= I\,(2K_1; x) \cdot I((P_n, \{a, b\}) + 2K_1; x) \\ &= (1 + x)^2 \cdot I((P_n, \{a, b\}) + 2K_1; x). \end{aligned}$$

In other words, each clique of Ω_m gives rise to $(1 + x)^2$. Since Ω_m has $(n-m)/2$ cliques of size two, the result follows. □

As a simple consequence, we obtain the following corollary.

Corollary 2.1. *If the clique covers Ω_m and Ω_k of P_n contain m and k vertices, respectively, and $m \geq k$, then*

$$I(\Omega_m\{P_n\};x) = (1+x)^{m-k} \cdot I(\Omega_k\{P_n\};x).$$

Let $H_n, n \geq 1$, be the graphs obtained according to the above construction from P_n, as one can see in Figure 4. By H_0 we mean the empty graph, i.e., $H_0 = (\emptyset, \emptyset)$.

Corollary 2.2. *If $J_n(x) = I(H_n;x), n \geq 1$, then for every clique cover Ω_m of P_n containing m vertices as cliques, it follows that:*

$$\begin{aligned} I(\Omega_m\{P_n\};x) &= (1+x)^m \cdot J_n(x), \text{for } n \text{ even},\\ I(\Omega_m\{P_n\};x) &= (1+x)^{m-1} \cdot J_n(x), \text{for } n \text{ odd}.\end{aligned}$$

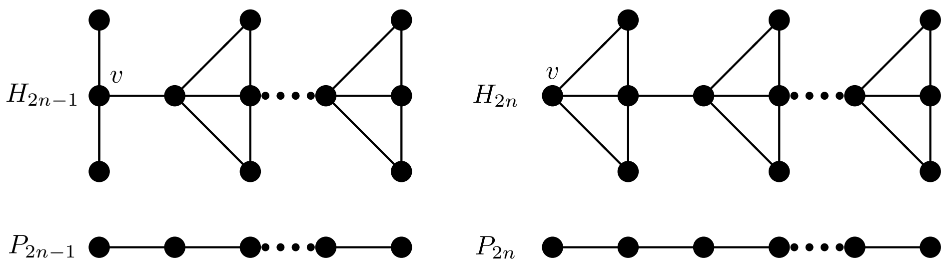

FIGURE 4. P_n and $H_n = \Omega\{P_n\}$.

Theorem 2.3. *If $J_n(x) = I(H_n;x), n \geq 0$, then*
(i) *J_{2n} and J_{2n-1} are both of degree $2n$;*
(ii) *$J_0(x) = 1, J_1(x) = 1 + 3x + x^2$ and $J_n, n \geq 2$, satisfies the following recursive relations:*

$$\begin{aligned} J_{2n}(x) &= J_{2n-1}(x) + x \cdot J_{2n-2}(x), \quad n \geq 1,\\ J_{2n-1}(x) &= (1+x)^2 \cdot J_{2n-2}(x) + x \cdot J_{2n-3}(x), \quad n \geq 2;\end{aligned}$$

(iii) *J_n is both palindromic and unimodal.*

Proof. (i) The assertion follows from the fact that the degree of J_n equals $\alpha(H_n)$, and, from the Figure 4, it is easy to see that $\alpha(H_{2n}) = \alpha(H_{2n-1}) = 2n$.

(ii) Clearly, $J_0(x) = 1$ and $J_1(x) = 1 + 3x + x^2$. Using Proposition 1.1, we deduce that (see Figure 4):

$$\begin{aligned} J_{2n-1}(x) &= I(H_{2n-1};x) = I(H_{2n-1} - v;x) + x \cdot I(H_{2n-1} - N[v];x) =\\ &= (1+x)^2 \cdot J_{2n-2}(x) + x \cdot J_{2n-3}(x),\end{aligned}$$

and also

$$\begin{aligned} J_{2n}(x) &= I(H_{2n};x) = I(H_{2n} - v;x) + x \cdot I(H_{2n} - N[v];x) =\\ &= J_{2n-1}(x) + x \cdot J_{2n-2}(x).\end{aligned}$$

(iii) According to Theorem 2.2, all J_n are palindromic. Consequently, in order to prove the unimodality of J_n, it is sufficient to check that the coefficients of the first half of J_n are in non-decreasing order.

We use induction on n. Clearly, $J_0(x)$ and $J_1(x)$ are unimodal. Assume that the assertion is true for $0 \leq k \leq 2n$. We have to validate it for $k \in \{2n+1, 2n+2\}$.

