Fixed points and the stability of some classes of functional equations

LIVIU CĂDARIU and VIOREL RADU

ABSTRACT.

We use a fixed point alternative in uniform spaces to prove the generalized Ulam-Hyers stability of a nonlinear single variable functional equation for mappings on groups with values in random normed spaces, as well as the generalized Ulam-Hyers stability of monomial functional equations, for mappings on Abelian groups taking values in sequentially complete locally convex spaces.

REFERENCES

[1] Agarwal, R. P., Xu, B. and Zhang, W., *Stability of functional equations in single variable*, J. Math. Anal. Appl. **288** (2003), 852–869

[2] Albert, M. H. and Baker, J. A., *Functions with bounded n-th differences*, Ann. Polonici Math. **43** (1983), 93–103

[3] Aoki, T., *On the stability of the linear transformation in Banach spaces*, J. M. Soc. Japan **2** (1950), 64–66

[4] Baker, J. A., *The stability of certain functional equations*, Proc. Amer. Math. Soc. **3** (1991), 729–732

[5] Bourbaki, N., *General topology. Chapters 5–10*, Translated from the French, Springer-Verlag, Berlin, 1998. iv+363 pp.

[6] Bourgin, D. G., *Classes of transformations and bordering transformations*, Bull. AMS **57** (1951), 223–237

[7] Cădariu, L. and Radu, V., *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure Appl. Math. **4(1)** (2003), Art.4 (http://jipam.vu.edu.au)

[8] Cădariu, L. and Radu, V., *Fixed points and the stability of quadratic functional equations*, Analele Universităţii de Vest din Timişoara, **41(1)** (2003), 25–48

[9] Cădariu, L. and Radu, V., *On the stability of the Cauchy functional equation: a fixed points approach*, Grazer Math. Ber., **346** (2004), 323–350

[10] Cădariu, L. and Radu, V., *Remarks on the stability of monomial functional equations*, Fixed Point Theory, **8**(2007), No. 2, 201–218

[11] Cădariu, L. and Radu, V., *Fixed points in generalized metric spaces and the stability of a cubic functional equation*, (Y. J. Cho, J. K. Kim & S. M. Kang - Eds.), Fixed Point Theory and Applications **7**, Nova Science Publ., 2007, 67–86

[12] Cădariu, L. and Radu, V., *Fixed points method for the stability of some functional equations*, Carpathian J. Math. **23** (2007), No. 1-2, 63–72

[13] Cădariu, L. and Radu, V., *Fixed point methods for the generalized stability of functional equations in a single variable*, Fixed Point Theory Appl., Art. ID 749392, (2008), 15 pp.

[14] Cain, G. L. and Kasriel, R. H., *Fixed and periodic points of local contraction mappings on probabilistic metric spaces*, Math. Syst. Theory **9** (1975), 289–297

[15] Forti, G. L., *Hyers-Ulam stability of functional equations in several variables*, Aeq. Math. **50** (1995), 143–90

[16] Forti, G. L., *Comments on the core of the direct method for proving Hyers-Ulam stability of functional equations*, J. Math. Anal. Appl. **295 (1)** (2004), 127–133

[17] Forti, G. L., *Elementary remarks on Ulam-Hyers stability of linear functional equations*, J. Math. Anal. Appl. 328 (2007), 109–118

[18] Gajda, Z., *On stability of additive mappings*, Internat. J. Math. Math. Sci. **14** (1991), 431–434

[19] Gilanyi, A., *A characterization of monomial functions*, Aequationes Math. **54** (1997), No. 3, 289–307

[20] Gilanyi, A., *Hyers-Ulam stability of monomial functional equations on a general domain*, Proc. Natl. Acad. Sci. USA **96** (1999), No. 19, 10588–10590

[21] Gilanyi, A., *On the stability of monomial functional equations*, Publ. Math. **56** (2000), No. 1-2, 201–212

[22] Hadžić, O. and Pap, E., *Fixed Point Theory in Probabilistic Metric Spaces*, Kluwer Academic Publishers, Dordrecht, 2001

[23] Hadžić, O., Pap, E. and Radu, V., *Generalized contraction mapping principles in probabilistic metric spaces*, Acta Math. Hungar., **101** (2003), No. 1-2, 131–148

[24] Hyers, D. H., *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA **27** (1941), 222–224

[25] Hyers, D. H., Isac, G. and Rassias, Th. M., *Stability of Functional Equations in Several Variables*, Basel, 1998

[26] Jung, S. M., *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press, Palm Harbor, Florida, 2002

[27] Jung, S. -M. and Kim, T. -S. *A fixed point approach to the stability of the cubic functional equation*, Bol. Soc. Mat. Mexicana **3 (12)** (2006), No. 1, 51–57

[28] Kuczma, M., Choczewski, B. and Ger, R., *Iterative Functional Equations*, Encyclopedia Math. Appl., **32**, Cambridge University Press, 1990

[29] Margolis, B. and Diaz, J. B. , *A fixed point theorem of the alternative for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc. **74** (1968), 305–309

[30] Miheţ, D. and Radu, V., *Generalized pseudo-metrics and fixed points in probabilistic metric spaces*, Carpathian J. Math., **23** (2007), No. 1-2, 126–132

[31] Mirzavaziri, M. and Moslehian, M. S., *A Fixed Point Approach to Stability of a Quadratic Equation*, Bulletin of the Brazilian Mathematical Society, **37(3)** (2006), 361–376

[32] Monna, A. F. *Sur un théorème de M. Luxemburg concernant les points fixes d'une classe d'application d'un espace métrique dans lui-même*, Nederl. Akad. Wet., Proc., Ser. A **64** (1961), 89–96

[33] Radu, V., *The fixed point alternative and the stability of functional equations*, Fixed Point Theory **4** (2003), No. 1, 91–96

[34] Rassias, J. M., *Alternative contraction principle and Ulam stability problem*, Math. Sci. Res. J., **9**, No. 7, (2005), 190–199

[35] Rassias, Th. M., *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300

[36] Rus, I. A., *Principles and Applications of Fixed Point Theory*, Ed. Dacia, Cluj-Napoca, 1979

[37] Šerstnev, A. N., *Sluchainye normirovanye prostranstva*, DAN SSSR 149 (1963), 280-283

[38] Tarafdar, E. U. and Chowdhury, M. S. R., *Topological methods for set-valued nonlinear analysis*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2008

[39] Ulam, S. M. *Problems in Modern Mathematics*, Chapter VI, Science Editions, Wiley, New York, 1964

[40] Zeidler, E., *Nonlinear functional analysis and its applications. I. Fixed-point theorems*, Springer-Verlag, New York, 1986

[41] Wolna, D., *Asymptotic stability of monomial functional equations*, Publ. Math. Debrecen **63** (2003), No. 1-2, 145–156

"POLITEHNICA" UNIVERSITY OF TIMIŞOARA
DEPARTMENT OF MATHEMATICS
PIAŢA VICTORIEI 2, 300006, TIMIŞOARA, ROMANIA
E-mail address: liviu.cadariu@mat.upt.ro, lcadariu@yahoo.com

WEST UNIVERSITY OF TIMIŞOARA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
DEPARTMENT OF MATHEMATICS
VASILE PÂRVAN 4, 300223, TIMIŞOARA, ROMANIA
E-mail address: radu@math.uvt.ro