CARPATHIAN J. MATH.
23 (2007), No. 1 - 2, 133 - 140

Dedicated to Professor Ioan A. RUS on the occasion of his 70^{th} *anniversary*

From Maia fixed point theorem to the fixed point theory in a set with two metrics

ANTON S. MUREŞAN

ABSTRACT. In this paper we give some results about Maia's fixed point theorem and we show how can be used these results to some types of applications in the sets with two metrics.

1. INTRODUCTION

In 1968, M. G. Maia [5] generalized Banach's fixed point theorem for a set X endowed with two metrics. Then there were obtained a serie of similar results and generalizations of this theorem in many papers [1] - [4], [6] - [16], [18].

In 1977, Ioan A. Rus [15] stated and proved an interesting fixed point theorem of Maia type by replacing one of the conditions in Maia's theorem.

B. Rzepecki using the idea from [15] proved in the papers [18], [19] some other fixed point theorem of Maia type.

Many papers deal with fixed point theorems of Maia type and with applications of these theorems in sets with two or three metrics [8], [9], [17].

Let (X, d) be a metric space and $T : X \to X$ an operator. We denote by $O(x) = \{x, T(x), T(T(x)), ...\}$ the **orbit** of T at the point $x \in X$. An element $x \in X$ is called **regular** for T if $O(x)$ is bounded.

The operator T is a **weakly Picard operator** (WPO) if the sequence of successive approximation, $(T^n(x))_{n\in\mathbb{N}}$ converges for all $x \in X$ and the limit (which may depend on x) is a fixed point of T.

If T is a WPO and $F_T = \{x^*\}$, then, by definition , T is a **Picard operator** (PO), where $F_T = \{x \,|x \in X, T(x) = x\}$.

The operator T is **O-continuous** if, for every subsequence $(x_{m_i})_{i\in\mathbb{N}}$ of sequence $(x_m)_{m\in\mathbb{N}}$, defined by $x_m = T(x_{m-1})$, convergent to u, we have that the sequence $(T(x_{m_i}))_{i\in\mathbb{N}}$ converge to $T(u)$, *i.e.*,

$$\lim_{i\to\infty} x_{m_i} = u \Rightarrow \lim_{i\to\infty} T(x_{m_i}) = T(u).$$

The metric space (X, d) is **O-complete** if every Cauchy sequence $(x_n)_{n\in\mathbb{N}}$ in $O(x)$ is convergent to a point of X.

For $A \subset X$ we denote $\delta_d(A) = \sup\{d(a, b) \,|a, b \in A\}$ and

$$I_{b,d}(T) = \{A \,|A \subset X, A \neq \emptyset,\ f(A) \subset A, \delta_d(A) < +\infty\}.$$

Received: 23.11.2006; In revised form: 05.02.2007; Accepted: 19.02.2007
2000 *Mathematics Subject Classification.* 54H25, 47H10.
Key words and phrases. *Fixed points, weakly Picard operators, comparison fractions, generalized contractions, Fredholm integral equations.*

A mapping $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is called a **comparison function** if the following conditions are satisfied:

(i) φ is upper semicontinuons from the right;

(ii) φ is increasing;

(iii) $\varphi(t) < t$ for $t > 0$.

The mapping $T : X \to X$ is a (δ, φ)**-contraction** if there exists a comparison function φ such that $\delta_d(T(A)) \leq \varphi(\delta_d(A))$, for all $A \in I_{b,d}(T)$.

2. Some known results

Let X be a nonempty set endowed with two metrics d, ρ and $T : X \to X$ a mapping. We have the following well know theorems.

Theorem 2.1. (M. G. Maia [5]) *If the metric space (X, d) is complete and*

(i) $d(x, y) \leq \rho(x, y)$ *for all* $x, y \in X$;

(ii) $\rho(T(x), T(y)) \leq a\rho(x, y)$ *for all* $x, y \in X$,*with* $0 < a < 1$;

(iii) *T is continuous in* (X, d),

then $T : (X, d) \to (X, d)$ *is a PO.*

Theorem 2.2. (I. A. Rus [14]) *If the metric space (X, d) is complete and*

(i) *there exists* $c > 0$ *such that* $d(T(x), T(y)) \leq c\rho(x, y)$ *for all* $x, y \in X$;

(ii) $\rho(T(x), T(y)) \leq a\rho(x, y)$ *for all* $x, y \in X$, *with* $0 < a < 1$;

(iii) *T is continuous in* (X, d),

then T is a PO.