Let us denote the sequences of coefficients of $J_{2n-1}(x), J_{2n}(x), J_{2n+1}(x), J_{2n+2}(x)$, respectively, by $(a_i), (b_i), (c_i), (d_i)$. Taking into account that, by *(ii)*,

$$J_{2n+1}(x) = (1 + 2x + x^2) \cdot J_{2n}(x) + x \cdot J_{2n-1}(x),$$

we get the following matrix of coefficients:

J_{2n}	1	b_1	b_2	b_3	...	b_i	b_{i+1}	...	b_n	b_{n-1}	...
$2x \cdot J_{2n}$		2	$2b_1$	$2b_2$	...	$2b_{i-1}$	$2b_i$	...	$2b_{n-1}$	$2b_n$	...
$x^2 \cdot J_{2n}$			1	b_1	...	b_{i-2}	b_{i-1}	...	b_{n-2}	b_{n-1}	...
$x \cdot J_{2n-1}$		1	a_1	a_2	...	a_{i-1}	a_i	...	a_{n-1}	a_n	...
J_{2n+1}	1	c_1	c_2	c_3	...	c_i	c_{i+1}	...	c_n	c_{n+1}	...

which implies that:

(a) $c_1 = 3 + b_1 \leq c_2 = 1 + 2b_1 + a_1$, because $1 \leq a_1, 1 \leq b_1$;

(b) for $i \leq n - 1$,

$$c_i = b_i + 2b_{i-1} + b_{i-2} + a_{i-1} \leq b_{i+1} + 2b_i + b_{i-1} + a_i = c_{i+1},$$

since $(b_i)_{i \leq n}, (a_i)_{i \leq n}$ are non-decreasing sequences;

(c) for $i = n$,

$$c_n = b_n + 2b_{n-1} + b_{n-2} + a_{n-1} \leq b_{n-1} + 2b_n + b_{n-1} + a_n = c_{n+1},$$

since $b_{n-2} \leq b_n$ and $a_{n-1} \leq a_n$.

Similarly, according to *(ii)*, we have

$$J_{2n+2}(x) = J_{2n+1}(x) + x \cdot J_{2n}(x),$$

and hence, we obtain:

J_{2n+1}	1	c_1	c_2	...	c_i	c_{i+1}	...	c_n	c_{n+1}	...
$x \cdot J_{2n}$		1	b_1	...	b_{i-1}	b_i	...	b_{n-1}	b_n	...
J_{2n+2}	1	d_1	d_2	...	d_i	d_{i+1}	...	d_n	d_{n+1}	...

which assures that:

(a) $d_1 = 1 + c_1 \leq d_2 = b_1 + c_2$, because $1 \leq b_1, c_1 \leq c_2$;

(b) in general, $d_i = b_{i-1} + c_i \leq b_i + c_{i+1} = d_{i+1}, i \leq n$, since $(b_i)_{i \leq n}, (c_i)_{i \leq n}$ are non-decreasing sequences.

Therefore, both polynomials J_{2n+1} and J_{2n+2} are unimodal. □

The product of two polynomials, one log-concave and the other unimodal, is not always log-concave, for instance, if $G = K_{40} + 3K_7, H = K_{110} + 3K_7$, then

$$\begin{aligned} I(G; x) \cdot I(H; x) &= \left(1 + 61x + 147x^2 + 343x^3\right)\left(1 + 131x + 147x^2 + 343x^3\right) \\ &= 1 + 192x + 8285x^2 + 28910x^3 + 87465x^4 + 100842x^5 + 117649x^6, \end{aligned}$$

which is not log-concave, because $100842^2 - 87465 \cdot 117649 = -121\,060\,821$. However, the following result, due to Keilson and Gerber, gives a sufficient condition for two polynomials to have a unimodal product.

Theorem 2.4. [9] *If P is log-concave and Q is unimodal, then $P \cdot Q$ is unimodal, while the product of two log-concave polynomials is log-concave.*

Corollary 2.3. *If the clique cover Ω_m of P_n contains m vertices as cliques, then $I(\Omega_m\{P_n\};x)$ is unimodal.*

Proof. Firstly, by Corollary 2.2, we obtain

$$\begin{aligned} I(\Omega_m\{P_n\};x) &= (1+x)^m \cdot J_n(x), \text{ if } n \text{ is even}, \\ I(\Omega_m\{P_n\};x) &= (1+x)^{m-1} \cdot J_n(x)), \text{ if } n \text{ is odd}. \end{aligned}$$

Secondly, $(1+x)^m$ is log-concave and $J_n(x)$ is unimodal. Consequently, the independence polynomial $I(\Omega_m\{P_n\};x)$ is unimodal, by Theorem 2.4. □

