Bivariate Schurer-Stancu operators revisited

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ABSTRACT.

Let $p \geq 0$, $q \geq 0$ be given integers and let $\alpha, \beta, \gamma, \delta$ be real parameters satisfying the conditions $0 \leq \alpha \leq \beta$, $0 \leq \gamma \leq \delta$. The Schurer-Stancu bivariate operators $\widetilde{S}_{m,p,n,q}^{(\alpha,\beta,\gamma,\delta)} : C([0, 1+p] \times [0, 1+q]) \to C([0, 1+p] \times [0, 1+q])$ are defined and then considering the Schurer-Stancu bivariate approximation formula, one studies its remainder term and one expresses them in terms of divided differences. When the approximated function is sufficiently smooth, an upper bound estimation for the remainder term is established. As particular cases, the remainder terms of Schurer, Stancu and respectively Bernstein bivariate approximation formulas are obtained.

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