Theorem 2.3. ([17]). *If the metric space (X, d) is complete and*

(i) *there exists* $c > 0$ *such that* $d(T(x), T(y)) \leq c\rho(x, y)$ *for all* $x, y \in X$;

(ii) $\rho(T^2(x), T(x)) \leq a\rho(x, T(x))$, *for all* $x \in X$;

(iii) *T is closed in* (X, d),

then $T : (X, d) \to (X, d)$ *is a WPO.*

Theorem 2.4. ([17]) *If the metric space (X, d) is complete and*

(i) *there exists* $c > 0$ *such that* $d(T(x), T(y)) \leq c\rho(x, y)$ *for all* $x, y \in X$;

(ii) *there exists* $\varphi : X \to \mathbb{R}_+$ *such that* $\rho(x, T(x)) \leq \varphi(x) - \varphi(T(x))$ *for all* $x \in X$;

(iii) *T is closed in* (X, d),

then $T : (X, d) \to (X, d)$ *is a WPO.*

We can generalize the above theorems in the case of the generalized metrics. For example in the case when $d(x, y), \rho(x, y) \in \mathbb{R}^m_+$ we obtain.

Theorem 2.5. (V. Mureşan [10]) *If the generalized metric space (X, d) is complete and*

(i) *there exists a matrix* $C \in M_m(\mathbb{R}_+)$ *such that*

$$d(T(x), T(y)) \leq C\,\rho(x, y) \text{ for all } x, y \in X;$$

(ii) *there exists a matrix convergent towards 0,* $A \in M_m(\mathbb{R}_+)$ *such that*

$$\rho(T(x), T(y)) \leq A\rho(x, y) \text{ for all } x, y \in X;$$

(iii) *T is continuous in* (X, d);

(iv) *there exists* $c_1 > 0$ *such that* $\rho(x, y) \leq c_1 d(x, y)$ *for all* $x, y \in X$,

then $T : (X, d) \to (X, d)$ *and* $T^p : (X, d) \to (X, d)$ *are PO, for any* $p \in \mathbb{N}^*$.

Theorem 2.6. ([17]) *If the generalized metric space* (X, d) *is complete and*
(i) *there exists a matrix* $C \in M_m(\mathbb{R}_+)$ *such that*

$$d(T(x), T(y)) \leq C\,\rho(x, y) \ \text{ for all } x, y \in X;$$

(ii) *there exists a matrix* $A \in M_m(\mathbb{R}_+)$ *convergent towards O, such that*

$$\rho\left(T^2(x), T(x)\right) \leq A\rho(x, T(x)), \ \text{ for all } x, \in X;$$

(iii) T *is closed in* (X, d)*;*
then $T : (X, d) \to (X, d)$ *is a WPO.*

Theorem 2.7. ([17]) *If the generalized metric space* (X, d) *is complete and*
(i) *there exists a matrix* $C \in M_m(\mathbb{R}_+)$ *such that*

$$d(T(x), T(y)) \leq C\,\rho(x, y), \ \text{for all } x, y \in X;$$

(ii) *there exists* $\varphi : X \to \mathbb{R}_+$ *such that*

$$\rho(x, T(x)) \leq \varphi(x) - \varphi(T(x)), \ \text{for all } x \in X;$$

(iii) T *is closed in* (X, d)*,*
then $T : (X, d) \to (X, d)$ *is a WPO.*

Theorem 2.8. ([9]) *If the generalized metric space* (X, d) *is* o *- complete and*
(i) $d(x, y) \leq \rho(x, y)$ *for all* $x, y \in X$*;*
(ii) *there exists a matrices* $A_i \in M_m(\mathbb{R}_+)$*,* $i = \overline{1, 5}$*, such that* $(I_m - A_3 - A_4)^{-1} \geq 0$*, the matrix* $A = (I_m - A_3 - A_4)^{-1}(A_1 + A_3 + A_5)$ *converge towards 0 and*

$$\begin{aligned}\rho(T(x), T(y)) \leq\ & A_1\rho(x, y) + A_2\rho(x, T(y)) + A_3\rho(y, T(x)) \\ & + A_4\rho(x, T(x)) + A_5\rho(y, T(y)), \ \text{for all } x, y \in X;\end{aligned}$$

