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Oscillation theorems for non-linear difference equation of the second order

E. M. ELABBASY and SH. R. ELZEINY

ABSTRACT.

We obtain some oscillation criteria for the solutions of the non-linear difference equation of the form

$$\Delta\left(r_n\psi\left(x_n\right)f\left(\Delta x_n\right)\right)+q_n\varphi\left(g\left(x_{n+1}\right),r_{n+1}\psi\left(x_{n+1}\right)f\left(\Delta x_{n+1}\right)\right)=0,\ n=0,1,2,...,$$

where $u\,\varphi\left(u,v\right)>0$ for all $u\neq 0$, $x\,g\left(x\right)>0$ and $xf\left(x\right)>0$ for all $x\neq 0$, $\psi\left(x\right)>0$ for all $x\in R$, $\{r_n\}_{n=0}^{\infty}$ is sequence of positive real numbers and $\{q_n\}_{n=0}^{\infty}$ is sequence of real values. The relevance of our theorems becomes clear due to a carefully selected examples.

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DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
MANSOURA UNIVERSITY
MANSOURA 35516, EGYPT.
E-mail address: emelabbasy@mans.edu.eg
E-mail address: shrelzeiny@yahoo.com

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