

Modified Szász-Mirakjan operators of integral form

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ABSTRACT.

We deal with approximation by a modified family of Szász-Mirakjan operators, with parameters $\rho > 0$, $\alpha > 0$, which reproduce linear functions:

$$L_{\alpha}^{\rho}(f, x) = e^{-\alpha x} \left(f(0) + \sum_{k=1}^{\infty} \frac{(\alpha x)^k}{k!} \int_0^{\infty} \frac{\alpha \rho^{k\rho}}{\Gamma(k\rho)} \cdot e^{-\alpha \rho t} (\alpha t)^{k\rho-1} f(t) dt \right),$$

where $x \geq 0$ and $f : [0, \infty) \rightarrow \mathbb{R}$ is taken such that this formula is well defined.

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