(iii) T *is O-continuous in* (X, d)*,*
then $T : (X, d) \to (X, d)$ *is a WPO.*

In the case when T is a (δ, φ) - contraction we have the following theorem.

Theorem 2.9. (A. S. Muresan [6]) *If the metric space* (X, d) *is complete and*
(i) *there exists* $c > 0$ *such that*

$$d(T(x), T(y)) \leq c\rho(x, y) \ \text{for all } x, y \in X;$$

(ii) $T : (X, \rho) \to (X, \rho)$ *in a* (δ_ρ, φ) *- contraction;*
(iii) T *is continuous in* (X, d)*;*
(iv) *every* $x \in X$*, is a regular element for* $T : (X, \rho) \to (X, \rho)$*,*
then $T : (X, d) \to (X, d)$ *is a* PO*.*

Remark 2.1. If the condition (iv) in Theorem 2.9 is replaced by the condition (iv′) (X, d) is a bounded complete metric space, then the conclusion is the same.

3. Main results

The purpose of this section is to give some fixed point theorems for expansion mappings on metric space endowed with two metrics.

Let X be a nonempty set endowed with two metrics d and ρ.

Let $T : X \to X$ a mapping . We suppose that

(1) there exists a constant $C > 0$ such that

$$d(T(x), T(y)) \leq c\rho(x, y) \text{ for all } x, y \in X,$$

(2) (X, d) is a complete metric space.

We have

Theorem 3.10. *We suppose that the conditions* (1) *and* (2) *holds. If*

(3) *f is surjective;*

(4) *there exists $a, b, c \in \mathbb{R}_+,\ a < 1, a + b + c > 1$ such that*

$$\rho^2(T(x), T(y)) \geq a\rho^2(x, T(x)) + b\rho^2(y, T(y)) + c\rho^2(x, y)$$

for all $x, y \in X$, $x \neq y$;

(5) *there exists $c_1 > 0$ such that $\rho(x, y) \leq c_1 d(x, y)$ for all $x, y \in X$,*

then $T : (X, d) \to (X, d)$ is a WPO.

Moreover, if $c > 1$ then $T : (X, d) \to (X, d)$ is a PO.

Proof. Let $x_0 \in X$. From (3) there exists $x_1 \in X$ such that $T(x_1) = x_0$.

We consider the sequence $(x_n)_{n\in\mathbb{N}}$ such that $T(x_{n+1}) = x_n$, for any $n \in \mathbb{N}$. We prove that this sequence is a Cauchy sequence in the metric space (X, ρ) . Without loss of generality, we can suppose that $x_{n-1} \neq x_n$ for any $n \in \mathbb{N}^*$. From (4), we have

$$\begin{aligned}\rho^2(x_{n-1}, x_n) &= \rho^2(T(x_n), T(x_{n+1})) \geq a\rho^2(x_n, T(x_n)) \\ &\quad + b\rho^2(x_{n+1}, T(x_{n+1})) + c\rho^2(x_n, x_{n+1}) \\ &= a\rho^2(x_n, x_{n-1}) + (b + c)\rho^2(x_n, x_{n+1}),\end{aligned}$$

therefore

$$(1 - a)\rho^2(x_{n-1}, x_n) \geq (b + c)\rho^2(x_n, x_{n+1}),$$

that is,

$$\rho(x_n, x_{n+1}) \leq \sqrt{\frac{1 - a}{b + c}}\rho(x_{n-1}, x_n).$$

As $\frac{1-a}{b+c} < 1$ we find that the sequence $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in the metric space (X, ρ) . From (1) the sequence $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in the complete metric space (X, d), too. Hence the sequence $(x_n)_{n\in\mathbb{N}}$ is convergent in the metric space (X, d) .

