

Fixed points for weakly compatible mappings satisfying an implicit relation in partially ordered metric spaces

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ABSTRACT.

Let (X, d, \preceq) be a partially ordered metric space. Let F, G be two set valued mappings and f, g two single valued mappings on X . We obtained sufficient conditions for existence of common fixed point of F, G, f and g satisfying an implicit relation in X .

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