

## Completely monotonic functions associated with gamma function and applications

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### ABSTRACT.

The aim of this paper is to study the behavior of the functions defined by  $\Gamma(x+1) = \sqrt{2\pi}(x/e)^x e^{s(x)}$  and  $\Gamma(x+1) = \sqrt{2\pi}((x+1/2)/e)^{x+1/2} e^{w(x)}$ . As applications, some inequalities involving the gamma function are obtained. Finally, an estimation of the digamma function is given.

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Received: 02.03.2009; In revised form: 04.06.2009; Accepted: 27.08.2009

1991 Mathematics Subject Classification. 33B15, 26A48, 41A60.

Key words and phrases. Factorial function, Stirling's formula, completely monotonicity, digamma and polygamma functions, inequalities.