Let $x^* = \lim\limits_{n\to\infty} x_n$. Then $d(x_n, x^*) \to 0$ for $n \to \infty$.

Let $y \in X$ such that $T(y) = x^*$. We have

$$\begin{aligned}\rho^2(x_n, x^*) &= \rho^2(T(x_{y+1}), T(y)) \\ &\geq a\rho^2(x_{n+1}, T(x_{n+1})) + b\rho^2(y, T(y)) + c\rho^2(x_{n+1}, y).\end{aligned}$$

Letting $n \to \infty$ we get $0 \geq b\rho^2(y, T(y)) + c\rho^2(x^*, y)$, that is, $x^* = y$. Thus $x^* \in F_T$, therefore T is a weakly Picard operator.

Now let $c > 1$. We show that $F_T = \{x^*\}$. We suppose that the fixed point is not unique. Let x^{**} be another fixed point for T. We have

$$\begin{aligned}\rho^2\left(x^{**}, x^*\right) &= \rho^2\left(T\left(x^{**}\right), T\left(x^*\right)\right)\\ &\geq a\rho^2\left(x^{**}, T\left(x^{**}\right)\right) + b\rho^2\left(x^*, T\left(x^*\right)\right) + c\rho^2\left(x^{**}, x^*\right),\end{aligned}$$

that is, $1 \geq c$. This contradicts $c > 1$.

So $F_T = \{x^*\}$, and therefore T is a Picard operator. □

Theorem 3.11. *We suppose that conditions* $(1)-(3)$ *and* (5) *in Theorem* 3.10 *hold. If there exists* $k \in \left[\frac{2}{3}, 1\right]$ *such that*

$$(6) \quad \rho\left(T\left(x\right), T\left(y\right)\right) \geq k\frac{\rho^2\left(x, T\left(x\right)\right) + \rho\left(x, T\left(x\right)\right)\rho\left(y, T\left(y\right)\right) + \rho^2\left(y, T\left(y\right)\right)}{\rho\left(x, T\left(x\right)\right) + \rho\left(y, T\left(y\right)\right)},$$

for any $x, y \in X,\ x \neq y$, *for which* $\rho\left(x, T\left(x\right)\right) + \rho\left(y, T\left(y\right)\right) \neq 0$, *then* $T : (X, d) \to (X, d)$ *is a WPO.*

Proof. We consider the sequence $\left(x_n\right)_{n\in\mathbb{N}},\ x_{n-1} = T\left(x_n\right)$ for $n \in \mathbb{N}^*$ with $x_0 \in X$. This sequence is convergent in the metric space (X, ρ). We prove that $T\left(x^*\right) = x^*$, where $x^* = \lim\limits_{n\to\infty} x_n$ in the complete metric space (X, d), hence in the metric space (X, ρ) from (5). We suppose that $T\left(y\right) = x^*$ and $y \neq x^*$. Then

$$\begin{aligned}\rho\left(x_n, x^*\right) &= \rho\left(T\left(x_{n+1}\right), T\left(y\right)\right)\\ &\geq k\frac{\rho^2\left(x_{n+1}, T\left(x_{n+1}\right)\right) + \rho\left(x_{n+1}, T\left(x_{n+1}\right)\right)\rho\left(y, T\left(y\right)\right) + \rho^2\left(y, T\left(y\right)\right)}{\rho\left(x_{n+1}, T\left(x_{n+1}\right)\right) + \rho\left(y, T\left(y\right)\right)}\\ &= k\frac{\rho^2\left(x_{n+1}, x_n\right) + \rho\left(x_{n+1}, x_n\right)\rho\left(y, T\left(y\right)\right) + \rho^2\left(y, T\left(y\right)\right)}{\rho\left(x_{n+1}, T\left(x_{n+1}\right)\right) + \rho\left(y, T\left(y\right)\right)}.\end{aligned}$$

Letting $n \to \infty$, we obtain $0 \geq k\rho\left(y, T\left(y\right)\right)$ therefore $y = T\left(y\right) = x^*$. So $x^* \in F_T$ and T is a weakly Picard operator. □

