A survey of different integer programming formulations of the generalized minimum spanning tree problem

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ABSTRACT.

In this survey paper, we discuss the development of the Generalized Minimum Spanning Tree Problem, denoted by GMSTP, and we focus on the integer programming formulations of the problem. The GMSTP is a variant of the classical minimum spanning tree problem (MST), in which the nodes of an undirected graph are partitioned into node sets (clusters) and we are looking for a minimum cost tree spanning a subset of nodes which includes exactly one node from every cluster. In this paper we describe twelve distinct formulations of the GMSTP based on integer programming. Apart from the standard formulations all the new formulations that we describe are 'compact' in the sense that the number of constraints and variables is a polynomial function of the number of nodes in the problem. Comparisons of the polytopes corresponding to their linear relaxations are established.

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