Remark 2.1. *The polynomials $L_1 = 1 + 100x + x^2 + x^3 + x^4 + x^5 + x^6$ and $L_2 = 1 + x + x^2 + x^3 + x^4 + 111x^5 + x^6$ are non-palindromic, but unimodal, while $L_3 = L_2 + x \cdot L_1 = 1 + 2x + 101x^2 + 2x^3 + 2x^4 + 112x^5 + 2x^6 + x^7$ is not unimodal, i.e., the recursions in Theorem 2.3 do not preserve unimodality.*

Remark 2.2. *The polynomials $M_1 = 1 + x + x^2 + 100x^3 + x^4 + x^5 + x^6$ and $M_2 = 1+x+11x^2+x^3+x^4$ are both palindromic and unimodal, while $M_3 = M_2+x\cdot M_1 = 1+2x + 12x^2 + 2x^3 + 101x^4 + x^5 + x^6 + x^7$ is neither palindromic, nor unimodal, i.e., the recursions of Theorem 2.3, without the initial values of J_0 and J_1, are not enough to conclude with unimodality and palindromicity of all J_n.*

Theorem 2.5. [2] *If P and Q are both unimodal and palindromic, then $P \cdot Q$ is unimodal and palindromic as well.*

However, the above result can not be generalized to the case when P is unimodal and palindromic, while Q is unimodal and non-palindromic; e.g.,

$$\begin{aligned} P &= 1 + x + 3x^2 + x^3 + x^4, Q = 1 + x + x^2 + x^3 + 2x^4, \text{ while} \\ P \cdot Q &= 1 + 2x + 5x^2 + 6x^3 + 8x^4 + 7x^5 + 8x^6 + 3x^7 + 2x^8. \end{aligned}$$

Using Theorems 2.5 and 2.3, we deduce the following corollaries.

Corollary 2.4. *If each connected component of G is isomorphic to some H_n, then $I(G;x)$ is palindromic and unimodal.*

Corollary 2.5. *If $G = H_n$, then $mG, m \geq 2$, has a palindromic and unimodal independence polynomial.*

Let us denote by $\uplus mG$ the Zykov sum of $m > 1$ copies of the graph G. Since $I(\uplus mG;x) = m \cdot I(G;x) - (m-1)$, we infer the following result.

Corollary 2.6. *If $G = H_n$ for some $n \geq 1$, then $\uplus mG$ has a palindromic and unimodal independence polynomial.*

Remark 2.3. *The following polynomials $P = 1 + 10x + 100x^2 + 10x^3 + x^4$ and $Q = 1 + x + x^2 + x^3 + x^4$ are log-concave and palindromic, but*

$$(1+x)^2 \cdot Q + x \cdot P = 1 + 4x + 14x^2 + 104x^3 + 14x^4 + 4x^5 + x^6$$

is not log-concave, since $14^2 - 4*104 = -220$. In other words, the recursions of Theorem 2.3, without the initial values of J_0 and J_1, are not enough to conclude with log-concavity of all J_n.

Using Theorem 2.3, we obtain successively,

$$\begin{aligned}
J_2(x) &= 1+4x+x^2, J_3(x) = 1+7x+13x^2+7x^3+x^4,\\
J_4(x) &= 1+8x+17x^2+8x^3+x^4,\\
J_5(x) &= 1+11x+41x^2+63x^3+41x^4+11x^5+x^6,\\
J_6(x) &= 1+12x+49x^2+80x^3+49x^4+12x^5+x^6,\\
J_7(x) &= 1+15x+85x^2+231x^3+321x^4+231x^5+85x^6+15x^7+x^8,\\
J_8(x) &= 1+16x+97x^2+280x^3+401x^4+280x^5+97x^6+16x^7+x^8.
\end{aligned}$$

It is easy to see that these polynomials are log-concave. In general, since $J_n(x)$ is unimodal and palindromic, we may deduce that $b^2 \geq a \cdot c$, where a, b, c are the middle coefficients of $J_n(x)$. In other words, $J_n(x)$ satisfies the log-concavity condition at least at this point. The examples presented above give support to the following conjecture: all the polynomials $J_n(x)$ are log-concave.

3. Conclusions

In this paper we proved that the independence polynomials of graphs H_n are both palindromic and unimodal. In [4] it is proved that the independence polynomial of a claw-free graph has only real roots. It is worth noticing that H_n are not claw-free graphs, but the examples of $J_n(x), n \leq 20$, (the first eight of them are listed above), have only real roots. Moreover, all their roots are located in the interval $(-6, 0)$, which we think is true in general.

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