Theorem 3.12. *We suppose that the conditions* $(1)-(3)$ *hold. If* $T : (X, d) \to (X, d)$ *is continuous and there exists a constant* $k > 1$ *such that*

$$\begin{aligned}(7) \quad &\rho^2\left(T\left(x\right), T\left(y\right)\right)\\ &\geq k\min\left\{\rho^2 x, T\left(x\right), \rho^2\left(y, T\left(y\right)\right), \rho\left(x, T\left(x\right)\right)\rho\left(x, y\right), \rho\left(y, T\left(y\right)\right)\rho\left(x, y\right)\right\},\end{aligned}$$

for all $x, y \in X$, *then* $T : (X, d) \to (X, d)$ *is a WPO.*

Proof. Let $x_0 \in X$. We consider the sequence $\left(x_n\right)_{n\in\mathbb{N}},\ x_{n-1} = T\left(x_n\right)$, for which $x_{n-1} \neq x_n, n \in \mathbb{N}^*$. From (7) we find that

$$\rho\left(x_n, x_{n+1}\right) \leq \frac{1}{\sqrt{k}}\rho\left(x_n, x_{n+1}\right),$$

that is, the sequence $\left(x_n\right)_{n\in\mathbb{N}}$ is a Cauchy sequence in the metric space (X, ρ). From the condition (1), $\left(x_n\right)_{n\in\mathbb{N}}$ is a Cauchy sequence in the metric space (X, d) therefore it is convergent. Let $x^* = \lim\limits_{n\to\infty} x_n$. From the continuity of mapping T it follows that

$$x^* = \lim_{n\to\infty} x_n = \lim_{n\to\infty} T\left(x_{n+1}\right) = T\left(\lim_{n\to\infty} x_{n+1}\right) = T\left(x^*\right),$$

i.e., $x^* \in F_T$. Thus T is a weakly Picard operator. □

Remark 3.2. The conclusions of Theorems 3.10 - 3.12 are valid when the metric spaces $(X, d), (X, \rho)$ are generalized metric spaces with the metrics d and ρ taking values in $\mathbb{R}^m_+$.

4. Applications

Let $\Omega \subset \mathbb{R}^m$ be a bounded domain. We consider the following functional - integral equation

(*) $$x(t) = h\left(t, x|_{\partial\Omega}\right) + \int_\Omega K(t, s, x(s))\, ds,\ t \in \overline{\Omega}.$$

We suppose that

(8) $K(t, s, x, u) = 0$, for all $t \in \partial\Omega$, $s \in \overline{\Omega}$, $u \in \mathbb{R}$;

(9) $h\left(\cdot, x|_{\partial\Omega}\right) \in C\left(\overline{\Omega}\right)$, for all $x \in C\left(\overline{\Omega}\right)$;

(10) $h\left(t, x|_{\partial\Omega}\right) = x(t)$ for all $t \in \partial\Omega$;

(11) $K \in C\left(\overline{\Omega} \times \overline{\Omega} \times \mathbb{R}\right)$;

(12) there exists $L \in C\left(\overline{\Omega} \times \overline{\Omega}\right)$ such that

$$|K(t, s, u) - K(t, s, v)| \leq L(t, s)\, |u, v|,\ \text{for all } t, s \in \overline{\Omega} \text{ and } u, v \in \mathbb{R};$$

(13) $\int_{\Omega\times\Omega} (L(t, s))^2\, dt\, ds < 1.$

Let $T : C\left(\overline{\Omega}\right) \to C\left(\overline{\Omega}\right)$ be the operator defined by

$$A(x)(t) = h\left(t, x|_{\partial\Omega}\right) + \int_\Omega K(t, s, x(s))\, ds.$$

We consider on $C\left(\overline{\Omega}\right)$ the following metrics d and ρ defined by

$$d(x, y) = \|x - y\|_{C\left(\overline{\Omega}\right)} \text{ and } \rho(x, y) = \|x - y\|_{L^2(\Omega)}.$$

It is clear that (see Theorem 2.2),

$$c = \sup_{t\in\overline{\Omega}} \left(\int_\Omega (L(t, s))^2\, ds \right)^{1/2},$$

and

$$a = \left(\int_{\Omega\times\Omega} (L(t, s))^2\, dt\, ds \right)^{1/2}.$$

We are in the conditions of Theorem 2.2 and so we have

Theorem 4.13. *We suppose that the conditions* $(8) - (13)$ *hold. Let* $S_T \subset C\left(\overline{\Omega}\right)$ *be the solution set of the equation* $(*)$.

Then $S_T \neq \emptyset$.

Remark 4.3. In the conditions of Theorem 4.13, moreover we have $Card\ S_T = Card\ C(\partial\Omega)$ (see Theorem 3.10 [17]).

Using Theorem 2.5 we can study existence and uniqueness for the solution of the of the system of Fredholm integral equations of the type:

$$x(t) = \int_\Omega K(t, s, x(s))\, ds + f(t), \quad t \in \overline{\Omega}. \quad (**)$$

Here $\Omega \subset \mathbb{R}^m$ is a bounded domain, $f \in C\left(\overline{\Omega}, \mathbb{R}^m\right)$ and $K \in C\left(\overline{\Omega} \times \overline{\Omega} \times \mathbb{R}^m, \mathbb{R}^m\right)$.

Consider the operator $T : C\left(\overline{\Omega}, \mathbb{R}^m\right) \to C\left(\overline{\Omega}, \mathbb{R}^m\right)$ defined by

$$A(x)(t) = \int_\Omega K(t, s, x(s))\, ds + f(t).$$

The space $X = C\left(\overline{\Omega}, \mathbb{R}^m\right)$ is a generalized metric space relative to the metrics

$$d(x, y) = \|x - y\|_{C\left(\overline{\Omega}, \mathbb{R}^m\right)} = \left(\|x_1 - y_1\|_{C\left(\overline{\Omega}\right)}, ..., \|x_m - y_m\|_{C\left(\overline{\Omega}\right)}\right),$$
$$\rho(x, y) = \|x - y\|_{L^2(\Omega, \mathbb{R}^m)} = \left(\|x_1 - y_1\|_{L^2(\Omega)}, ..., \|x_m - y_m\|_{L^2(\Omega)}\right),$$

where $\|u_i\|_{C\left(\overline{\Omega}\right)} = \max_{t \in \overline{\Omega}} |u_i(t)|$, $\|u_i\|_{L^2(\Omega)} = \left(\int_\Omega u_i(t) |^2 dt\right)^{1/2}$.

We have

Theorem 4.14. *If the following conditions are fulfilled*

(14) *there exists* $L : \Omega \times \Omega \to M_m(\mathbb{R}_+)$, *with* $\sup_{t \in \overline{\Omega}} \left(\int_\Omega |L_{ij}(t, s)|^2 ds\right)^{1/2} < \infty$ *for all* $i, j = \overline{1, m}$ *such that* $|K(t, s, u) - K(t, s, v)| \leq L(t, s)|u - v|$, *for all* $t, s \in \Omega$, $u, v \in \mathbb{R}^m$ *and* $|K(t, s, 0)| \leq r(t, s)$ *where* $r \in C\left(\overline{\Omega} \times \overline{\Omega} \times \mathbb{R}^m_+\right)$;

(15) *there exists a matrix* $A \in M_m(\mathbb{R})$, *convergent towards* 0, *such that*

$$\left(\left\{\int_{\Omega \times \Omega} |L_{ij}(t, s)|^2 dt\, ds\right\}\right)^{1/2} \leq A,$$

then the system of equations $(**)$ *has in* $C\left(\overline{\Omega}, \mathbb{R}^m\right)$ *a unique solution* x^*, *which can be obtained by the successive approximation method starting from any element of* $C\left(\overline{\Omega}, \mathbb{R}^m\right)$.

References

[1] Albu, M., *A fixed point theorem of Maia-Perov type*, Studia Univ. "Babeş-Bolyai", Mathematica, **23** (1978), 76-79

[2] Berinde, V., *A fixed point theorem of Maia type in K-metric spaces*, Sem. on Fixed Point Theory, Preprint **3** (1991), "Babeş-Bolyai" Univ., Cluj-Napoca, 7-14

[3] Dhage, B. C., *An extension of a fixed point theorem of Maia*, Pure Appl. Math. Sci., **XXIV**, 1-2 (1986), 65-69

[4] Dhage, B. C. and Dhobale, V. V., *On generalization of Maia's fixed point theorem*, J. Indian Acad. Math., **8** (1986), 25-30

[5] Maia, M. G., *Un'osservatione sulle contrazioni metriche*, Rend. Sem. Mat. Univ. Padova, **40** (1968), 139-143

[6] Mureşan, A. S., *Some fixed point theorems of Maia type*, Sem. on Fixed Point Theory, Preprint **3** (1988), "Babeş-Bolyai" Univ., Cluj-Napoca, 35-42

[7] Mureşan, A. S., *Fixed point theorems of Maia type for expansion mappings*, Studia Univ. "Babeş-Bolyai", Oeconomica, **34** (**1**) (1989), 81-84

[8] Mureşan, A. S. and Muresan, V., *A generalization of Maia's fixed point theorem*, Conferinta de matematica aplicata si mecanica, 20-23 oct., 1988, **II**, Institutul Politehnic Cluj-Napoca, Sem. Th. Angheluţă, 185-190

[9] Mureşan, A. S. and Mureşan, V., *Some applications of weakly Picard operators*, The Tenth International Conference on Applied Mathematics and Computer Science, Cluj-Napoca-Baişoara, May 31 - June, 2006, ACAM, **15** (2006), 7-13

[10] Mureşan, V., *Basic problem for Maia-Perov's fixed point theorem*, Sem. on Fixed Point Theory, Preprint **3** (1988), "Babeş-Bolyai" Univ., Cluj-Napoca, 43-48

[11] Petruşel, A., *Fredholm-Volterra integral equations and Maia's theorem*, Sem. on Fixed Point Theory, Preprint **3** (1988), "Babeş-Bolyai" Univ., Cluj-Napoca, 79-82

[12] Precup, R., *A fixed point theorem of Maia type in syntopogenous spaces*, Sem. on Fixed Point Theory, Preprint **3** (1988), "Babeş-Bolyai" Univ., Cluj-Napoca, 49-70

[13] Ray, B. K., *On a fixed point theorem in a space with two metrics*, The Mathematics Education, **9** (1975), No. 3, 57-58

[14] Rus, A. I., *On a fixed point theorem of Maia*, Studia Univ. "Babeş-Bolyai", Mathematica, **1** (1977), 40-42

[15] Rus, A. I., *On a fixed point theorem in a set with two metrics*, Mathematica, Revue d'analyse mathematique et de la theorie de l'approximation, **6** (1977), 197-201

[16] Rus, A. I., *Basic problem for Maia's theorem*, Sem. on Fixed Point Theory, Preprint **3** (1981), "Babeş-Bolyai" Univ., Cluj-Napoca, 112-115

[17] Rus, A. I., Mureşan, A. S. and Mureşan, V., *Weakly Picard operators on a set with two metrics*, Fixed Point Theory, **6**, No. 2 (2005), 323-331

[18] Rzepecki, B., *A note on fixed point theorem of Maia*, Studia Univ. "Babeş-Bolyai", Mathematica, **25** (**2**) (1980), 65-71

[19] Rzepecki, B., *Remarks on the Banach fixed point principle and its applications*, Rev. Roum. Math. Pures at Appl., **27** (1982), 81-93

[20] Singh, S. P., *On a fixed point theorem in metric space*, Rend. Sem. Mat. Univ. Padova, **43** (1970), 229-232

FACULTY OF ECONOMICS
DEPARTMENT OF STATISTICS, ECONOMICAL ANALYSIS, FORECASTING AND MATHEMATICS
"BABEŞ-BOLYAI" UNIVERSITY OF CLUJ-NAPOCA
M. KOGĂLNICEANU ST. 1, ROMANIA
E-mail address: asmuresan@econ.ubbcluj.ro